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The Cooling of Particle Beams

Andrew M. Sessler

Accelerator and Fusion Research Division
Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720

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Andrew M. Sessler

*Center for Beam Physics
Lawrence Berkeley Laboratory
Berkeley, CA 94720

Abstract. A review is given of the various methods which can be employed for cooling particle beams. These methods include radiation damping, stimulated radiation damping, ionization cooling, stochastic cooling, electron cooling, laser cooling, and laser cooling with beam coupling. Laser Cooling has provided beams of the lowest temperatures, namely 1 mK, but only for ions and only for the longitudinal temperature. Recent theoretical work has suggested how laser cooling, with the coupling of beam motion, can be used to reduce the ion beam temperature in all three directions. The majority of this paper is devoted to describing laser cooling and laser cooling with beam coupling.

INTRODUCTION

By the cooling of particle beams we mean the violation, or circumvention, of Liouville's theorem (1). That is, we do not consider, adequate, the simple "cooling" of a group of particles (reducing the "temperature") with an associated increase of the group of particle's physical dimensions. What is meant by "cooling", for particle beams, is an actual reduction in phase space volume.

A derivation, and discussion, of Liouville's theorem may be found in standard texts (2). Recently two of us considered the subject in some detail (3). In brief, if the particles can be described, adequately, by a continuous fluid *Hamiltonian* then Liouville's theorem is rigorously valid. Thus it is necessary to have dissipative forces (such as in radiation damping) in order to cool a beam.

There are a number of methods for reducing phase space volume. The primary methods will be reviewed in this paper, and then attention will be turned to laser cooling. Notice that different methods may be restricted to operate only on particular species; thus radiation cooling works only on electrons, ionization cooling on μ mesons, and laser cooling on ions.

RADIATION DAMPING

The understanding of radiation damping was pioneered by Kenneth Robinson and Mathew Sands (4). Their work was motivated by the desire to turn the Cambridge Electron Accelerator (CEA), which had been constructed so that it didn't damp in all three directions (because that was of no importance in a synchrotron), into a storage ring. They succeeded in this program of conversion of the CEA, by installing special magnets so as to make the ring damp in all three directions.

A rather complete derivation can be found in the treatise by Matt Sands (5). An electron moving in a circular accelerator is, of course, accelerated and it will thus radiate. The amount of radiation is

$$P = \frac{2}{3}\frac{e^2\beta^4\gamma^4 c}{R^2} \quad . \tag{1}$$

Hence the radiated energy per turn is

$$\delta E = \frac{4}{3}\frac{\pi e^2}{R}\beta^3\gamma^4 \quad , \tag{2}$$

or, in practical units (and taking $\beta = 1$):

$$(\delta E)(\mathrm{MeV}) = 8.85\times 10^{-2}\,\frac{E^4(\mathrm{GeV})}{R(\mathrm{m})} \quad . \tag{3}$$

The emission of radiation has an effect on the radiating particle. It is only for electrons that this is a significant effect, but the effect is vital in determining the property of beams in an electron storage ring. The radiation reaction can cause either damping or undamping of the electrons' oscillations (transversely and in energy) about the equilibrium orbit. If we characterize this exponential damping rate by rate constants α_x, α_y, α_E then

$$\alpha_{\begin{pmatrix} x \\ y \\ z \end{pmatrix}} = \frac{c\gamma^3 e^2 J}{3R^2 mc^2} \; , \tag{4}$$

where the damping partition numbers satisfy:

$$J_y = 1, \; J_x + J_E = 3 \quad . \tag{5}$$

One can arrange by proper lattice design, as one must in a storage ring, to have damping in all three directions.

Thus, on the basis of the above analysis, an electron beam in a storage ring will just damp and damp so that its transverse size becomes smaller and smaller. This is approximately true, and beams become very small, indeed, but they do not become arbitrarily small. Why not? Because quantum effects need to be taken into account; i.e., that electrons radiate discrete photons and that the hard photons, which are radiated statistically, kick the electron. In fact, the size of electron beams is determined by these quantum mechanical effects. The energy spread of the beam, which damps to zero classically, is (in a uniform field):

$$\left(\frac{\sigma_E}{E}\right)^2 = \left(\frac{55}{32\sqrt{3}}\right)\left(\frac{\hbar}{mc}\right)\frac{\gamma^2}{j_E R} \quad , \tag{6}$$

and one can see that *the finiteness of σ_E is due to a quantum mechanical effect*; i.e. to the non-zero nature of Phanck's constant $\hbar$. A similar formula can be given for the radial size of the beam. The vertical size is, clearly, determined by coupling to the horizontal motion.

