MEASUREMENT ACCURACY, BIT-STRINGS, MANTHEY'S QUATERNIONS, and RRQM*

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ABSTRACT

We continue the discussion started last year. By now three potentially divergent research programs have surfaced in ANPA: (1) the Bastin-Kilmister understanding of the combinatorial hierarchy (Clive's "Menshevik" position); (2) my bit-string "Theory of Everything" (which Clive has dubbed "Bolshevik"); (3) Manthey's cycle hierarchy based on co-occurrence and mutual exclusion that Clive helped him map onto quaternions (an as yet unnamed heresy?). Unless we can find a common objective, these three points of view will continue to diverge. We suggest the reconstruction of relativistic quantum mechanics (RRQM) as a reasonable, and attainable, goal that might aid convergence rather than divergence.

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1. INTRODUCTION

So far, ANPA has thrived without making a specific research program with a stated objective our main reason for existence. In a general sense, we are interested in providing a forum for coherent presentations of foundational theories. In our statement of purpose we use as an example a code word for research arising from the Bastin-Kilmister program of the 50's[9-14] which started from an examination of Eddington's FUNDAMENTAL THEORY, the discovery of the combinatorial hierarchy by Fredrick Parker-Rhodes in 1961[9], and the grounding of his construction on discriminately closed sets by John Amson in 1965[9,10] Other than this, a vague consensus that a new scientific revolution is in the air and that we wish to encourage it has sufficed.

I once thought that the core research, and objective, were so clear that what was needed to advance the revolution was a more specific focus and presented my proposals in two papers whose titles used the English translation of two polemics by Lenin[9,10]. My temerity was gently labeled "Bolshevik" by Clive. I assume his intended implication was that my Bolshevik (which means majority in Russian) approach in fact did not command a numerical majority acceptance in our ANPA community. Historically, Lenin seized an occasion when his faction had a majority on the Central Committee of the Second International to make his policies dominant. Probably many (most?) members of the organization did not agree with him; yet from then on those who did not follow his “party line” were defined as Menshevik, which means the “minority”. So when Clive called his position “Menshevik”, he implied that in fact it was much closer to the center of gravity of the ANPA membership than mine.

I still find it remarkable that in an organization like ours which espouses radical viewpoints, a debate on what should be the foundational ideas of our enterprise has taken so long to emerge. In fact, Clive once told me that he was wary of looking too closely at the foundations of the combinatorial hierarchy because of a fear that they might dissolve under critical examination. By now I am sure he agrees both
that the combinatorial hierarchy has firm foundations and that we now have a solid enough organization to survive internal criticism and self-criticism.

Naturally, when it comes to details, individuals in ANPA may fall outside or in between the three main groups I discuss. There is also controversy within them. For example, Ted (Bastin) and Clive (Kilmister) disagree as to the importance of the sequence $2^2, 4^2, 16^2, 256^2$ compared to the sequence $3, 10, 137, 2^{127} - 1$ which we all aim at understanding. David McGoveran insists that without both sequences, the construction is too general to be applied to physics, as do I. Yet he also insists[8] that my detailed application of his corrections to coupling constants and mass ratios is illegitimate because I am not working in the ordering operator calculus context in which they were derived. I have noted his objections in a recent paper. Clive started out last year[6] from a remark of Tony Deakin’s at ANPA 14 that Mike Manthey had provided the mathematical theory for Alison Watson’s metaphysics. Yet he ended up concluding that there are detailed mathematical connections between Manthey’s construction and the original Parker-Rhodes construction (P-R), and detailed philosophical connections between P-R and Alison’s thesis, but not between Mike Manthey and Alison Watson. And so on.

In this paper I attempt to spell out some of the critical differences underlying this debate, and propose a way to allow us to still have a common objective.

2. The Traditional (“Menshevik”) Position

Fortunately, a book presenting the traditional position will be available shortly[13] I will defer detailed criticism of the ideas presented there until the printed version is available to all of us. The earlier versions of this text I have seen still left me puzzled as to how the work presented is supposed to be related in detail to the actual practice of laboratory physics and observational cosmology.

Here I will respond to Ted Bastin’s remarks at ANPA 15. I agree with Ted that — in contrast to Parker-Rhodes — all the current protagonists of core
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tioned by Ted.

