ELASTO-PLASTIC FINITE ELEMENT ANALYSIS OF AXIAL SURFACE CRACK IN PHT PIPING OF 500MWe PHWR

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Abstract: The leak before break (LBB) approach in nuclear piping design envisages demonstrating that the pressurized pipe with a postulated flaw will leak at a detectable rate leading to corrective action well before catastrophic rupture would occur. This requires analysis of cracked pipe to study the crack growth and its stability. This report presents the behaviour of a surface crack in the wall of a thick Primary Heat Transport (PHT) pipe of 500 MWe Indian PHWR. The Line Spring Model (LSM) finite element is used to model the flawed pipe geometry. The variation of crack driving force (J-integral) across the crack front has been presented. The influence of crack geometry factors such as depth, shape, aspect ratio, and loading on peak values of J-integral as well as crack mouth opening displacement has been studied. Several crack shapes have been used to study the shape influence. The results are presented in dimensionless form so as to widen their applicability. The accuracy of the results is validated by comparison with results available in open literature.

Keywords/Descriptors: CRACKS; PHWR TYPE REACTORS; DEPTH; SHAPES; PIPES; FRACTURE MECHANICS; REACTOR COMPONENTS; INTEGRALS; LEAKS; RUPTURES; FINITE ELEMENT METHOD; MATHEMATICAL MODELS; PRIMARY COOLANT CIRCUITS; CRACK PROPAGATION; SURFACES
ABSTRACT

The leak before break (LBB) approach in nuclear piping design envisages demonstrating that the pressurized pipe with a postulated flaw will leak at a detectable rate leading to corrective action well before catastrophic rupture would occur. This requires analysis of cracked pipe to study the crack growth and its stability. The analysis of the part through 3-D surface cracks is prohibitively expensive as well as time consuming. The approximation of crack to semi-elliptical shape has been commonly accepted in the design codes. However, crack shape is known to become non-elliptic after the growth, raising some questions regarding influence of crack shape geometry on results. A simplified line spring model (LSM) offers an economic technique for analysis of surface cracks.

Under the LBB development undertaken by Reactor Engineering Division of B. A. R. C, this report presents the behaviour of a surface crack in the wall of a thick Primary Heat Transport (PHT) pipe of 500 MWe Indian PHWR. The LSM finite element is used to model the flawed pipe geometry. The variation of crack driving force (J-integral) across the crack front has been presented. The influence of crack geometry factors such as depth, shape, aspect ratio, and loading on peak values of J-integral as well as crack mouth opening displacement has been studied. Several crack shapes have been used to study the shape influence. The results are presented in dimensionless form so as to widen their applicability. The accuracy of the results is validated by comparison with results available in open literature.
1. INTRODUCTION:

Leak before break (LBB) design approach is well established in nuclear industry to reduce the conservatism and resultant costs without sacrificing safety. Additional advantage is further safety promotion due to elimination of pipe-whip restraints for large diameter PHT piping. Based on recent advances in elastic plastic fracture mechanics the approach envisages demonstrating that pressurised components will not fail catastrophically; at worst, they will leak at a detectable rate and corrective action can/could be taken well before catastrophic rupture could occur.

Recent developments in fracture mechanics enable analysis of two dimensional elastic as well as elastic-plastic cracked structures without any difficulty. In a wide majority of engineering applications, however, the fracture problem usually consists of part through surface cracks that are three dimensional in nature and should be treated accordingly. Out of the three levels of confidence which need to be demonstrated in LBB approach, the first level is inherent in the design philosophy of ASME Boiler and Pressure Vessel Code, Sec.III, which assumes that factor of safety will take care of enhanced stresses due to flaws which are acceptable by code. Level 2 confidence is obtained by postulating a part through crack at the most stressed region of a pressurized component and then to
demonstrate that it will not grow through wall during the entire working life of reactor. Level 3 postulates the initial part through flaw to grow through wall due to some unforeseen events. For Level 3 it should be demonstrated that final flaw size at the end of evaluation period is sufficiently smaller than the critical flaw.

In order to assess the safety margin against the unstable growth of the flaw in most vulnerable region, it becomes essential to obtain the stress intensity at the crack front and compare it with material crack growth resistance. The details of the steps involved in development of LBB approach are discussed in an earlier report\cite{1} and the present report concentrates on part-through cracks, obtaining fracture mechanics parameters and their use in the assessment of possibility of leak, stable growth or break. In this report we develop the methodology for the application of EPFM to axially cracked pipe subjected to internal pressure. The procedure is developed in terms of J-integral concept discussed in the earlier report.

2. Mechanics of Surface Crack:

The pressurized components in nuclear industry are made of highly ductile materials. The stable crack growth and ductile instability phenomenon of part-through cracks are the most important fracture modes from the structural integrity point of view. In the conventional design, a number of pipe-whip restraints were employed against guillotine break hypothetical accident of piping. In the LBB concept, however, whip restraints are eliminated in the design, which is considered to economise the construction and maintenance \cite{6}. Before we present the application of analysis techniques to cracked pipes it may be beneficial to review the previous practice and evolution of current techniques. In what follows the limitations of the practice are also outlined.

Recent structural integrity assessment procedures have been based on the use of fracture mechanics in one form or other.
Linear Elastic Fracture Mechanics (LEFM) with small local plastic deformation was the basis of ASME III and XI approaches. Stress Intensity Factor (SIF) of LEFM retain their importance for aspects of the overall structural integrity programs such as fatigue crack growth. However, in the ductile materials used for pressurized components, a large scale yielding occurs around the crack front before the crack starts to grow and then LEFM cannot describe the fracture phenomenon adequately. The Elasto-Plastic Fracture Mechanics (EPFM) theory can describe the fracture process under large scale yielding condition.

In the engineering approach to fracture mechanics of crack extension and stability, two measures are introduced. One is the J-integral crack driving force and the other is tearing modulus featuring the stability of crack growth. Under the condition that deformation theory is valid, the J-integral preserves its path independency and represents the strength of the Hutchinson-Rice-Rosengreen (HRR) singularity of the stress-strain fields. In the experimental fracture mechanics field, the simple evaluation procedure of the J-integral has been developed and contributed to establish the fracture toughness testing method based on the EPFM. This has given impetus to the application of the J-integral and tearing modulus concept to a wide variety of integrity studies of nuclear structures.

Despite wide application of J-integral and tearing modulus concept for assessment of integrity of flawed structures, it must be noted that J-integral concept has some theoretical limitations as well. When the cyclic load is applied to a cracked structure of elasto-plastic material or when the stable crack grows, the crack, under the large scale yielding, is necessarily subjected to non-proportional loading and unloading; then, the deformation theory of plasticity is not valid any more. As a result, tearing modulus concept (J/T theory) can deal with only short crack growth, in which monotonic and proportional loading conditions hold near the crack tip [27]. As such, the results presented in this work can be assumed to apply only for small crack growth.
There is another important limitation of the J/T theory for mechanics of crack extension. The simple engineering approach to characterize crack initiation, growth and stability in terms of J-integral and tearing modulus suffices for through wall crack. The crack growth is known to depend on triaxility distribution for its role in creation of voids and aiding the fracture process. While J-integral accounts only for energy travelling towards the crack tip and thus governing the stress singularity for the crack driving force, the voids act as catalysts and yield additional growth. This has been established by both theoretically and experimentally [31]. The approach developed in the present work cannot give the triaxility distribution and therefore, the approach can be termed adequate for engineering design only; the detailed prediction of growth of a surface crack along the front and its passage through the ligament causing the break through will need the development of detailed triaxility distribution through 3-D finite element analysis.