STIMULATED RADIATION COOLING

It has recently been proposed, by E.G. Bessonov that in ion beams, one can stimulate radiation damping (6). Firstly, the magnetic field (which accelerates the charged particle and causes it to radiate) is replaced by the field of a laser beam. Secondly, the laser frequency is choosen so that there is resonance with a bound electron on the ion.

Thus, realizing that ions don't radiate significantly and therefore radiation damping is negligible for them, it is proposed to employ the electrons in ions to do the radiation and transfer the resulting momentum kick to the ion as a whole. If the magnetic field (from a laser) is in resonance with an atomic spacing of levels, then the cross section can be very large; that is, an electron can readily be excited to an upper level.

For an incident laser photon, off resonance, the cross section is simply the Compton cross section:

$$\sigma = (8/3)\pi r_0^2 \tag{7}$$

where r_e is the classical electron radius. However, on resonance, the cross section is

$$\sigma = \frac{\lambda^2}{2\pi} \tag{8}$$

where λ is the wavelength of the laser (in the ion's frame). This enhancement is a well-known effect, reminding one of slow neutron scattering and, currently, used in laser cooling. However, the laser line width must be rather broad in this case, since the spread in frequencies within the ion beam is rather large, and thus most ions are not in resonance. This means that the average cross section is greatly reduced from the maximum value given above, but since the maximum is $(\lambda/4\pi r_e)^2 \approx 10^{18}$ times the Compton cross section, even with an averaging factor one still gains a great deal.

For a relativistic ion the radiation reaction is large (γ^2 times the non-relativistic case) and thus the efficacy of damping is considerable. Putting all this together one obtains (approximately) the damping rate for vertical oscillations (similar formulas can be obtained for each degree of freedom)

$$\tau = \frac{SR}{c\lambda\ell\gamma}\left(\frac{\Delta f}{f}\right)\frac{P_A}{P} \qquad (9)$$

where S is the cross section of the laser beam, of frequency f and spread Δf, and power P, R is the radius of the storage ring, l is the length of the interaction region, and P_A is $m_e m_i c^5/e^2$. In a numerical example Bessonov finds the damping time, for 500 GeV He ions to be 160 sec.

Notice that this scheme, in contrast with the laser cooling scheme to be described later, requires a laser of wide line width (rather than a very narrow line width) and an rf system on the storage ring capable of restoring the ion's energy a few times (for damping requires that the ion lose its energy a number of times, just as in radiation damping, but unlike laser cooling where the change in ion energy is very very small).

IONIZATION COOLING

The concept of ionization cooling was developed by Sasha Skrinsky (7). It is exactly the same as radiation damping, but the energy loss mechanism is now the ionization of a medium. Of course this must be balanced against the medium scattering, which makes the method of no value for strongly interacting particles, but very useful for μ mesons (8). The following discussion is taken from Ref. 8.

The basic mechanism of transverse μ cooling is quite simple; muons passing through a material medium lose energy (and momentum) through ionization interactions. The losses are parallel to the particle motion, and therefore include transverse and longitudinal momentum losses; the transverse energy losses reduce (normalized) emittance. Reacceleration of the beam (in rf cavities) restores only longitudinal energy. The combined process of ionization energy loss plus rf reacceleration reduces transverse momentum and hence reduces transverse emittance. However, the random process of multiple scattering in the material medium increases the emittance.

The equation for transverse cooling can be written in a differential-equation form as:

$$\frac{d\varepsilon_\perp}{dz} = -\frac{\frac{dE_\mu}{dz}}{E_\mu}\varepsilon_\perp + \frac{\beta_0}{2}\frac{d\langle\theta^2_{rms}\rangle}{dz} , \qquad (10)$$

where $\varepsilon_\perp$ is the (unnormalized) transverse emittance, dE_μ/dz is the absorber energy loss per cooler tansport length z, β_0 is the betatron function in the absorber and Θ_{rms} is the mean accumulated multiple scattering angle in the absorber. Note that $dE_\mu/dz = f_A dE/ds$, where f_A is the fraction of the transport length occupied by the absorber, which has an energy absorption coefficent of dE_μ/ds. Also the multiple scattering can be estimated from:

$$\frac{d\langle\theta^2_{rms}\rangle}{dz} \cong \frac{f_A}{L_R}\left(\frac{0.014}{E_\mu}\right)^2 \qquad (11)$$

where L_R is the material radiation length and is E_μ in GeV. (The differential-equation form assumes the cooling system is formed from small alternating absorber and reaccelerator sections; a similar difference equation would be appropriate if individual sections are long.)