One basic difficulty is that any published description of PU would allow it to be implemented on a Turing machine with a finite tape. The finite tape, which may be a loop, goes part way toward meeting the McGoveran requirement that before any calculation is started we must name a "largest integer", a requirement that I also adopt. If one drops the finite tape restriction (which may even be irrelevant to the points at issue), one immediately encounters two "theorems" usually accepted by computer scientists. The first is that adding a random component to a Turing machine does not increase its computational power; perhaps this was the objection McGoveran had in mind that Ted mentions on p.8 of his talk. The second is that parallel processing using a number of Turing machines can increase the speed of computation but, again, not the computational power. The validity of this second theorem is explicitly denied by Manthey in Ref. 18. We examine briefly, in Chapter 4, his claim to know how to construct a genuine learning and goal-seeking computer which is not a Turing machine. Here we note that if Topsy works as advertised, and PU can be implemented on it, Ted will have to re-examine his objections.

However, even within the framework of PU as it now exists as a sequential program, one is guaranteed a finite number (not an "indefinite" number, as Ted says) of examples of different levels of the hierarchy construction existing and interacting "concurrently". Nevertheless, he is correct in assuming that the step from this to a spatially distributed universe is not an easy one. I admit that I have been too cavalier in making that assumption in the past. What my recent work with Kauffman, discussed in the next chapter, has established is that the "space" we arrive at is non-commutative at a fine grained but finite level, yet implicates the Maxwell and Einstein fields as representing a well-defined approximation to the forces on a single test-particle under appropriate restrictions.

As to whether my neo-Bridgmanian "operationalism" would have been legitimated or denied by Bridgman, neither Ted nor I knows. My claim to inherit a small fold of Bridgman's mantle is presented elsewhere. When it comes to whether or not I can make contact with the current practice of physicists, however, I think I am on firm ground in the particle physics community, or at least the experimental wing of it. "All" I need to do is to present them with a computer algorithm that will, using the same notation and observables they are accustomed to use in relating cross sections computed from Feynman diagrams to the digital results of their experiments, make a different or more easily tested prediction than currently accepted theories, and they will be just as happy trying to shoot me down as they are any other theorist. At their level of practice, there will be no problem communicating once I have given my theory the precision I, myself, require. Of course, I cannot expect many theorists to take me seriously until after I succeed where they fail. But I insist that my language of "counts in detectors", Feynman diagrams, and cross sections is common with that used by current practitioners of particle physics, and understood by non-negligible numbers of theorists. In particular, a recent communication[29] summarizes Tini Veltman's position as

"... the Feynman rules and Feynman diagrams are the theory because quantum field theory has failed in the end to produce them, and is itself inconsistent. That is THE Veltman."

Next Ted talks about "quantum objects" and "reality". Although Stein thinks he knows what he means by a "quantum object",[59] for me the term is an oxymoron. I never use that term and also avoid talking about "reality" if at all possible. My laboratory "spatial relationships" are defined by standard metrological practice. This includes measurement accuracy bounded from below by finite space and time intervals, specified in advance and in context. Generalizing Chew and Heisenberg, I am not allowed to extrapolate them down below where scale invariance is broken at $\hbar/2m_e$, either directly or by inference from observable effects. So I am not allowed "... all sorts of spatial relationships which go with a conjectural 'particle' which is associated with those counts." Like most practicing physicists, I take short cuts in talking about particles within a community where this will not get me into trouble. I accept the criticism that I am not always careful to drop this cavalier attitude when I engage in ANPA discussions. I hope in the next chapter.
will be more acceptable on that score.

3. The Radical ("Bolshevik") Position

3.1 PARTICLES, NO-YES EVENTS AND MEASUREMENT ACCURACY

My conceptual foundations for reconstructing relativistic quantum mechanics and physical cosmology are the coupled concepts of particle, event and conserved quantum number. I join them together in the following way:

A particle is a conceptual carrier of conserved quantum numbers between events.

An event is a finite spatial region which particles enter and leave during a finite time interval. Both the spatial dimensions and the time interval are fixed in the context of a particular application of the definition.

The algebraic sum of each type of quantum number carried into the region by the entering particles is equal to the algebraic sum of that type of quantum number carried out of the region by the leaving particles. This statement defines a conserved quantum number. Note that the number of particles entering the region need not equal the number of particles leaving the region; in other words, particle number is not necessarily conserved.

The paradigm for an event we have in mind is a counter firing in which a counter of relevant spatial size \( \Delta x \) at a specified location in the laboratory does not fire during a time interval \( \Delta t \), which we call a NO-event, or does fire during that time interval, which we call a YES-event.