3. NUMERICAL SOLUTIONS IN FRACTURE MECHANICS OF PART-THROUGH CRACKS;

To understand the usefulness of the present approach to the treatment of part-through cracks, it may be useful to review the previous work which has led to the current level of analysis techniques. The application of fracture mechanics theory to complicated structures such as pressure vessels, nozzles, piping components (bends, tees etc.) has become possible mainly due to remarkable improvement of the accuracy and efficiency of the evaluation technology of the fracture mechanics parameters [29]. This technology is supported by finite element method and the boundary element method (BEM). To obtain the accurate solution of the stress intensity factors and J-integral distributions for semi-elliptical surface cracks, several numerical techniques, such as FEM combined with singular element, the BEM, the influence functions, alternating method and line spring method have been developed in the past [9,11,20].
An efficient FEM where beam, plate, shell elements are employed simultaneously to compute the SIF in a pipe structure was proposed since fully three dimensional finite element analysis of practical problems are very time consuming even with supercomputers [28].

The surface cracks in pressurised vessels and piping have attracted a number of researchers over last twenty years. Earliest attempts of obtaining engineering estimates for SIF for internally pressurised thick walled cylinder did not include the effect of wall thickness [2]. The wall thickness effect was included by Kobayashi et.al. [4] later. The effect of both internal and external cracks on SIF was presented by Kobayashi [4]. The variation of SIF along the crack front obtained by use of three dimensional finite element analysis for both cracks has been presented by Atluri [5] while the results for arbitrary loadings for only internal cracks for a limited range of configuration parameters has been carried out by McGowan and Raymund [6].

The SIF for internal surface cracks for thin and thick pipes for a wide range of crack shapes using 3-D finite element analysis has been presented by Newman and Raju [7] while those for internal cracks using BIE method have been obtained by Heliot et.al. [8]. Recently, Raju and Newman [9] have obtained SIF for internal and external cracks under pressure as well as thermal shock and for a range of crack depths ratios (0.2 to 0.8), crack aspect ratios (0.2 to 1.0). They used singularity elements along the crack front and obtained SIF using nodal force method. Close agreement with the results obtained using BIE method were obtained. The SIF for surface cracks in flat plates were also presented by Raju and Newman, and solution convergence was studied with finer meshes [12,13].

Another approach to the surface crack solution was by alternating method based on FEM, originally proposed by Atluri [5]. It used cubic pressure distribution on crack surface. Though it gave relatively poor result, it had the potential of cheaper
technique if it could be improved [6]. Smith et al. [10] introduced FEM into the alternating method to analyse surface cracks emanating from a fastener holes. A major improvement in this technique was made by Nishioka and Atluri [11] by implementing the complete analytical solution for an elliptical embedded crack in an infinite solid in conjunction with the FEM. This procedure resulted in accurate evaluation of SIF and computational effort saving by an order of magnitude as compared to the hybrid FEM or 3-D FEM. The results on internal surface cracks of various shapes and external semi-elliptical crack in a thick cylinder were compared with those obtained by 3-D FEM and also by other researchers.

The geometric and parametric complexities of surface crack remain a formidable obstacles for "exact" continuum solutions, even if they are obtained with the aid of powerful computers executing sophisticated computational algorithms. In LEFM, through exact elasticity solutions have been obtained for many simple configurations, the complexity of 3-D crack geometry has required the use of numerical methods. In engineering practice, there is a great need for simplified mechanics models of surface crack behaviour which approximately account for major observed response. However, the scope and quantitative effects of assumptions made in a simplified model may be difficult to determine a priori. There are only few simplified models which have had great impact on the understanding of surface crack behaviour. The line spring was the earliest and therefore well developed model while some recent models need accumulation of experience and confidence.

The line spring concept was proposed by Rice[18,19] almost concurrently as alternating method, for LEFM analysis. It proved to very attractive and cost effective tool for the analysis of surface cracks in plates. The initial aim was to obtain approximate fracture mechanics solutions for the class of structures broadly termed "Surface cracked plates and shells". The problem of a three dimensional surface crack is reduced to that of stretching and bending of a plate with a series of line
springs to represent stiffness of a part-through crack. Later Parks [22,26] extended the model to the analysis of J-integral for infinite plates where boundary conditions are simple enough to lead to the simple dominating integral equations.

The generalization of the concept within the FEM framework was attempted by Shiratory and Miyoshi [23] and Parks. Parks [20-22] embedded the line spring concept within the structural models of either shallow shell theory or shell type finite elements. In the linear elastic case it was found that LSM provides SIF distribution all along the crack front which have acceptable accuracy (few percent) compared to those obtained from detailed numerical models. Greatly reduced complexity of the LSM as compared to the fully three dimensional models caused substantial saving both in data preparation costs and computer time. The potential savings were expected to multiply in nonlinear analyses [21,22] which motivated Parks to develop elastic-plastic line spring model along the basic outline suggested by Rice. Parks presented the theory for combining the elastic stiffness of LSM with the description of a nonhardening yield surface to obtain elastic plastic rate stiffness matrix of the LSM, using slipline theory and upper bound analysis of Rice [18,19]. By restricting (ideally) plastic response to the line spring only, Parks obtained [22] approximate J-analyses of various elastic-plastic surface cracked geometries. The LSM was later incorporated into ABAQUS by Parks, Lockett and Brockenbrough [32] after implementing incremental elastic-plastic strain-hardening constitutive model. Shiratori and Miyoshi [23] have used it for LEFM studies. White et.al.[24] embedded the LSM in an elastic plastic FEM program capable of admitting plastic response in the structure as well. Using LSM, White et.al. [24] carried out EPFM studies on surface cracked plates under bending and compared the J values obtained with those from experiments on CT specimens. While the initiation of cracks occurred at similar J's for both through-crack and surface-crack specimens, but the part-through crack specimen exhibited a higher apparent tearing modulus. Recently, similar observations have been made by Roos et.al.[40]. Parks and White presented
(24) elastic plastic J- analysis of a long axial crack in pressurized cylinder using LSM. Semi-elliptical flaw of aspect ratio 3 and the effect of varying depth was studied.

Besides a number of elastic plastic fracture analyses based on incremental theory, the fully plastic solutions for various geometries of semi-elliptical surface cracks have been obtained with the fully plastic finite element analysis by Yagawa et al. (29). The flow theory of plasticity is an incremental theory and the most commonly used. The fully plastic solutions employ deformation plasticity theory with power law hardening constitutive model. The deformation theory is used quite widely in Soviet union. In applications where the unloading and strong deviation from proportional loading is restricted to small region of the structure, deformation theory is just as valid as flow theory, and for problems where exact proportional deformation is satisfied, the prediction of flow and deformation theory are identical. Deformation theory is more cost effective in computational solutions as the deformations are not history dependent. The most advantageous feature of the deformation theory is scalability of the solutions, almost like linear solutions. This feature coupled with so called engineering approach of elastic plastic estimation scheme was evolved by Shih, Kumar and others (27,30). It has led to a simple, rational and sufficiently accurate engineering approach for fracture mechanics solutions of engineering interest. J2 deformation theory of plasticity formulation of LSM for obtaining fully plastic solutions in the framework of shell FEM was presented by Kumar and German (30). Results for 2-D axial and circumferential cracks in cylinders agreed closely with those obtained by 3-D crack analyses. Some discrepancies found in the case of axially cracked cylinders under internal pressure which led to the development of continuum LSM yielding better results. The shell LSM was later implemented in ADINA code by delorenzi (33). The thin shell finite element approach does not model an internally pressurized shell under fully plastic conditions in an accurate manner. The major causes are the ignored shell normal stress in theory and increased importance of this stress due to
incompressibility constraint posed by deformation theory of plasticity. Such discrepancy, however, does not appear when using flow theory of plasticity. This can be considered an additional limitation of deformation theory.