If the parameters are constant, Eqs. (12) and (13) may be combined to find a minimum cooled (unnormalized) emittance of

$$\varepsilon_\perp \to \frac{(0.014)^2}{2E_\mu}\frac{\beta_0}{L_R\frac{dE_\mu}{dz}} \tag{12}$$

or, when normalized

$$\varepsilon_N \to \varepsilon_\perp\gamma \to \frac{(0.014)^2}{2m_\mu c^2}\frac{\beta_0}{L_R\frac{dE_\mu}{dz}} \tag{13}$$

(all energies are in GeV).

Longitudinal (energy-spread) cooling is also possible, if the energy loss increases with increasing energy. The energy loss function for muons, dE/ds is rapidly decreasing (heating) with energy for $E_\mu < 0.3$ GeV, but is slightly increasing (cooling) for $E_\mu >$ 0.3 GeV. This natural dependence can be enhanced by placing a wedge-shaped absorber at a "non-zero dispersion" region where position is energy-dependent. Longitudinal cooling is limited by statistical fluctuations in the number and energy of muon-atom interactions. An equation for energy cooling is:

$$\frac{d\left\langle(\Delta E)^2\right\rangle}{dz} \approx -2\frac{\partial\frac{dE_\mu}{ds}}{\partial E_\mu}\left\langle(\Delta E)^2\right\rangle + \frac{dE_\mu}{dz}I \ , \tag{14}$$

where I is the mean energy exchange (~12Z eV) and the derivative with energy combines natural energy dependence with dispersion-enhanced dependence. An expression for this enhanced cooling derivative is:

$$\frac{\partial\frac{dE_\mu}{dz}}{\partial E_\mu} = f_A\left(\frac{\partial\frac{dE_\mu}{dz}}{\partial E_\mu}\right)_0 + f_A\frac{dE_\mu}{ds}\frac{d\delta}{dx}\frac{\eta}{E_\mu\delta_0} \ , \tag{15}$$

where η is the dispersion at the absorber, and δ and $d\delta/dx$ are the thickness and tilt of the absorber. Note that using a wedge absorber for energy cooling will reduce transverse cooling; the sum of transverse and longitudinal cooling rates is invariant.

STOCHASTIC COOLING

Early in the 70's Simon van der Meer realized that it was practical to "beat Liouville" by means of a device that works on the fluctuations from equilibrium. He proposed operating (with pickups and kickers) on individual particles (or a rather small number of particles, where the finiteness of the number is vital). Thus he invented stochastic cooling (9). The main difficulty was technological; that is, the development of sufficiently sensitive pickups, good amplifiers, and excellent filters. A comprehensive treatment may be found in the article by Joseph Bisognano (10).

With development, stochastic cooling proved to be remarkably effective and thus allowed for the construction of proton-antiproton colliders. For these colliders cooling was essential, for the antiprotons are produced in a very warm state; i.e., with a density which was completely inadequate to give the desired luminosity. With cooling the energy spread was reduced by a factor of 10^4, while the transverse emittance was also reduced by large factors. It was this very powerful cooling that made proton-antiproton colliders possible.

Stochastic cooling is the damping of transverse and energy oscillations by means of feedback. A pick-up electrode detects (say), the transverse position of an electron and sends this signal, after amplification, to a kicker downstream. The time delay is such that a particle is subject to its own signal, which is done by cutting across an arc of the accelerator.

Clearly if there is only one particle this will work. Equally clearly, by Liouville's theorem, if there are many particles so that the beam can be treated as a fluid, then there will be no damping. For a finite, but very large number of particles there is a residue of the single particle effect; i.e., some damping, as was first realized by van der Meer.

Consider N particles in a ring where f = 1/T is the revolution frequency of particles. Suppose the electronics has a band width W. Then the pick-up electrode effectively "sees" a number of particles.

$$n = \frac{N}{2WT} \quad . \tag{16}$$

Under the influence of the pick-up and kicker this particle will have its transverse displacement, x_i, changed

$$x_i \rightarrow x_i - g \sum_{j=1}^{n} x_j \quad , \tag{17}$$

where g is the effective gain of the system. Consequently, the value of $(x_i)^2$ will change by:

$$\Delta(x_i^2) \equiv \left(x_i - g \sum_{j=1}^{n} x_j \right)^2 - x_i^2 \quad , \tag{18}$$

$$\Delta x_i^2 = -2g\ x_i \left(\sum_{j=1}^{n} x_j \right) + g^2 \sum_{j=1}^{n} \sum_{k=1}^{n} x_j \cdot x_k \ . \tag{19}$$