We further assume that these NO-YES events can be recorded, using a clock at the counter (or calibrated in such a way that it can be thought of as "in" the counter) which has been synchronized to the laboratory clock using the Einstein convention in relation to spatial coordinates of the counter position fixed relative to the position of the laboratory clock ("origin") and three fixed, independent (i.e. non-coplanar) directions.

This allows us to represent the record made by a single counter as an ordered sequence of two distinct symbols such as "0" and "1". When we have specified how two such ordered sequences of symbols of the same length combine, we will call them bit-strings. A general representation of our bit-strings is given in Appendix I.

We take as our paradigm for measurement accuracy the smallest counter size \( \Delta x \) and time resolution \( \Delta t \) which we can either construct, or infer from the theory we are in the process of constructing.

This is a very powerful and restrictive definition, because it prohibits us from considering fractional space and time intervals. Once we have developed the theory far enough to give meaning to interference, as in optical interferometry, this assumption of a minimum distance also implies a maximum distance and time, which we can call the event horizon.

Up to this point we have treated length and time measurement as distinct. But the System International, employed universally by physicists in reporting the results of measurement, and establishing the meaning of "fundamental constants" takes time measurement to be primary and defines the unit of length:

"The meter is defined to be the length of path traveled by light in vacuum in \( 1/299 \, 792 \, 458 \) seconds. See B.W. Petley. Nature, 303, 373 (1983)."

Thus, following current practice, we are no longer allowed to define \( \Delta x \) and \( \Delta t \) separately when specifying our lowest bound on measurement accuracy. In fact, we must make the scale invariant statement that

\[
\frac{\Delta x}{c \Delta t} = 1
\]  

(3.1)

in any system of units which allows us to talk about NO-YES events in a precise way.

We can summarize the content of this section by the phrase:

PHYSICS IS COUNTING
3.2 Postulates for the Feynman-Dyson-Tanimura-Kauffman Proof

The derivation of Maxwell's Equation's using the discrete ordered calculus (DOC) of Etter and Kauffman has been presented elsewhere. My version of the physics underlying this piece of mathematical physics was presented at ANPA 15, and discussed again more recently. My ANPA presentation was uncharitably called a “South Sea Bubble” by Clive, a derogatory characterization more appropriately applied to an entrepreneur pitch than to a Bolshevik platform. Nevertheless, he has subsequently vetted the most rigorous piece of the work (Ref. 26) for submission to the Proceedings of the Royal Society A. Lou (Kauffman) and I are most grateful to him for this informal refereeing; his comments undoubtedly enabled us to tighten up the argument.

We are now on firm ground in claiming that the postulates

1. \[ [X_i, X_j] = 0; \]
2. \[ [X_i, \dot{X}_j] = \kappa \delta_{ij} \]

and the assumption that the acceleration of a test charge is related to \( E \) and \( H \) by the Lorentz law lead to the conclusion that

\[ \sum_i \nabla_i H_i = 0; \quad \frac{\partial H_i}{\partial t} + c_{ijt} \nabla_j E_k = 0 \] (3.2)

in the context of the DOC. Skeptics can consult Appendix II and purists Ref. 26. We concentrate in this section on understanding the postulates in the context of measurement accuracy bounded from below (i.e. \( \Delta t = \text{int} \Delta t \) with \( n, n_T \) integer and \( \Delta x = c \Delta t \)). We explore some of the implications of the proof in the next section.

In our context, we can think of \( 2K_{\text{max}} + 1 \) macroscopic counters of size \( \Delta x \) lined up in some direction. Take the “origin” as the central counter in the array, \( X^{(0)} \), and define

\[ X^{(k)} = k \Delta x; \quad k \in -K_{\text{max}}, -K_{\text{max}} + 1, ..., +K_{\text{max}} - 1, +K_{\text{max}} \] (3.3)

Since the recording clocks associated with these counters are synchronized using the Einstein convention, any pattern of NO-YES events in these counters defines a pattern which can be associated with bit-strings, the 1’s representing, for example, YES events and 0’s representing NO events. This association can be made in a number of ways.