The line spring model is now available in most nonlinear mechanics codes. It is established as an approximate but cost-effective tool for surface flaw analysis.

More recent approaches to the treatment of surface flaws consist of Sub-Region Model (SRM) and Rigid Body Spring Models (RBSM). SRM is based on the sub-region mixed FEM using sub-region variational principal proposed by Cheung [34]. It has been established [34] for LEFM and scope for its extension to EPFM exists. RBSM proposed by Suzuki [35] assumes element nodes at centroids, elements of arbitrary shapes and there is no distinction between beam/plate/shell elements. It is very effective for 3-D cracks, especially nonlinear problems with plastic deformation and limit analyses. Total plastic deformation is treated as slip of one part of rigid body relative to other and spring stiffness which resists this slip is derived from shear modulus of elasticity; for elastic deformation stiffness is derived from hardening properties and yield function. Both Mises and Mohor—Coulomb yield theories can be used. The application to the 2-D crack growth propagation studies reveal that Mohor—Coulomb law appeared to be more appropriate. Inspite of computational efficiency of the two techniques described here, the experience on these recent techniques is limited and their widespread acceptability may take time.

For the results of engineering importance in nuclear industry the authors made a choice of line spring model for its proven accuracy. Before the theory underlying the finite element modelling is discussed it may be appropriate to discuss the theoretical development of line spring concept and its finite element implementation. In the next section, the formulation of line spring concept outlined[41].
The basic features of the line spring model have been described in a number of recent works [18-22]. The implementation in ABAQUS slightly differs from the formulations in the literature. Therefore, only a brief description will be presented here for the sake of completeness.

The part-through surface crack of length $2c$ and depth $a$ in plate/shell of thickness $t$ is considered as shown in Figure 4.1 and 4.2. The position along the crack is denoted by $x$, coordinate parallel to the surface crack and in the midplane of the shell, such that crack region is given by $|x| \leq c$. Local depth of crack is given by $a(x)$. The model is constructed using two main assumptions.

1. The additional flexibility introduced into the shell by the presence of crack is modelled by a through-crack of length $2c$ in the plate/shell with a continuously distributed stiffness connecting the two sides of fictitious through crack (Figure 4.3). The generalised forces and displacement across the two sides are stress resultants and work conjugate displacements at midplane of the shell. The distributed stiffness depends on local crack depth $a(x)$, and relates generalised membrane forces and bending moments to cracked displacements and rotation in a plane strain single edge-notched plate of thickness $t$ and crack depth $a$ (SENP) (Figure 4.4).

2. When the distributed stiffness is used, the solution of combined shell/spring model leads to the displacement and force distribution. The crack front stress intensity at position $x$ ($K_I(x), J(x)$) is estimated as the same value which would occur in the plane strain edge-cracked plate (SENP) when subjected to the combined load histories $\{N(x), M(x)\}$.

The basic building block of LSM is then the stiffness and $J$-response of SENP subjected to simultaneous bending and tension. For linear elastic material, the stiffness and $J$-
calibration of the SENP is obtained \([22,32]\) from \(K_i\) calibrations such as those of Tada, et al \([36]\) by using the method of Rice \([37]\). That is,

\[
\begin{bmatrix} N \\ M \end{bmatrix} = [S] \begin{bmatrix} \delta \end{bmatrix}
\]

\[
\begin{bmatrix} N \\ M \end{bmatrix} = Q \quad \text{and} \quad \begin{bmatrix} \delta \end{bmatrix} = a
\]

Where \(N\) = Axial force per unit width

\(M\) = Moment per unit width

\(\delta\) = Displacement between two faces at midplane

\(\Theta\) = Angular displacement between two faces at midplane

For elastic-plastic response, we need rate moduli \(S_{ij}\) connecting increments

\[Q = S \cdot q\]

We assume \([22]\) that displacement increments are decomposed into as elastic and plastic part

\[q = q^e + q^p\]

We follow standard flow plasticity theory. Yield surface \(\phi(Q,a,t,\infty)=0\) is defined in the generalised force space, wherein \(\infty\) is meant to be an average flow stress. Particular form of the function \(\phi\) are given by Rice \([19]\) and in details by others \([23,25]\). In the elastic region of the stress space \(\phi < 0\), the rate response is linear so that \(S = S^e\). Similarly if the stress state is on the yield surface \(\phi = 0\) and if it is unloading (either \(q_j(\delta\phi/\delta Q_j) \leq 0\)) then the rate modulii are elastic. Here \(\delta\phi/\delta Q_j\) is the outward normal to the yield surface.

If the generalised stress state is on the yield surface and \(q_j(\delta\phi/\delta Q_j) > 0\), for the active plastic loading phase the displacement increment is taken as

\[q^p_j = \Lambda \phi, j\]
where $\phi_j$ is normal to surface $\phi$ and nonnegative scalar $\lambda$ is evaluated by imposing the "consistency condition", that, during flow, the stress state remains on the yield surface $\phi = 0$. In the rate form

$$\dot{\phi} = 0 = \dot{\phi}_j Q_i + (\partial \phi / \partial \sigma_o) \dot{\sigma}_o$$

(4)

$Q_j$ is now written in terms of elastic stiffness and $\dot{q}_k$ as

$$Q_j = S_{jk} q_k = S_{jk} (q_k - q_k^p)$$

(5a)

$$= S_{jk} q_k - \lambda S_{kj} \phi_{i_k}$$

(5b)

Where eqn. 5(b) follows from eqn. 5(a) and (3). The rate of change of $\dot{\sigma}_o$ is related to the rate of change of $\dot{\sigma}_o^p$ through a plastic material modulus $h$ which may vary with the level of plastic deformation. In ABAQUS implementation $h$ can be specified in discrete steps such that

$$\dot{\sigma}_o = h \dot{\sigma}_o^p$$

(6)

The total plastic work rate $\dot{W}_p$ per unit thickness of the edge-cracked plate can be expressed in two ways. At the macroscopic level, it is the work rate of generalised forces through plastic generalised displacement rates.

$$\dot{W}_p = Q_i \dot{q}_i = \lambda Q_i \phi_{i_k}$$

(7)

The $\dot{W}_p$ is also the integral of continuum plastic work rates over the area $A^*$ of the edge cracked specimen where plastic dissipation is occuring

$$\dot{W}_p = \int_{A^*} \sigma_{ij} \dot{\epsilon}_{ij}^p dA = \int_{A^*} \sigma \dot{\epsilon}_{ij}^p dA$$

(8)

where the quantities $\sigma$ and $\dot{\epsilon}_{ij}^p$ are local values of tensile equivalent stress and plastic strain rate in the section $A^*$. 

12
If it is assumed that the area $A^*$ is restricted to remaining ligament of length $c=t-a$ then one would expect that the $A^*$ will be proportional to $C^2$. Thus an approximate evaluation of the second integral of eq. (8) can be made in terms of $\sigma_0$ and $\varepsilon_0^p$ as

$$ W^p = f * \sigma_0 * \varepsilon_0^p * C^2 $$

(9)

Where dimensionless scalar $f$ is expected to be of the order unity.