Initially there are no correlations between particles' positions and hence, on averaging over all particles we have

$$< \Delta x_i^2 > = -2g\ < x_i^2 > + ng^2 < x_i^2 > \ , \tag{20}$$

This minimizes at $g = 1/n$ and the rate of damping of rms betatron amplitudes is

$$\frac{1}{\tau} = \frac{1}{4}\frac{1}{n}\frac{1}{T} = \frac{W}{2N} \tag{21}$$

where the T appears because the system works on any one particle once per turn and the factor of 4 reduction comes about from taking the rms (1/2) and the fact that phase space is two-dimensional and only x (not x') is being damped (1/2).

The assumption of no correlations is not valid, in general, and is especially complicated if one has bunches. Much of the literature is devoted to analyzing this case, which will not be discussed, here, further. Typically, $W \sim 1$ GHz and N varies from 10^7 to 10^{22}. Thus the cooling time varies from a few milliseconds to an hour.

In the above analysis, which is, of course, very simple, we have assumed no noise in the electronics. In real life there is noise and one might think that if the amplifier noise is greater than the stochastic beam signal then there will be no cooling because the feedback system will heat the beam faster than it cools it. Not so. All one needs to do is select a lower gain and there is cooling (of course, at a reduced rate). (An analogy with a refrigerator is, perhaps, more correct.)

If there is noise, then x_i is changed

$$x_i \to x_i - g\left(\sum_{j=1}^{n} x_j + r \right) \ , \tag{22}$$

where r is the amplifier noise expressed as apparent average x-amplitude at the pick-up. The analysis now proceeds exactly as before:

$$\Delta\left(x_i^2\right) = -2\ < x_i^2 > \ + ng^2\ < x_i^2 > \ + ng^2 r^2 \ . \tag{23}$$

Assuming no correlations

$$\frac{1}{\tau} = -2g + ng^2\left(1 + \frac{r^2}{< x_i^2 >} \right) \ , \tag{24}$$

and minimizing this one obtains

$$\frac{1}{\tau}=\frac{W}{2N(1+\mathcal{N})} \quad , \tag{25}$$

where the factor

$$\mathcal{N}=\frac{<r^2>}{<x_i{}^2>} \quad ; \tag{26}$$

is simply noise power over signal power.

ELECTRON COOLING

The formalism for beam scattering was employed to analyze electron cooling, which was invented by G. I. Budker in 1966 (11). MURA physisists had proposed tapered foils which worked in principle, but not in practice (because of too much scattering in the foil). But Budker's idea replaced a fixed foil with electrons so there was little scattering. Furthermore, he proposed moving electrons of very cold temperature, so that the interaction between protons and the cooled electrons would lead to a cooling of the protons. A simple discussion of electron cooling is given by Hugh Herward (12).

The rate of cooling is given by

$$\tau_e=\frac{C}{L_e}\frac{F_1\gamma_0^2}{r_e r_i n_e c \ln\Lambda}\left[\left(\frac{k_B T_{be}}{m_e c^2}\right)^{3/2}+\left(\frac{k_B T_{bi}}{m_i c^2}\right)^{3/2}\right] . \tag{27}$$

Here n_e is the electron density, assumed to be the same as the ion density n_i, re and r_i are the classical electron and ion radii, F_1 is a constant that for a smooth focusing system has the value $F_1=3/4\sqrt{2\pi}\approx 0.3$, and γ_0 is the relativistic energy factor (identical for both beams); the electron and ion temperatures are measured in the beam frame. L_e/C is the fraction of the storage ring occupied by the cooling section, and ln Λ is the Coulomb logarithm. When equilibrium is reached, the two beam temperatures are the same (i.e., $T_{bi} \approx T_{be}$). Assuming that both beams have identical transverse cross sections, one obtains an emittance ratio of $\frac{\varepsilon_i}{\varepsilon_e} \approx (m_e/m_i)^{1/2}$; *that is, the ion-beam emittance would be* considerably smaller than that of the electron beam in view of the inverse square root mass ratio.

LASER COOLING

Laser cooling of ions, which is the primary subject of this paper was first proposed by a number of laser/atomic physicists (13). Actual realization, experimentally, of the cooling has been achieved by groups in Heidelberg and in Aarhus (14). The following disucssion is taken from Ref. (15).