We defer the examination of Lorentz boosts and rotations of the array (within a larger context) until we have introduced more structure. Here this array is a fixed (one dimensional) spatial coordinate system, and we can define a simultaneous firing pattern \( F(K_{\text{max}}; k, n) \) relative to tick \( n \) on the clock at \( X^{(0)} \) as the pattern of firings or non-firings of the counters located at grid position \( k \) at tick \( n \). Note that to examine this data locally, we will have to wait at least \((K_{\text{max}} + 1)\Delta t \) seconds to collect it all in one place. To restrict the maximum size of this finite and discrete model of “space-time” symmetrically, we require that

\[ n \in -K_{\text{max}}, -K_{\text{max}} + 1, ..., +K_{\text{max}} - 1, +K_{\text{max}} \] (3.4)

Once the data are collected, we can map a single firing pattern onto a bit-string (see Appendix I for notation) by

\[ F(K_{\text{max}}; k, n) \Rightarrow F(S; f_s); \quad S = 2K_{\text{max}} + 1; \quad s = k + K_{\text{max}} + 1 \] (3.5)

where

\[ f_S(n) = 1 \text{ if } X^{(k)}(n) \text{ fires, else } f_s = 0; \quad f(n) = \sum_{s=1}^S f_s(n) \] (3.6)

In the absence of further information, this block of data could easily have been generated by PU. We call a single firing pattern \( f_S(n) \) a measurement, and a sequence of firing patterns parameterized by a sequence of ordered ticks \( n_i \)

\[ -K_{\text{max}} \leq n_1 < n_2 ... \leq +K_{\text{max}} \] (3.7)

a sequence of measurements.
This model is still too general for our current purpose. We restrict ourselves to single particle trajectories, which have one and only one bit for any firing pattern \( f(n) = 1 \) for all \( n \), and which pass through the origin (i.e. \( X(0)(0) = 0 \), or stated another way \( f_{k_{\text{max}}+1}(0) = 1 \)). We further require them to be “piecewise continuous” by imposing the constraints

\[
\text{if } f_s(n) = 1 \text{ then } f_{s+1}(n+1) = 1 \text{ or } f_{s-1}(n+1) = 1 \text{ but not both}
\]

\[
\text{else } f_s(n+1) = 0
\]

(3.8)

In the absence of further information, we have modeled a “random walk”.

In order to restrict ourselves to “particles” \((-c < v < +c)\) and exclude “photons” we require that at the initial time the initial firing does not lie at either of the extreme values of \( X \) and similarly for the final firing. Assume that we make only these three measurements, which we call \( X(t)(-T) \), \( X(0)(0) \), and \( X(t)(+T) \). These three measurements then specify two average velocities:

\[
\nu^{(-)} \equiv \frac{X(t)(0) - X(0)(-T)}{T} \quad \nu^{(+)} \equiv \frac{X(t)(+T) - X(t)(0)}{T}
\]

(3.9)

and a change in average velocity:

\[
\Delta \nu = \nu^{(+)} - \nu^{(-)} = \frac{X(t)(-T) + X(t)(+T) - 2X(t)(0)}{T}
\]

(3.10)

If we now extend our counter array by any number of blocks of counters of length \((K_{\text{max}} + 1)\Delta x\), run a piecewise continuous single particle trajectory through them all, and define

\[
X \equiv X^{(-)}(-T); \quad X' \equiv X(0)(0); \quad X'' \equiv X^{(+)}(+T); \quad X''' \equiv X(2T); \ldots
\]

(3.11)

we have now provided a preliminary bit-string and measurement accuracy model for what I will call a Kauffman trajectory.

In his preliminary remarks, Kauffman points out that postulate 2 (which is formally equivalent to Feynman’s, if \( \kappa = -i\hbar/m \)) looks like quantum mechanics. In fact Feynman’s proof cannot refer to quantum mechanics because \( X \) and \( \dot{X} \) are subsequently treated as continuous functions of \( t \), and hence are defined at the same time; this is incompatible with the uncertainty principle. Our derivation does not fall into this trap because Kauffman introduces a new symbol \( X' \) which means \( X \) shifted by one time step, which he takes to be 1 and in the model articulated above is obviously \( T \). He further interprets the symbol \( XX' \) as the instruction, “first measure \( X \), then measure \( X' \).” This means that “velocity”, however introduced, is not measured at the same time as \( X \) and further that the symbol \( XX' \) is meaningless as it stands. We are therefore able to invest it with meaning, if we use care to do so consistently. Then (Ref. 7, p. 3) making the hypothesis that

\[
X \dot{X} - \dot{X} X = \kappa
\]

“is regarded as a hypothesis about the structure of their non-commutativity”.