Combining eqs. (4) through (9) there results

$$ A = \left( \phi_s S_{i j} q_j \right) / \left( \phi_m S_{i j} q_j (-\delta \phi / \partial \sigma_0) \right) \left[ n Q_k \phi_k / (f \sigma_0 C^2) \right] $$

(10a)

$$ = n_i q_j / D $$

(10b)

with the scalar $D$ represents denominator of eq. (10a) and $n_{i j}$ is the plastic normal multiplied by elastic stiffness. With $A$ thus determined, the rate moduli for active plastic straining are given by

$$ S_{i j} = S_{i j}^e - (1/D)n_i n_j $$

The value of the dimensionless scalar $f$ has been adjusted by comparing results of a single spring with continuum solution of the SEN of corresponding dimension, material properties and load histories. White et.al.[24] noted that for crack depth $a/t = 0.5$ subjected to bending loads and fully plastic range, a reasonable value of $f = 0.4$. Moreover, this value correlated well for predominantly bending loads and Parks found that this value holds good for pure tension loading also. In order to obtain an estimate of $J$-integral in SEN plate, $J$ is expressed as sum of an elastic and plastic part:

$$ J = J^e + J^p $$

$J^e$ is related to current loads $Q_i$ and crack length.
stress intensity factor through $K$ produced by loads according to

$$J^* = K^2 (1 - v^2)/E$$  \hspace{1cm} (12)$$

Where $K = Q_k \kappa_{i} (a, t)$

The stress intensity factors $k_i$ (i=1: tension, i=2: bending) can be obtained from reference [36].

An evolution law for plastic contribution $J^P$ is expressed in terms of plastic crack tip opening displacement

$$J^P = m \alpha \delta^P$$

$m$ being scalar, expected to depend on the deformation pattern in the plastic regime.

Fig.4.5 shows loci of yield surface $\phi = 0$ for various crack depths. The average shear stress $\tau_0$ is related to $\sigma_o$ according to Mises criterion $\tau_0 = \sigma_o/(3)^{1/2}$. The yield loci shown in Fig.4.5 are generated from the family of ideally plastic upper-bound solutions shown in Fig.4.5. This approach was originally proposed by Rice [19] who provided an explicit approximate form for the yield surface under predominantly tensile load

$$\phi = (((N/2)T_o c - 0.3)/0.7)^2 + 9(M + N (t-c)/Z)/(2 \tau_o c^2)^2 - 1$$ \hspace{1cm} (15)$$

Fig.4.6, the kinematics of the assumed flow field provides a relation between the macroscopic displacement increments $\delta_i$ and $\delta^P$

$$\delta_i = \delta_i^P + (t/2 - a) \phi_i^P$$

$$= \Lambda [\phi_i + (t/2 - a) \phi_i]$$ \hspace{1cm} (16)$$

Under ideally plastic conditions, the scalar $m$ is given by

$$m = \phi_i a /[\alpha_0 (\phi_i + (t/2 - a) \phi_i)]$$
This completes the approximate J analysis of edge-cracked plate, as proposed by Rice and further developed by Parks. In ABAQUS implementation while scalar f has been retained as 0.4 the yield surface definition is slightly different. Yield surface is defined w.r.t. generalised stress variables $N_i$ and $M_i$ as defined by Rice

\[ X = (3)^{\frac{5}{2}} \frac{N_i}{\sigma_o(t-a)} \]

\[ Y = \left( (3)^{\frac{5}{2}} \frac{M_i + (t/2) N_i}{\sigma_o(t-a)^{\frac{3}{2}}} \right) \]

Then for $X \leq 2Y$ the yield function $\phi$ is taken as an envelope to limit analysis results as proposed by Rice

\[ \phi = (X - 0.3)^2 + 4.41 (Y - X/2) - 0.49 = 0 \]

otherwise we use

\[ \phi = \alpha_1 X + \alpha_2 Y + \alpha_3 X^3 Y + \alpha_4 X^5 Y^3 - 1 = 0 \]

with

\[ \alpha_1 = 2.264568665 \]
\[ \alpha_2 = 2.79974917 \]
\[ \alpha_3 = -0.99731078 \]
\[ \alpha_4 = 5.0627466 \]

This surface is chosen to blend continuously with $\phi_4$ at $X = 2Y$ and as a reasonable estimate of the behaviour for $X > 0, Y < 2X$. Rice pointed out that at $X = 1, Y = 0.5$ the yield surface will have a vertex. ABAQUS, however, adopts the smooth surface shown in Fig.4.7 for numerical stability. This restricts the possible flow behaviour at $(X=1, Y=0.5)$; however it is assumed that this is not a critical issue. While using LSM in ABAQUS a limitation that arises is due to closing of the crack. ABAQUS issues warning when such a case arises, and stops computation.

5.0 ANALYSIS OF PHT PIPING;
5.1 INTRODUCTION:

The heat generated in the core of PHWR is removed by
Primary Heat Transport (PHT) system. The heavy water coolant is circulated through the core using centrifugal pumps to extract the nuclear heat from the fuel bundles. The water from the pump is directed to inlet headers from where it enters inlet feeders. The coolant after passing through channel is collected in outlet header. The hot coolant then passes through the steam generator, where secondary heat transport system absorbs the heat of PHT system. This coolant then goes back to inlet header. The inlet header distributes the flow by inlet feeders.

The PHT circuit functions as a barrier for release of coolant radioactivity. This is made possible in PHWR by having two separate circuits. The secondary system takes away heat from PHT in steam generator without getting mixed. Therefore it is clear that structural integrity of PHT system is very essential to avoid release of radioactivity to secondary system and in case of failure to surrounding atmosphere. In the PHT system a large volume of piping is involved. Hence PHT piping is a major concern to qualify the system and maintain the safety. If the pipe has any crack at any location, it may grow with time depending upon the applied loading and water chemistry. The crack may also get initiated and grow due to corrosion if pH and oxygen level in coolant are not controlled. The crack growth stability will depend on loading, crack shape, size and material resistance capability. The answers to all these questions is only possible if detail fracture mechanics analysis of piping is carried out.

5.2 GEOMETRY, MATERIAL PROPERTIES AND LOADING:

The cracks in pipe may be orientated in different directions. For a thick pipe subjected to internal pressure the maximum tensile stress occurs at internal surface and is in the hoop direction. When the pipe is subjected to bending as a result of inertial forces during earthquake loads or bending forces due to thermal distortion, the maximum stresses occur in axial direction at outer radius. In the present work, internal pressure is the only load under consideration. Two extreme possible directions for crack are axial and circumferential. In the
present work, an internal axial surface crack in straight portion of pipe is considered. The PHT pump inlet pipe has outer diameter of 610mm, which is higher than PHT pump outlet pipe and hot leg. So it is decided to consider the PHT pump inlet pipe dimensions for fracture mechanics analysis. Figure 5.1 shows the geometrical details of PHT pump inlet pipe with assumed internal axial surface crack. The material of PHT pump inlet pipe is SA-333 Grade 6. The yield strength and ultimate strength of this material is 18.58 Kg/mm$^2$ and 40.84 Kg/mm$^2$ respectively\cite{39,43}. Its modulus of elasticity is 18318 Kg/mm$^2$.