Laser cooling is the result of the velocity-selective transfer of photon momentum from a laser beam to a moving ion. In the most basic laser cooling scheme, known as Doppler cooling, particles having a closed transition (i.e. the population is confined to two levels) between internal energy levels are utilized. Those particles which are in resonance with a laser beam absorb photons. Each absorbed photon transfers momentum of magnitude hv/c to the particle, which recoils in the direction of the laser beam propagation. When a photon is spontaneously emitted by the excited particle, the particle again recoils, but the average momentum transfer to a particle after many spontaneous emissions is negligible, because the angular distribution of the emission is symmetric. There is thus a net radiation pressure force, directed along the laser beam, on resonant

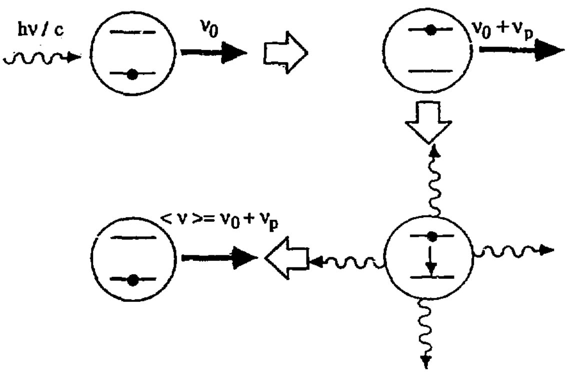

FIGURE 1. The principle of laser cooling.

particles. The process is shown in Fig.1. Due to the small magnitude of the photon momentum (optical photons have at most a few eV/c), it is necessary for an ion to absorb many photons to achieve macroscopically significant acceleration. It is for this reason that the optical transition must be closed. The force is dependent on the velocity of the particle, by virtue of the Doppler shift. By tuning the frequencies of two counterpropagating lasers to accelerate slow particles and decelerate fast ones, it is possible to reduce the velocity spread (in on dimension) of a collection of particles; hence the name "cooling". The minimum temperature T_{min} achievable by Doppler cooling is given as

$$k_B T_{min} \approx \frac{\hbar\Gamma}{2} , \tag{28}$$

where k_B is the Boltzmann constant, and Γ is the spontaneous decay rate of the upper level back to the lower level; $\Gamma/2$ is the radiative linewidth of the transition.

The maximum laser cooling force is

$$F_{LC}^{max} = \left(\frac{\Delta p}{\Delta t}\right)_{max} = \frac{\hbar k_0 \Gamma}{2} , \tag{29}$$

where k_0 is the wave number.

A more rigorous analysis yields

$$F_{LC} = \frac{\hbar k_0 \Gamma}{2} \frac{S}{\left[1 + 2\frac{2\Delta(v_\parallel)}{\Gamma}\right]} , \tag{30}$$

where S is the ratio between laser intensity I and saturation intensity I_{sat}, i.e. $S = I / I_{sat}$, and the detuning $\Delta(v_\parallel)$ is given by

$$\Delta(v_\parallel) = \omega_L \gamma \left(1 - \frac{v_\parallel}{c}\right) - \omega_0 . \tag{31}$$

The longitudinal temperature $T_\parallel$ is

$$\frac{1}{2} k_B T_\parallel = \frac{1}{2} m (\delta v_\parallel)^2 , \tag{32}$$

while the transverse temperature $T_\perp$

$$k_B T_\perp = \frac{1}{2} m \left\langle v_x^2 + v_y^2 \right\rangle . \tag{33}$$

The above formula lead to

$$\frac{1}{2} k_B T_\parallel = E_0 \left(\frac{\delta p}{p_0}\right)^2 \rightarrow T_\parallel[K] = 2.32 x 10^4 x \left(\frac{\delta p}{p_0}\right)^2 E_0[eV] , \tag{34}$$

$$k_B T_\perp = 2E_0 \frac{\varepsilon_\perp}{\langle\beta\rangle} \rightarrow T_\perp[K] = 2.32 x 10^4 x \frac{\varepsilon_\perp[m \cdot rad]}{\langle\beta\rangle[m]} E_0[eV] , \tag{35}$$

In a storage ring, laser cooling is sometimes achieved by using two laser beams, overlapping the ion beam for up to a few meters in the machine. One laser propagates opposite to the ions' direction of travel and can decelerate particles with velocities above the mean. The second laser is copropagating and acts to accelerate slow particles. The initial energy spread in a stored beam is usually much higher than the linewidth of the cooling transition. The lasers must then be tunable over a wide frequency range in order to interact with all of the ions.