In the model given above, all we can say about \( \kappa \) so far is that dimensionally it is an area per unit time, and hence in a broader measurement context can be related to Kepler’s second law. We have discussed this in references given here and cited in them, but an axiomatic treatment is still lacking. For the purposes of the Dyson-Feynman-Tanimura-Kaufman proof, nothing prevents us from fixing \( \kappa \) by the scale invariant definition

\[
\frac{\Delta x^2}{\kappa \Delta t} = 2\pi
\]

(3.12)

The mathematical step next taken is to introduce a “shift operator” \( U \) defined by \( X' = UX \) and treat \( U \) as an algebraic element of the same algebraic type as the sequence of \( X \)’s we already have. If we now assume three independent directions \( i, j, k \), postulate 1 simply asserts their independent measurability, and is consistent with setting up a three dimensional version of our counter array. We also must
define
\[ \frac{\partial X_i}{\partial X_j} = \delta_{ij}; \quad \nabla_t = \frac{\partial}{\partial X_t} \] (3.13)

For details, see Ref. 26, and for subsequent steps in the actual proof, see Appendix II.

3.3 SOME IMPLICATIONS

Asserting that the Maxwell Equations specify a necessary connectivity between a piecewise continuous trajectory and the "field" which specifies the acceleration (change in velocity) of a "particle" passing through the events (of volume \( \Delta x^3 \Delta t \)) where (in a punctiform language) the changes occur to be a consequence of accepting finite and discrete measurement accuracy bounded from below is a powerful conclusion. But it does not mean that we have "derived" classical fields from measurement accuracy for a number of reasons.

One of the problems is that classical fields have ontological existence, and are not supposed to change no matter how many "test-particles" pass through the space-time region in which they exist. But our model contains only one particle which "exists" in the sense of having demonstrable properties only when it produces YES events. Further, we can only state that it interacts with some electromagnetic field when a sequence of at least three YES events (at time intervals \( T \)) show that its velocity changes. To show that these fields "propagate", which is implied in the usual classical interpretation of Maxwell's Equations, will take a lot more work.

A second problem is that the "fields" so derived do not commute. This is actually to be expected. In 1933 Bohr and Rosenfeld derived the uncertainty principle version of the \( E, H \) commutation relations from the restrictions on measurability imposed by the non-relativistic uncertainty principle applied to the measuring apparatus. That we have been able to arrive at a (non-commutative, finite and discrete) version of the Maxwell Equations by postulating fixed measurement accuracy is consistent with their result. How this works out in detail might be worth exploring.

A third problem is that we can take the field as given and compute the trajectory, or the trajectory as given and compute the field, but not both at once. Further, once we introduce momentum conservation, we can treat the particle as either a source or sink of the field, but not both at once. This may be a virtue in disguise, because this "self-energy" problem has proved to be insoluble in classical theory, and intractable in a rigorous, mathematical sense in QED. We return to this issue below.

One way to get off the ground and start moving toward the many particle problem is to consider a closed trajectory which is a regular polygon, implying a "field" directed toward the center acting at the counters where the direction but not the velocity of the particle changes. This is a scale invariant version of the bit-string "Bohr atom" that McGoveran and I used to discuss the fine structure of hydrogen. Extended to a polygonal approximation of an elliptical orbit, it is reminiscent of diagrams in the Principia, and can be extended to hyperbolic orbits if finite and discrete boundary conditions are supplied. Further, by making step-lengths inversely proportional to masses, we could replace this "motion around a center" by a well defined solution of the two body problem.

In contrast to the classical equations in the usual framework, which render these orbits unstable due to "radiation", they are perfectly acceptable solutions to the "trajectory-field" equations we have derived and do not radiate. So we have a scale invariant version of the generalized Bohr-Sommerfeld two body system "derived" from measurement bounded from below! Interchanges of energy between two such systems would be quantized. If there are no other sources or sinks, this would extend our "coherence" to at least some four-body systems.

The differences between gravitation and electromagnetism in this context should prove to be of interest. At least if Tanimura's extension is any guide, gravitation will require closure conditions, even for a single orbit and massive center, which will depend on the area of the orbit, and hence go beyond the quasi-local "trajectory-field" interaction we have examined in this paper.
Returning to the self-energy problem, it will be seen that our underlying "random walk" model could be attributed to a "background radiation" and in "free space" required to yield the finite and discrete version of the Dirac equation which McGoveran, Karmanov, Stein and I kicked around half a decade ago. Consistency between this and our quantized Maxwell Equations might even lead to a solution of the basic problem in the relativistic quantum theory of fields and particles. This speculation could well prove to be analogous to the "South Sea Bubble", so I offer no stock for sale.