As per ASME code for qualifying the component, semi-elliptical surface crack with size as $a/t = 0.25$ and $2c/a = 6$ should be considered. The particular size is also considered in the current analysis to meet the ASME requirement. Due to fatigue loading or transient loadings the crack may extend. The bigger sizes of cracks are also considered to assess the safety margin available with grown up cracks such that available ligament thickness of pipe is sufficient to avoid yielding as per membrane formula.

5.3 FINITE ELEMENT MODELLING OF PIPE WITH CRACK;

Finite element modelling of PHT pump inlet straight pipe with one axial internal surface crack is carried out. This geometry has symmetry about two different planes. First symmetricity plane is perpendicular to the axis of the pipe and cutting the pipe at centre of the axis of crack. Second symmetric plane is passing through the axis of pipe and crack front length. Due to symmetry, one-fourth of the pipe is modelled. Eight noded thick shell elements are used for modelling the pipe. Fine mesh is taken near the crack while away from the crack location, the mesh is coarse. Multipoint constraint facility of ABAQUS is utilised at interface of smaller and bigger elements. Using this option of ABAQUS it was possible to increase the density of elements in region near the crack tip. Model is checked by plotting.
The axial internal surface crack is modelled using line spring elements. Near the crack end, the smaller line spring elements are employed. LS3S line spring elements are used for modelling. These are symmetric line spring elements having 3 nodes per element and have utility at location of symmetric plane only. These elements take care of symmetric structure existing at other side of symmetricity plane.

For analyses with changing size of crack front length, the model is modified for different cases. The modification is carried out in such a way that aspect ratio of different elements should be within limit. Figure 5.2 shows, one of the different finite element models of PHT pump inlet pipe. Zoomed finite element mesh near the crack is shown in Figure 5.3. Line spring elements are shown by dotted lines. Appropriate symmetric boundary conditions are used at two symmetric planes. The length of pipe for modelling is taken 14 times the length of crack front in different models. For different cases of axial crack, the crack depth along the crack front is calculated depending upon the shape and size of crack. A computer program was used to generate the crack depth at different points along the crack for elliptical and circular axial crack. Different shapes and sizes of cracks are considered while analysing the pipe. Figure 5.4 and 5.5 show the different shapes of cracks considered for the analysis.

6.0 COMPUTATIONAL RESULTS FOR ELLIPTICAL CRACK:
6.1 INTRODUCTION:

In this chapter we discuss the computational results obtained by application of line spring element to the finite element discretization of PHT pipe. The results obtained are verified by comparing the results with those obtained by Parks[26]. The variation of J-integral with loading, the distribution of J-integral across the crack front, the variation of crack mouth opening displacement with loading as well as its distribution across crack length has been studied. The influence of crack geometry factors such as aspect ratio, depth ratio and shape on Peak J-integral values as well as their distribution.
across the crack front has been presented and the crack behaviour explained on physical grounds.

6.2 THE J-INTEGRAL VALUES AND THEIR DISTRIBUTION FOR VARYING INTERNAL PRESSURE LOADING FOR ELLIPTICAL CRACKS:

To verify the validity of subsequent results, the J-integral values, crack mouth opening displacement and their distribution across crack front has been obtained. The peak J-values are compared with those obtained by Parks [26] in Figure 6.1a for depth ratio 0.5 at centre of crack front. The agreement appears to be quite close. The comparison for J-integral values along the crack front is carried out for external axial surface crack with that of a 3-D (with solid elements) elasto-plastic finite element results of Brocks and Kunecke[42]. The Figure 6.1b shows that the two results are having satisfactory agreement. The present line spring model results are approximately 20 percent higher at centre of crack and for most of central portion of crack. Also the peak J-integral values are found to be at centre only. So it can be said that line spring results are conservative. But at the ends, the 3-D solid element result shows finite J-integral values while LSM results are zero. The differences are at the ends of the crack where the J-values decrease and thus are less critical. The LSM is known to be poor in accuracy at crack ends[24,25,26].

At the end of the crack where crack front intersects with the traction free surface, there exists an elastic singularity. This problem has been studied by a number of investigations[44-46]. For mode I loading of a crack making normal incidence with a free surface, it appears, that the order of the corner singularity is generally weaker than square root singularity. Thus a strict view of $K_1$ as the strength of the square-root singularity indicates that at such points $K_1$ should vanish in a thin boundary layer. The zone of the dominance of this singularity or boundary layer thickness has been estimated to be approximately two to three percent of thickness by Nakamura and Parks [47]. The estimated region of dominance was
found to be only \((a/t)_c (a/c)_c (t/33)\). The validity of this conjecture awaits sufficient refined analysis. It may be noted that general finite element models will miss this boundary layer. The LSM predicts zero values of \(J\) at crack end. As LSM concept is comparable to three dimensional finite element models, it should predict non-zero finite values at end, though theoretically \(J\) at end should vanish in this layer.

The shallow cracks are simulated by \(a/t=0.25\) while average crack is simulated by \(a/t=0.5\). For the deep cracks, the authors decided to use \(a/t\) ratio =0.75 throughout this study as generally cracks deeper than this ratio tend to be unstable. It can be seen from Figure 6.1a that the stress intensity at the crack tip slowly increases linearly up to normalised pressure \((P =pr/\sigma t)\) unity for shallow cracks. For deeper crack, however, the behaviour becomes nonlinear even for lower pressure. After yielding commences, which can be seen by a characteristic change in the slope of the curves, SIF curve takes distinct transition point at knee. The \(J\)-values increase rapidly with pressure. The rate of increase depends on crack depth ratio. While \(J\)-integral increases gradually after yielding for shallow cracks, the rate becomes very large for deeper cracks. Thus range of pressure which cracks can withstand before initiation reduces for deeper cracks.

The distribution of crack driving force across the crack front for a shallow crack can be seen in Figure 6.2. The distribution is shown for various values of pressure loading. It can be seen that the crack driving force is distributed uniformly across the front for low values of pressure 30 degree away from the edge of crack. As soon as the pressure exceeds the one required for yielding to develop near the crack, the \(J\)-distribution changes significantly. The maximum value is attained in the centre of crack where depth is maximum; however, the spread of the peak value is appreciably larger when pressure exceeds yield value. Moreover, the change in the magnitude is large indicating that, if \(J\)-values were to exceed the material toughness, the crack would grow across a large section of front.
The opening through the wall would be for a large area rather than a small opening widening gradually. These observations remain applicable to deep crack (Fig. 6.3) also. For deep crack, the opening area will be larger if applied $J$-value exceeds the material toughness. Further, the slope of the $J$-distribution curve is sharp. Thus, the spread of peak values is wider making instability spread large enough across the front.

6.3 CRACK MOUTH OPENING DISPLACEMENT FOR VARYING PRESSURE:

The crack mouth opening displacement is an important input to the calculation of area available for fluid leakage, should the crack become through-wall. Further, the CMOD becomes an important mechanical parameter for measurement of crack extension in experiments. The distribution of crack mouth opening displacement (CMOD) for elliptical shallow and deep crack are shown in Figures 6.4 and 6.5 respectively. The normalized CMOD for these cracks are shown in Figures 6.6, 6.7 respectively.

The growth of crack driving force with pressure and sudden growth of CMOD near yield pressure can be seen in Figure 6.4. However, the change from small opening near the end of crack to large opening in the centre of crack is gradual in the case of shallow crack while in case of deep crack the opening spreads almost to the end. Deep cracks are thus likely to open wider when they become through-wall cracks.