In Figures 2, 3 and 4 we show four different ways in which to achieve laser cooling, all of which have been used in practice. That is, it is not necessary to employ two laser beams if: (1) one laser can be swept in frequency, or an auxiliary force is employed which is (2) constant, (3) varies linearly, or (4) is produced by an RF bucket. Of course, much theoretical work, employing the Fokker-Planck equation, etc. has been done on this subject. The essence of the laser cooling, however, has been described here.

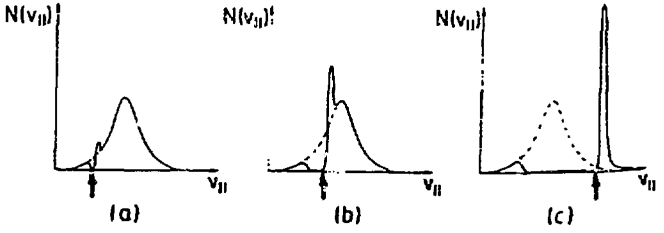

FIGURE 2. (a) The spontaneous force exerted by a laser copropagating with a fast, hot ion beam is able to accelerate ions within a narrow velocity region. When scanning the laser frequency (resonant velocity is marked by an arrow), the ions are collected in a narrow velocity distribution (b), which finally results in a logitudinally cooled ion beam (c).

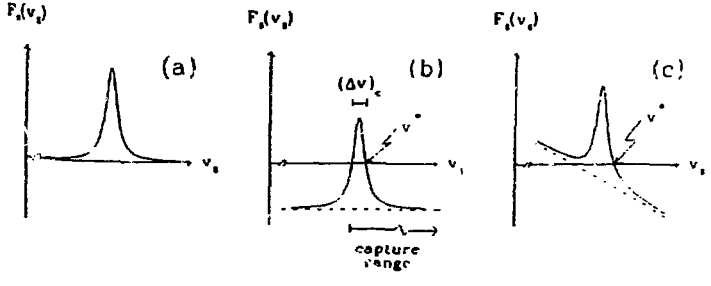

FIGURE 3. (a) The spontaneous force itself has no stable point v*. With the help of a (b) constant or (c) linear auxiliary force F_{aux} (dashed lines), such a point can be generated for the combined forces (solid line).

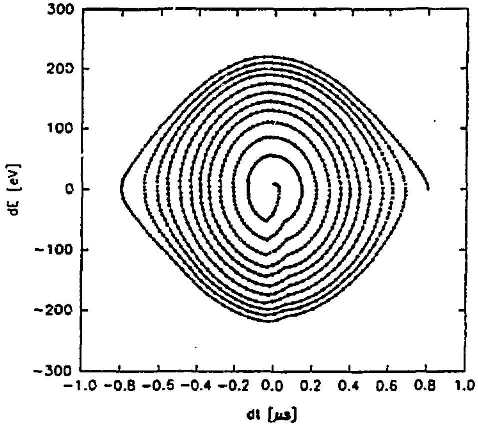

FIGURE 4. Longitudinal phase space plot illustrating the principle of bunched-beam laser cooling. Energy deviation and time deviation are plotted for a single particle in an RF bucket.

LASER COOLING AND BEAM COUPLING

As we have seen, laser cooling has, to date, only been employed for longitudinal cooling. A method of transverse cooling (essentially by having mirrors and sending the laser light transverse to the beam) has been proposed (16). A method of coupling the ion beam transverse and longitudinal motion has recently been proposed (17). This method involves the use of "regular cavities" in dispersive regions or special "coupling cavities" (similar to ones suggested in a different context (18)), (19).

The idea is based upon developing a forced synchro-betatron resonance where the transverse tune ν_T and the longitudinal tune ν_L satisfy the resonance condition $\nu_T - \nu_L =$ integer. The coupling is induced by a coupling rf cavity set on a storage ring. The cavity is excited with a specific mode whose longitudinal field component has a transverse-coordinate dependence; here we consider the TM_{210} mode which gives very effective coupling.

For the coupling cavity, consider a rectangular rf cavity which has a width of 2a and the height of 2b. For the TM_{210} mode, the longitudinal electric field component is obtained from Maxwell's equations as

$$E_z = -V_c \sin\left(\frac{\pi x}{a}\right) \cos\left(\frac{\pi y}{2b}\right) \cos(\omega_c t + \phi_c) \qquad (36)$$

$$\Gamma_c \equiv \frac{qV_c}{2\beta_0 c p_0}\frac{R}{a} \quad , \tag{41}$$

and we have simply assumed that the storage ring studied here has been designed such that the dispersion η, and $d\eta/ds$, vanish at the rf cavity positions. In addition, the betatron motion has been smoothed out introducing the transverse tune ν_T, while $\bar{\nu}_L$ is a constant which roughly corresponds to the longitudinal tune ν_L, and is given by the relation $\cos(2\pi\nu_L) = 1 - 2\pi^2\bar{\nu}_L{}^2$.