4. Manthey's coexistence and mutual exclusion

Mike (Manthey) did an excellent job at ANPA 15 in making clear the connection between his computational model and the Parker-Rhodes construction (not the current Kilmister construction) of the combinatorial hierarchy. The fact that "There is a 'phase difference' of one level between the two constructions, as revealed by the syntactic match of \( \{a, b, a[b]\} \) with \( \{1, 1, 1\} \), versus its semantic match with \( a_1 a_2 \) which appears one level later" is undoubtedly significant. It may be connected with the fact that the computational approach uses "\( \phi \)" as well as "\( 0 \)" as a basic symbol. Note that Parker-Rhodes makes essentially the same distinction in getting his "inevitable universe" to start up. The fact that Manthey has this new symbol, and positive as well as negative elements makes it easier for him to demonstrate that he has constructed quaternions than it is using only discrimination and the Parker-Rhodes matrix mapping. Clive and Mike did some work together exploring this connection, but more work along this line might prove to be fruitful. Similarly, exploring what it would take to get to the same formalism using discrimination and concatenation as basic would tighten connections to the PU approach.

Mike notes that in his approach, as in some versions of the Menshevik program, the second Parker-Rhodes sequence does not appear. This is a genuine problem if we wish to make contact with particle physics and weak-electromagnetic unification using the first three levels, gravitation at the fourth level and physical cosmology using the Parker-Rhodes closure to both being particle physics into contact with gravitation and fix the baryon and lepton number of the universe in rough agreement with observation. Until quantitative contact is made with experience by either program in this sense, each will remain speculative metaphysics in my evaluation. In contrast the Bit-String, PU approach leads to a detailed modeling of the standard model of quarks and leptons, and results similar to those obtained in the standard "big bang" cosmology, in both cases with much less effort (see Ref. 13).

The computational model itself was described by Manthey at ANPA 14, and more recently in Ref. 18. Here Mike's work has importance for quite a different reason. He claims, and I believe with considerable cogency, that a von-Neumann computer with a CPU, or its abstraction as a Turing machine, cannot deal with mathematical problems based on set theory and functional composition, but not with the critical concepts of mutual exclusion and concurrency used in his cycle hierarchy. In particular, he claims that the "synchronization sticks" which occur when one couples two wait-signal nodes from the Hewett actor model in a specific way conceal a bit which is not a "bit of information" in the Turing machine sense. I suspect he is right, but reserve judgment until he can make a formal argument that is understandable by, for example, Tom Etter.

A lot of contemporary discussion of "AI" and consciousness hinges on whether or not a computer can model consciousness, or quantum mechanics in the sense of EPR, let alone "be conscious", whatever that means. I am almost prepared to accept the arguments that claim that a Turing machine cannot be conscious, but Mike Manthey's "Topsy" may be another matter. So I hope that Mike can use Topsy to both model EPR and a concurrent version of PU in a way that satisfies both him and David McGoveran. I am trying hard to do EPR in my own fashion, but am not there yet.
5. CONCLUSION

All three approaches produce the first Parker-Rhodes sequence. A better understanding of what makes this necessary or contingent will help all three programs. I have argued above that the second Parker-Rhodes sequence is needed for particle physics and physical cosmology, so continued attention as to whether this also emerges "naturally" in the Menshevik and Topsy programs is clearly a matter of great importance.

As second area of tension between the three programs arises in the use of bit-strings. For Manthey they are the interface between his hierarchical computer program and the "external world". For me they can be "firing patterns" of counters, with essentially the same sort of interpretation. So bringing later steps in the construction into congruence should be vigorously pursued. The Menshevik program is more abstract, and only arrives at bit-strings as a possible representation at a fairly advanced stage of the construction. Here I urge its protagonists to show us more explicitly how they view their strings as relating to "experience".

I think it is clear that one possible way to bring the three programs together is to concentrate on the reconstruction of relativistic quantum mechanics (RRQM). But, so far, this has not been a priority for others. I suggest for a topic at ANPA 17 a discussion of (a) whether it is even desirable for the three programs to have a common goal and (b) if so, what it should be. Alternatives to RRQM would certainly be of interest to me.