The crack mouth opening, when plotted normalized with reference to crack driving force, shows an interesting feature of the result. For both shallow (Figure 6.6) and deep (Figure 6.7) cracks the normalized CMOD decreases in the region near centre as the pressure increases. This indicates that in the central region crack driving force $J$ increase for more rapidly due to yielding than crack mouth opening displacement. For shallow crack there are two distinct zones in which the CMOD falls. In the region near the end (approximately $\phi = 0$ to 25 degree) the normalized CMOD increases with pressure indicating that the actual CMOD increases more rapidly than $J$. This is the
result of natural selection of the fracture process to open the crack where it is easiest. The vulnerability of the crack is maximum in the centre for shallow cracks. For deep cracks however, the point of peak CMOD shifts (Figure 6.7) away from centre. Thus crack driving force is more effective at point slightly away from centre.

6.4 INFLUENCE OF CRACK DEPTH:

In the earlier section we discussed the results as affected by depth of crack, either deep or shallow. In order to systematically understand the depth influence, Figure 6.8 and 6.9 show the variation of CMOD and normalized CMOD for various pressures. The CMOD curve becomes nonlinear (increases rapidly) as pressure increases beyond yield pressure. The pressure at which nonlinearity sets in reduces as depth of crack increases. Moreover the growth rate of opening increases with depth. As pressure increases, normalized CMOD reduces and near yield pressure, a transition point is seen. Though curves for different depth are distinct before yielding, however they almost coalesce after yielding. The deep crack curve keeps reducing more rapidly than shallow crack curve. This indicates that growth rate of J-integral exceeds that of CMOD. These observations have been brought out with more clarity in Figure 6.10 and 6.11 which show variation of CMOD with depth ratio. The growth rate of CMOD with depth increases very rapidly with pressure as seen from Figure 6.8. Figure 6.10 depicts an interesting feature. Beyond depth ratio 0.6, there is a general reduction in normalized CMOD. Each curve peaks at a certain depth ratio. The peak of the curve changes with pressure and for higher pressure the peak point increases towards higher depth ratios. The reduction in normalized CMOD with increasing pressure is much more obvious in Figure 6.11. In the Figure 6.11, we have already seen variation of J with three discrete crack depths. The variation of peak J-integral at centre of crack with smoothly varying crack depth can be seen in Figure 6.12. For low pressure, the peak J-integral value is insensitive to the depth. As pressure increases, however, the influence of depth becomes significant. The rapid
changes occur after the yield pressure. The effectiveness of pressure loading gets increased by higher crack depth. This indicates that if a crack grows it is likely to become unstable. This Figure is also termed 'applied J' curve. The fracture toughness of the material can be plotted as a J-R curve on this figure to find regions of unstable and stable crack growth. To generate this curve several analyses with different crack depth ratios have to be carried out. These curves form an important starting point in fracture mechanics analysis of a structure. The application of this result to PHT pump inlet pipe is discussed later.

6.5 THE INFLUENCE OF ASPECT RATIO:

It is generally believed that having large aspect ratio cracks simulate plane strain behaviour. The important question that needs to be answered is how long a crack should be to be treated under plane strain behaviour. ASME code recommended an aspect ratio $2c/a = 6$ for analysis of structural integrity of pressure vessels. This ratio is important as can be seen from the result plotted in Figures 6.13 and 6.14 for shallow and deep crack respectively. For the case of shallow cracks, aspect ratio is almost inconsequential as operating pressure is generally lower than yield pressure. At higher pressure (which might arise due to certain abnormal conditions) the aspect ratio of crack appears to have strong influence on normalized values. For deep cracks, however, the differences appear right at low pressures (Figure 6.14) and they widen as pressures increase. The aspect ratio appears to be more important for deep crack ($a/t=0.75$) as compared to a shallow crack. At high pressures, for the constant crack depth longer cracks show more driving force (J-integral value) as compared to shorter cracks.

7.0 STUDIES ON VARIOUS CRACK SHAPES:

7.1 INTRODUCTION:

In realistic situation the surface cracks will have irregular geometry. They need not be even plane. Fully three
Dimensional cracks are not easily amenable to any theoretical treatment. However, for design codes, semi-elliptic plane crack is the universal assumption. For this shape, the depth is smoothly varying and the crack front ends normal to the surface. The question arises regarding the sensitivity of the fracture mechanics results for non-elliptic shape. If one generates a semi-elliptic shape by least square fit to the actual geometry, the influence of local spikes extending beyond the smoothly varying depth remains to be investigated. Authors carried out few studies with various crack geometries and the investigated above uncertainties. Triangular, circular shape with various radii of curvature and rectangular geometries form the group of candidate shapes.

7.2 Influence of Crack Shape on Crack Driving Force:

Normalized J-integral values at centre of shallow crack in Figure 7.1 suggest that triangular crack produces least crack driving force while rectangular crack, which is limiting case, is most damaging. It is interesting to note that the difference due to shape are negligible during elastic deformation. They become significant only after yielding commences. This implies that the shape will have strong influence on crack growth and final crack shape when crack breaks out or gets arrested due to release of loading. The distribution of crack driving force across the crack front and peak value obtained is a characteristic signature of each crack shape. The circular crack result lie close to those for triangular, while elliptical crack results lie close to rectangular crack. Approximating a crack by a semi-elliptical geometry is not most conservative approach. However, conservatism is only slightly conceded by elliptic approximation.

J-integral results for deep internal cracks are presented in Figure 7.2 bring out the differences between shapes more strikingly. In the elasto-plastic deformation zone, the crack shape identity is revealed with more clarity. In what follows, we discuss each crack shape separately for ease of understanding.
7.3 CIRCULAR CRACK: EFFECT OF RADIUS OF CURVATURE:

The effect of radius of curvature on the circular crack geometry can be seen in Figure 7.3 and 7.4. The crack yields higher crack driving force $J$ as its radius is increased. For large radius, the results approach those for rectangular crack as they should. The differences in the result appear only in elasto-plastic deformation zone. The deep crack shows the predominant influence of radius of curvature. This can be explained by noting that the depth of the crack at its end rapidly increases with increased radius of curvature (Figure 5.3) and crack approaches a rectangular shape. The rate of increase of depth at end of the crack depends on the crack depth at centre.

The change in the distribution of normalized crack driving force $J$ across the crack front as influenced by radius of curvature is depicted in Figures 7.5 and 7.6 for shallow and deep crack respectively. The normalization of crack front coordinate is w.r.t half crack length. It can be seen that the $J$-value at centre increases with radius of curvature; the rate of growth depends on crack depth ratio. The smooth peaked distribution of $J$ becomes more and more flat as radius of curvature increases. For deep crack, the distribution in the centre is nearly flat and increase in the radius of curvature of crack makes the $J$-distribution more uniform across the crack front. The radius of curvature influence is more pronounced for deep crack as noticed earlier.

7.4 RECTANGULAR AND TRIANGULAR CRACKS:

The progress of crack driving force distribution with loading for rectangular crack is depicted in Figure 7.7 (for shallow crack) and 7.8 (for deep crack). The deep crack is seen to become more sensitive in comparison with shallow crack after pressure increase beyond yield pressure. The curves are not smooth at crack end because line spring approximation accuracy deteriorates in this region.
The above results for triangular crack (Figure 7.9 and 7.10) show an interesting feature. The feature is more prominently seen for deep crack (Figure 7.10) as compared to shallow crack (Figure 7.9). The crack driving force increases with increased pressure, but the peak does not occur at centre of the crack where depth is maximum. After the pressure exceeds yield pressure, the peak force occurs at approximately $x/c = 0.5$ and the peak point starts moving towards the centre gradually. The $J$ at centre is approximately 20 percent less than that at peak point for normalized pressure 1.176. This indicates that growth of crack will not occur at centre but away from centre, say at $x/c = 0.55$. This can be explained by following arguments.