The ions susceptible to laser cooling are heavy particles for which the synchrotron radiation loss is negligible and, therefore, it is unnecessary to accelerate to compensate for energy loss. However, we need the ordinary rf cavity as a bunching cavity. The energy of stored heavy-ion beams is, in general, below transition, i.e. $\xi_0<0$, and ψ_b must then be positive in the definition introduced here. Then, to have the maximum bunching effect, we choose the synchronous phase $\psi_b=\pi/2$. Similarly, ψ_c is chosen to be zero, so that the coupling effect becomes maximum. Under these simplifications, the Hamiltonian can be re-written as

$$H = \frac{1}{2}\left(p_x{}^2 + \nu_T{}^2 x^2\right) - \frac{\xi_0 W^2}{2} - \frac{\pi\bar{\nu}_L{}^2\psi^2}{\xi_0}\delta_p(\theta-\theta_b) - 2\pi\Gamma_c x\psi\delta_p(\theta-\theta_c) \tag{42}$$

where the higher order terms in x and ψ have been neglected, and the tilde has been dropped. This Hamiltonian leads to the equations of motion

$$\begin{aligned} &\frac{d^2x}{d\theta^2} + \nu_T{}^2 x = 2\pi\Gamma_c\psi\delta_p(\theta-\theta_c), \\ &\frac{d^2\psi}{d\theta^2} + 2\pi\bar{\nu}_L{}^2\psi\delta_p(\theta-\theta_b) = -2\pi\xi_0\Gamma_c x\delta_p(\theta-\theta_c) \end{aligned} \tag{43}$$

These linear equations can be solved by employing matrix methods. Before doing that, we add a term which replicates the laser cooling; namely a term on the left-hand side of the ψ-equation of which is $\Lambda(d\psi/d\theta)$ over the laser cooling section. A numerical study of these equations has been made, and some results from the matrix approach, are shown in Fig. 5. It can be seen that when one operates on the coupling resonance and makes Γ_c adequately large, then there is damping in both dimensions. Figure 6 shows numerical results from tracking (which includes non-linear effects). Again, damping in both degrees of freedom can be obtained.

Coupling can also be achieved by means of a regular rf cavity in a region of dispersion. In fact, that results in precisely the same equations as we have obtained above. Also, the vertical motion can be coupled to the horizontal motion and it is possible, in this manner, to achieve damping of all three degrees of freedom. These extensions have been shown in Ref. 19.

Here, V_c corresponds to the maximum voltage and φ_c is the initial rf phase. The oscillation angular frequency ω_c is given by $(\pi/a)^2 + (\pi/2b)^2 = (\omega_c/c)^2$, where c is the speed of light. These electromagnetic fields, having the above E_z are derivable from the vector potential

$$A_c = \left(0, 0, \frac{V_c}{\omega_c}\sin\left(\frac{\pi x}{a}\right)\cos\left(\frac{\pi y}{2b}\right)\sin(\omega_c t + \phi_c)\right) \quad . \qquad (37)$$

In addition, we also have an rf bunching cavity whose vector potential is

$$A_b = \left(0, 0, \frac{V_b}{\omega_b}\sin(\omega_b t + \phi_b)\right) \quad , \qquad (38)$$

where V_b and ω_b are, respectively, the voltage amplitude and angular frequency of the bunching cavity, and ϕ_b is the initial rf phase.

The Hamiltonian for the coupled motion caused by the coupling cavity can be readily obtained. Taking the distance s along the reference particle orbit in a storage ring as the independent variable, instead of time, and considering only dipole and quadrupole magnets installed on the ring, we obtain, together with the vector potentials, the approximate Hamiltonian

$$\begin{aligned} H_1 = & -p + (p_0 - p)\frac{x}{\rho} + \frac{p_x{}^2}{2p} + \frac{p_0 K(s) x^2}{2} \\ & -\frac{qV_b}{\omega_b}\sin(\omega_b t + \phi_b)\delta_p(s - s_b) - \frac{\pi q V_c}{\omega_c}\frac{x}{a}\sin(\omega_c t + \phi_c)\delta_p(s - s_c), \end{aligned} \qquad (39)$$