With regard to my own research program, we believe that the discussion started last year of the derivation of the classical relativistic field equations from scale invariance bounded from below and Kepler's Second Law is achieving the status of mathematical rigor in an appropriately limited context. We believe that to use this as a basis for both establishing a "correspondence limit" for relativistic quantum mechanics and some sort of finite and discrete "quantum gravity" is now only a matter of time. We have made progress in nailing down the bit-string framework for the finite and discrete transformation laws which keep \( E(E+1) - p^2 = m^2 \) and \( j(j+1) - j_z^2 = j_x^2 + j_y^2 \) invariant under Lorentz boosts and rotations, respectively. No surprises have showed up, and closure should not lie too far in the future. These are, of course, formulated in terms of discrimination between bit-string states. Results may be available for ANPA 17. By then we could also relate these states to the two positions occupied by no, one, or two blocks used as the basis for Manthey's construction of quaternions. With these tools in hand, we believe that the time would be ripe for the reconstruction of relativistic quantum mechanics (RRQM) which has long been our goal.

6. APPENDIX I: Bit-string Basics

Specify a bit-string \( a(S) \) by its \( S \) ordered elements \( a_s \):

\[
a_s \in 0, 1; \quad s \in 1, 2, 3, \ldots, S
\]

If we interpret the symbols "0" and "1" in the strings as integers, we can calculate the norm, or Hamming measure, \( a(S) \) by the formula

\[
a \equiv \sum_{s=1}^{S} a_s = |a(S)| = a(S)
\]

Because we interpret the symbols "0" and "1" as integers rather than bits, we can define the operator XOR, which combines two strings to form a third and is symbolized by "\( \oplus \)", in terms of the elements of the resulting string:

\[
(a \oplus b)_s \equiv (a_s - b_s)^2
\]

This is isomorphic to the usual meaning of XOR, addition mod 2 or boolean symmetric difference in the sense that the element is 1 if \( a_s \) and \( b_s \) differ, and 0 if they are the same.
We introduce the null string $0(S)$ with elements $0_s = 0$, the anti-null string $1(S)$ with elements $1_s = 1$, and the complement to $a$ defined by $\bar{a} = 1 \oplus a$; clearly this string is $a$ with the “0”’s and “1”’s interchanged. We note that

$$a \oplus a = 0; \quad a \oplus \bar{a} \oplus 1 = 0 \quad (6.4)$$

We introduce a second bit-string operation called concatenation, symbolized by “||” and defined by

$$c_k^{ab} = a_k, \ k \in 1, 2, ..., S_a; \ c_k^{ab} = b_j, \ j \in 1, 2, ..., S_b, k = S_a + j \quad (6.5)$$

7. APPENDIX II: Formal derivation of finite and discrete Maxwell Equations

When I recently showed Ref. 26 to my colleague, M. Peskin, he noted that the “shift operator $J$” defined by Kauffman is, in our context of a single particle, isomorphic to the operator $U = \exp(-iHT)$ representing a finite time shift in the Heisenberg representation. Then the formal steps in Kauffman’s rigorous version of Feynman-Dyson-Tanimura “proof” go through easily. The difficulty with adopting Peskin’s approach is that what operational context the Heisenberg formalism fits into is by no means obvious. So, for mathematical and physical clarity, one needs to invoke the DOC and discuss the relationship between measurement accuracy and the DOC. I am indebted to Peskin for allowing me to quote his shortened version of the Kauffman proof below.

Define

$$\dot{X} = UX - UX = [X, U] \quad (7.1)$$

where $U$ is the time shift operator from $X$ to $X'$ in time $\Delta t$ (e.g. $U = e^{-iH\Delta t}$).

Notice that

$$(AB) = [AB, U] = [A, U]B + A[B, U] = \dot{A}B + A\dot{B} \quad (7.2)$$

as required.