1. For a crack growth involving area $A$ the energy release is given by $J dA$, $J$ being maximum at $x/c = 0.55$ the maximum energy release for a given growth will occur there.

2. The growth area $A$ for a given crack advance at a given crack position along front is maximum not at the centre but away from the centre. As crack advance has to be normal to the front and normal is not uniquely defined at centre of triangular crack, the $J$-integral value reduces there.

3. The crack advance has to be such that maximum energy release must occur for a given advance area. The driving potential $J$ attains maximum value away from the centre due to nonuniqueness of normal at centre. Should the crack geometry become smooth due to blunting at centre the peak point will start shifting towards centre. The crack growth process thus tries to make a triangular crack elliptic or circular.

8.0 APPLICATION TO PHT PIPING:

8.1 SCOPE OF ASSESSMENT VALIDITY:

In this section we attempt application of computed results to the case of PHT piping. For this purpose, we need material properties for the piping material. The range of
The applicability of fracture mechanics methods is restricted by limits of transferability of fracture mechanics material laws. The crack initiation value \( J_i \) determined on the basis of "Streched zone" evaluation can be considered as a transferable material characteristic\(^{[35]}\). Beyond crack initiation the crack resistance curve depends on the geometry as well as specimen dimensions and are influenced mainly by the gradient of triaxility of stress state across the ligament\(^{[38]}\). It is possible to make an assessment of the transferability of crack—resistance curve of the specimen to the component by comparing the triaxiality of stress state across the ligament of component and specimen\(^{[8]}\). As the triaxiality result is not possible to obtain in the LSM approach, the predictions on the crack growth using material crack—resistance curve can not be made reliable. The triaxiality results require that full three dimensional model be generated for crack geometry. As this is very expensive computationally, the limitation on the crack growth predictions can be considered as the price to be paid for computational economy of LSM approach. In what follows, therefore, we will attempt prediction of crack initiation only. Another limitation of the present work arises from the limitation posed by LSM methodology. The transient thermal stresses cannot be considered. The LSM implementation in ABAQUS does not permit secondary thermal stresses. However, the material properties at operating temperature of 300°C have been used for calculations\(^{[39]}\). The pipe is assumed to be stress free at operating temperature and only internal pressure is considered to be the main load.

8.2 INTEGRITY ASSESSMENT

The crack driving force due to the internal pressure for various crack sizes has been plotted in Figure 8.2 for PHT pipe. This figure is obtained by conversion of Figure 6.12 to dimensional parameters. Similarly, crack mouth opening displacement is shown in Figure 8.3. In Figure 8.2 we also plot the material Jin property, initiation J-value, given as 152 KJ/m\(^2\) for 300°C temperature\(^{[39]}\). The design pressure (ignoring
transients) for the PHT piping is 1.26 Kg/mm$^2$. As per ASME, to qualify the component, the crack depth to thickness ratio of 0.25 (and $2c/a = 6$) should be considered. From Figure 8.2 it can be seen that for crack $a/t$ ratio of 0.25 (crack depth=12.5mm) and $2c/a = 6$ with design pressure of 1.26 Kg/mm$^2$, the computed peak $J$-integral value ($J_{app}$) is less than 5 KJ/m$^2$. The ratio of $J_{in}$ to $J_{app}$ is more than 30. Therefore it can be said that quite a large safety margin is available in PHT piping with axial internal surface crack due to internal pressure. In other words, it can be stated that axial internal surface crack with $a/t=0.25$ and $2c/a=6$ has no scope of extending due to internal pressure loading. From Figure 8.2 it can be seen that 23mm deep crack requires 4.0 Kg/mm$^2$ pressure for initiation. Similarly 31mm deep crack requires 3.8 Kg/mm$^2$ pressure for extension. If the crack depth is more than 31 mm then the ligament thickness will not be sufficient to withstand the design pressure of 1.26 Kg/mm$^2$ as per membrane stress formula and will start yielding.

An interesting observation that can be made from Figure 8.3 is that for the cases considered earlier, when the cracks initiate, the crack mouth opening displacements are less than one mm. A 31 mm deep crack ($a/t=0.62$) is hardly 0.4 mm across its mouth when it starts growing, at 3.4 Kg/mm$^2$ pressure. On the other hand, 23 mm deep ($a/t=0.46$) crack shows a mouth opening of 0.7 mm at 4.0 Kg/mm$^2$ pressure. It is important to see that deep cracks which grow at lower pressure do not exhibit large mouth opening. Thus limitation of visual examination to detect growing cracks can be understood. ASME design procedure requires crack with $a/t=0.25$ for evaluation. As 23 mm deep crack considered herein has $a/t$ more than 0.25, it can be seen that for surface crack with $a/t=0.25$ requires very high pressure for extension.

9.0 CONCLUSIONS:

9.1 INTRODUCTION:

This work has attempted the assessment of line spring model (LSM) technique for structural integrity evaluation of a axial surface cracked pipe under internal pressure. Transient
loading, especial thermal transients are not considered in the present study. Using 3-D solid elements and elasto-plastic deformation takes computer time which is approximately 150-200 times large than LSM technique.

After reviewing the previous research, the validity, complexity and applicability of the technique is discussed. The technique is applied for PHT pipe under pressure. The results are obtained in dimensionless form for wider applicability.

The variation of peak crack driving force, represented by J-integral, and its distribution across the front is studied for a variety of parameter such as loading, crack depth and crack geometry. The crack shape parameter such as aspect ratio, shape (circular/rectangular/elliptical/triangular) are studied. Following specific conclusions are drawn.

9.2 CONCLUSIONS:

There is a marked increase in crack driving force after yielding commences near the cracktip. The rate of increase depends on ratio of crack depth to wall thickness. Deeper crack tend to open out wide since peak crack driving force. Crack mouth opening displacement (CMOD) grows slower than driving force along most of the crack front. CMOD grows more rapidly as compared to driving force near crack end. Thus with increased loading crack tries to grow deeper at point of maximum depth, while at its ends it elongates.

The aspect ratio of cracks is not very important for shallow cracks. For deeper cracks, however, aspect ratio decides the criticality, long crack tending to be more critical. As aspect ratio reduces, the peak J point shifts towards end. For sufficiently small aspect ratio, it is known that crack grows along length rather than depth.

The crack shape has strong influence only if elasto-plastic deformation takes place. The distribution of peak J and
peak value itself is a characteristic signature of crack shape.

The application of results, to PHT pipe as an example, leads to the conclusion that up to 200 mm long crack with 50 % depth will not grow unless pressure is at least three times of design operating pressure. This builds up the confidence in the statement that pipe will leak before break.

The line spring model appears to be very cost effective for part-through crack analysis tool.