where q and ρ are, respectively, the charge state of stored ions and the local curvature of the orbit, K(s) corresponds to the quadrupole field strength, $\delta_p(s)$ denotes a periodic delta function, and we have assumed that the bunching and coupling cavity are located at the position s_b and s_c respectively. Writing the total energy of a particle as W, the total momentum p is $p=[(W/c)^2-m_0{}^2c^2]^{1/2}$ where m_0 is the rest mass of the ions. Applying several canonical transformations and scalings, we eventually find, changing the independent variable to $\theta = s/R$ (R = average ring radius),

$$\begin{aligned} \tilde{H}_1 = & \frac{\tilde{p}_x{}^2}{2} + \frac{\nu_T{}^2\tilde{x}^2}{2} - \frac{\xi_0 \tilde{W}^2}{2} + \frac{2\pi\bar{\nu}_L{}^2}{\xi_0}\sin(\tilde{\psi} + \psi_b)\delta_p(\theta - \theta_b) \\ & -\frac{2\pi h_b \Gamma_c}{h_c}\tilde{x}\cdot\sin\left(\frac{h_c}{h_b}\tilde{\psi} + \psi_c\right)\delta_p(\theta - \theta_c), \end{aligned} \qquad (40)$$

where $\theta_b = s_b/R$, $\theta_c = s_c/R$, $\xi_0 = \alpha - 1/\gamma_0{}^2$ where α is the momentum compaction factor, and ψ_b and ψ_c are the so-called synchronous phase at the bunching and coupling cavity whose harmonic numbers are, respectively, h_b and h_c. The coupling constant Γ_c has been introduced as

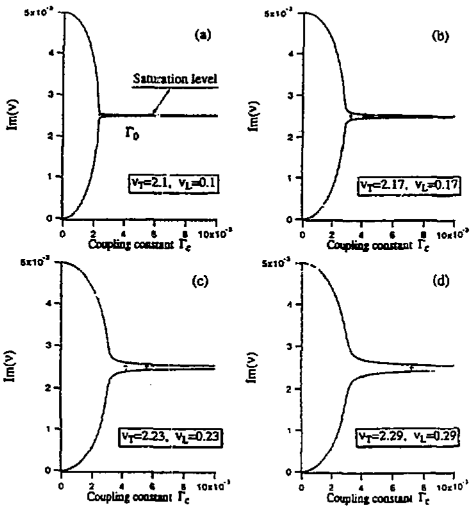

Figure 5. Imaginary part of the eigenvalues describing longitudinal and transverse motion as a function of the coupling strength between the modes. The bunching and coupling cavities are next to each other and 180° from the laser cooling section. The longitudinal and transverse tunes are varied in the four figures, keeping the resonance condition satisfied. The damping rate was held fixed such that $\Lambda_d/2\pi$=0.01.

CONCLUSION

We have seen that there really are quite a number of effective ways to circumvent Liouville's theorem. In retrospect it is remarkable, or perhaps I should say instructive, that the MURA Group, well aware of the limit that theorem imposed, trying very hard to beat it, and not totally incompetent, were unable to devise an effective method to do so. It was only 10 years later, that Budker with electron cooling, and then van de Meer with stochastic cooling, devised effective cooling methods. In recent years a number of other methods have been proposed, the most recent, and the most effective (in terms of the low temperature achieved), is laser cooling. In the future, we may expect ever colder ion beams, maybe even the achievment of crystalline beams (20).

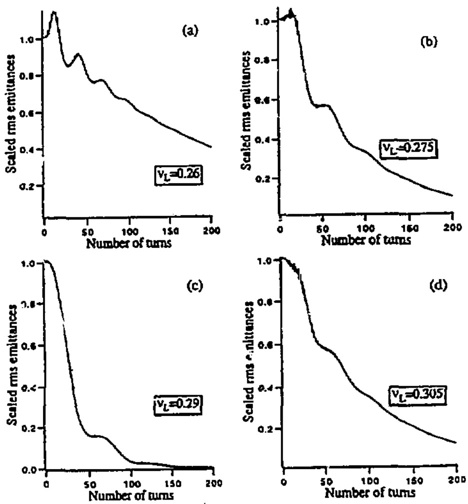

Figure 6. Tracing results, i.e., solutions of Eq. ( ), in which 500 particles are followed and, from them, transverse (solid line and longitudinal dotted line) scaled rms emittances are evaluated. The effective damping of both degrees of freedom is seen in all the figures, but most dramatically in (c) where the operating point is exactly on resonance. The transverse turn and coupling constant are fixed, respectively, at $\nu_T = 2.29$ and Γ_C=0.015 in all cases and the damping rate is taken as $\Lambda_d/2\pi$=0.01.

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