Postulate:

1. $[X_i, X_j] = 0$; 2. $[X_i, \dot{X}_j] = \kappa \delta_{ij}$

Rewrite 2 as

$$[X_i, [X_j, U]] = -[X_j, [U, X_i]] - [U, [X_i, X_j]] \quad (7.3)$$

and noting that $[U, [X_i, X_j]] = [U, 0] = 0$ we find that

$$\kappa \delta_{ij} = [X_i, [X_j, U]] \text{ symmetric in } i, j \quad (7.4)$$

Now define

$$H_i = \frac{1}{2\kappa} \epsilon_{ijk} [\dot{X}_j, \dot{X}_k] \quad (7.5)$$

Then

$$\nabla_i H_i = \frac{1}{2\kappa} \epsilon_{ijk} [[\dot{X}_j, \dot{X}_k], \dot{X}_l] \quad (7.6)$$

But this cyclic sum vanishes by the Jacobi identity. Thus

$$\nabla_i H_i = 0 \quad (7.7)$$

which is one of the two Maxwell equations we set out to derive.
Finally, define
\[ E_i = \dot{X}_i - \epsilon_{ijk} H_k \]  
(7.8)

We wish to prove that
\[ \frac{\partial H_i}{\partial t} + \epsilon_{ijk} \nabla_j E_k = 0 \]  
(7.9)

First we need to define \( \frac{\partial}{\partial t} \) by
\[ \dot{H} = \frac{d}{dt} H = \frac{\partial H}{\partial t} + (\dot{X} \cdot \nabla) H \]  
(7.10)

Then
\[ \frac{\partial H_i}{\partial t} = \dot{H}_i - \dot{X}_j \nabla_j H_i \]
\[ = \frac{1}{2\kappa} \epsilon_{ijk} ([\dot{X}_k, [\dot{X}_l, X_j]) - \dot{X}_j \frac{1}{\kappa} \epsilon_{ijk} [\dot{X}_k, X_i], X_j] \]
\[ = \frac{1}{\kappa} \epsilon_{ijk} [\dot{X}_k, X_i] - \frac{1}{2\kappa^2} \dot{X}_j \epsilon_{ikl} [\dot{X}_k, X_i], X_j] \]  
(7.11)

\[ \epsilon_{ijk} \nabla_j E_k = \epsilon_{ijk} \frac{1}{\kappa} \left( \left( \dot{X}_k - \epsilon_{klm} \dot{X}_l H_m \right), X_j \right) \]
\[ = \frac{1}{\kappa} \epsilon_{ijk} \left[ \dot{X}_j, \dot{X}_k \right] (-1) - \epsilon_{ijk} \epsilon_{klm} \epsilon_{mab} \frac{1}{2\kappa^2} \left[ \dot{X}_l \left[ X_a, \dot{X}_b \right], X_j \right] \]
\[ = - \frac{1}{\kappa} \epsilon_{ijk} \left[ \dot{X}_j, \dot{X}_k \right] \]
\[ - (\epsilon^{ij} g^{jm} - \delta^{im} \chi^{jl}) \epsilon_{mab} \frac{1}{2\kappa^2} \left( \left[ \dot{X}_l \left[ X_a, \dot{X}_b \right], X_j \right] + \dot{X}_l \left[ X_a, [X_k, X_j] \right] \right) \]
\[ = - \frac{1}{\kappa} \epsilon_{ijk} \left[ \dot{X}_j, \dot{X}_k \right] + \frac{1}{2\kappa^2} \epsilon_{iab} \dot{X}_j \left[ [X_a, X_b], X_j \right] \]
\[ - \epsilon_{jba} \frac{1}{2\kappa^2} \left[ \dot{X}_i, \dot{X}_j \right] \left[ X_a, \dot{X}_b \right] \]  
(7.12)

Now
\[ \epsilon_{iab} \left[ \dot{X}_i, \dot{X}_j \right] \left[ X_a, \dot{X}_b \right] = \left[ \dot{X}_i, X_j \right] \left[ X_2, X_3 \right] \]
\[ + \left[ \dot{X}_i, \dot{X}_2 \right] \left[ X_3, X_j \right] + \left[ \dot{X}_i, \dot{X}_3 \right] \left[ X_1, X_j \right] \]  
(7.13)

for \( i = 1 \), eg
\[ \left[ \dot{X}_1, \dot{X}_2 \right] \left[ X_3, X_1 \right] + \left[ \dot{X}_1, \dot{X}_3 \right] \left[ X_1, X_2 \right] = 0 \]  
(7.14)

so
\[ \epsilon_{ijk} \nabla_j E_k = - \frac{1}{\kappa} \epsilon_{ijk} \left[ \dot{X}_j, \dot{X}_k \right] + \frac{1}{2\kappa^2} \epsilon_{iab} \dot{X}_j \left[ [X_a, X_b], X_j \right] \]
\[ = \frac{\partial H}{\partial t} QED. \]  
(7.15)

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