REFERENCES:


Figure 4.1 Surface crack notation and reference system

$\vec{n}$ - normal vector to the surface
$\vec{q}$ - displacement vector

$A$ - point on the crack
$B$ - point on the crack
$n$ - angle on an inscribed circle for locating a point on the crack

Figure 4.2 Elliptical surface crack geometry

$a = \text{maximum flaw depth}$

$2c = \text{surface length of crack}$

$t = \text{shell thickness}$

$\phi = \text{angle on an inscribed circle for locating a point on the crack}$
Fig. 4.3 Line spring distribution

Fig. 4.4 Line spring stiffness from single edge cracked plate (SECP)
Figure 4.5 Yield surfaces $\phi = 0$ used in line-spring calculations
Figure 4.6 Upper-bound velocity field used to generate crack yield surfaces of Figure 4.5 indicating the kinematics of plastic crack tip opening displacement increment in terms of load point plastic generalized displacement increment.

\[ \dot{\delta}^p = \dot{\delta}^p + (t/2 - a) \dot{\delta}^p \]

Figure 4.7 Generalized stress yield surface assumed for line springs.

\[ Y = \frac{\sqrt{3}}{2} \sigma_s \left[ (M + N_t) \left( \frac{t-a}{2} \right) \right] \]

Region where crack opening does not occur

Surface obtained from smooth continuation assumption

Rice's [1972] envelope limit analysis solutions
Figure 5.1 Geometry of PHT pipe with assumed crack

Figure 5.2 Finite element model of pipe with an axial surface crack.

Figure 5.3 Zoomed finite element mesh near surface crack
Figure 5.4 Different shapes of surface crack considered

Figure 5.5 Varying radius of curvature of circular crack
Fig. 6.1a Crack driving force \( \frac{EJ}{K^2t} \) versus pressure \( \frac{pR}{\sigma^2 t} \) for internal axial surface crack (elliptical crack, \( 2c/a = 6 \)).

Fig. 6.1b Crack driving force along the crack front for external axial surface crack as compared with 3-D results of Ref. (42). (\( \phi \) as per Figure 4.2)
Fig. 6.2 Crack driving force \((EJ/\sigma^2)\) along the crack front for shallow internal axial surface crack (elliptical crack, \(a/t=0.25, 2c/a=6\)).

(\(\theta\) as per Figure 4.2)

Fig. 6.3 Crack driving force \((EJ/\sigma^2)\) along the crack front for deep internal axial surface crack (elliptical crack, \(a/t=0.75, 2c/a=6\)).

(\(\theta\) as per Figure 4.2)
Fig. 6.4 Crack opening displacement along the crack front for shallow internal axial surface crack (elliptical crack, $a/t=0.25$, $2c/a=6$). ($\theta$ as per Figure 4.2)

Fig. 6.5 Crack opening displacement along the crack front for deep internal axial surface crack (elliptical crack, $a/t=0.75$, $2c/a=6$). ($\theta$ as per Figure 4.2)
Fig. 6.6 Crack Opening Displacement ($\delta = \sigma / J$) along the crack front for shallow internal axial surface crack (elliptical crack, $a/t=0.25$, $2c/a=6$). ($\theta$ as per Figure 4.2)

Fig. 6.7 Crack Opening displacement ($\delta = \sigma / J$) along the crack front for deep internal axial surface crack (elliptical crack, $a/t=0.75$, $2c/a=6$). ($\theta$ as per Figure 4.2)
Fig. 6.8 Crack opening displacement versus applied pressure ($pR/\sigma_3 t$) for internal axial surface crack (elliptical crack, $2c/a=6$).

Fig. 6.9 Crack opening displacement ($\delta_{\alpha_3}/J$) versus pressure ($pR/\sigma_3 t$) for internal axial surface crack (elliptical crack, $2c/a=6$).
Fig. 6.10 Crack opening displacement versus crack depth for internal axial surface crack (elliptical crack, $2c/a=6$).

Fig. 6.11 Crack opening displacement ($\delta x/\delta y$) versus crack depth for internal axial surface crack (elliptical crack, $2c/a=6$).
Curve \( pR/(\sigma y^2) \), \( \phi=90^\circ \), Ref. Fig. 4.2

1. 0.8232
2. 0.9996
3. 1.1172
4. 1.176

Fig. 6.12 Crack driving force \( (EJ/\sigma y^2) \) versus crack depth for internal axial surface crack (elliptical crack, \( 2c/a=6 \)).
Fig. 6.13 Crack driving force \( (EJ/\sigma_S^2t) \) versus pressure \( (pR/\sigma_S t) \) for shallow internal axial surface crack (elliptical crack, \( a/t=0.25 \)).

Fig. 6.14 Crack driving force \( (EJ/\sigma_S^2t) \) versus pressure \( (pR/\sigma_S t) \) for deep internal axial surface crack (elliptical crack, \( a/t=0.75 \)).
Fig. 7.1 Crack driving force \( (EJ/\sigma_{Y}^2t) \) at centre of crack versus pressure \( (pR/\sigma_{Y}^3t) \) for different shapes of shallow internal axial surface crack \( (a/t=0.25, 2c/a=6) \).

Fig. 7.2 Crack driving force \( (EJ/\sigma_{Y}^2t) \) at centre of crack versus pressure \( (pR/\sigma_{Y}^3t) \) for different shapes of deep internal axial surface crack \( (a/t=0.75, 2c/a=6) \).
Fig. 7.3 Crack driving force ($\frac{EJ}{\sigma_0^2 t}$) at centre of crack versus pressure ($pR/\sigma_0^2 t$) for different radius of curvature of circular shallow internal axial surface crack ($a/t=0.25, 2c/a=6$).

Fig. 7.4 Crack driving force ($\frac{EJ}{\sigma_0^2 t}$) at centre of crack versus pressure ($pR/\sigma_0^2 t$) for different radius of curvature of circular deep internal axial surface crack ($a/t=0.75, 2c/a=6$).
Fig. 7.5 Crack driving force ($\frac{EJ}{\sigma_0^2}$) along the crack front for different radius of curvature of circular shallow internal axial surface crack ($pR/\sigma_0^2=1.176$, $a/t=0.25$, $2c/a=6$). ($x/c$ as per Figure 5.4)

Fig. 7.6 Crack driving force ($\frac{EJ}{\sigma_0^2}$) along the crack front for different radius of curvature of circular deep internal axial surface crack ($pR/\sigma_0^2=1.176$, $a/t=0.75$, $2c/a=6$). ($x/c$ as per Figure 5.4)
Fig. 7.7 Crack driving force \( (EJ/\sigma_y t) \) along the crack front for rectangular shallow internal axial surface crack \( (a/t=0.25, 2c/a=6) \).
\((x/c\) as per Figure 5.4)\\

Fig. 7.8 Crack driving force \( (EJ/\sigma_y t) \) along the crack front for rectangular deep internal axial surface crack \( (a/t=0.75, 2c/a=6) \).
\((x/c\ as per Figure 5.4)\)
Fig. 7.9 Crack driving force (EJ/σ_yt) along the crack front for triangular shallow internal axial surface crack (a/t=0.25, 2c/a=6).
(x/c as per Figure 5.4)

Fig. 7.10 Crack driving force (EJ/σ_yt) along the crack front for triangular deep internal axial surface crack (a/t=0.75, 2c/a=6).
(x/c as per Figure 5.4)
Figure 8.1 Fracture mechanics evaluation procedure by consideration of multiaxiality of stress state.
Fig. 8.2 Crack driving force J at centre of crack versus crack depth for internal axial surface crack in PHT pipe (elliptical crack, $2c/a=6$).

Fig. 8.3 Crack opening displacement at centre of crack versus crack depth for internal axial surface crack in PHT pipe (elliptical crack, $2c/a=6$).