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**Couplings at LEP \***

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\* Invited talk given by S.D. at the *Joint U.S.-Polish Workshop on Physics from Planck Scale to Electroweak Scale*, Sept. 21-24, 1994, Warsaw, Poland, and by G.V. at the *1994 DPF Meeting*, Aug. 2-6, 1994, Albuquerque, New Mexico.

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# PROBING ANOMALOUS GAUGE BOSON COUPLINGS AT LEP<sup>1</sup>

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## ABSTRACT

We bound anomalous gauge boson couplings using LEP data for the  $Z \rightarrow \bar{f}f$  partial widths. We use an effective field theory formalism to compute the one-loop corrections resulting from non-standard model three and four gauge boson vertices. We find that measurements at LEP constrain the three gauge boson couplings at a level comparable to that obtainable at LEP II.

## 1 Introduction

High precision measurements at the  $Z$  pole at LEP have been used to place stringent limits on new physics beyond the standard model. Under the assumption that the dominant effects of the new physics would show up as corrections to the gauge boson self-energies, the LEP measurements have been used to parameterize the possible new physics in terms of three observables,  $S$ ,  $T$ ,  $U$ . [1, 2] A fourth observable corresponds to the partial width  $Z \rightarrow b\bar{b}$ .

In view of the extraordinary agreement between the standard model predictions and the observations, it seems reasonable to assume that the  $SU(2)_L \times U(1)_Y$  gauge theory of electroweak interactions is essentially correct and that the only sector of the theory lacking experimental support is the symmetry breaking sector. There are many extensions of the minimal standard model that incorporate different symmetry breaking possibilities. One large class of models is that in which the interactions responsible for the symmetry breaking are strongly coupled and their effects would show up in experiments as deviations from the minimal standard model couplings. In this paper we use the measurements of the partial decay widths of the  $Z$  boson to place bounds on anomalous gauge boson couplings.

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<sup>1</sup>Invited talk given by S.D. at the *Joint U.S.-Polish Workshop on Physics from Planck Scale to Electroweak Scale*, Sept 21-24, 1994, Warsaw, Poland and by G.V. at the *1994 DPF Meeting*, Aug. 2-6, 1994, Albuquerque, New Mexico.

## 2 Formalism

We assume that the electroweak interactions are given by an  $SU(2)_L \times U(1)_Y$  gauge theory with spontaneous symmetry breaking to  $U(1)_{EM}$  and that we do not have any information on the symmetry breaking sector except that it is strongly interacting and that any new particles have masses higher than several hundred GeV. This scenario can be described with an effective Lagrangian with operators organized according to the number of derivatives or gauge fields they have. If we call  $\Lambda$  the scale at which the symmetry breaking physics comes in, this organization of operators corresponds to an expansion of amplitudes in powers of  $(E^2 \text{ or } v^2)/\Lambda^2$ . The lowest order effective Lagrangian for the symmetry breaking sector of the theory is: [3]

$$\mathcal{L}^{(2)} = \frac{v^2}{4} \text{Tr} \left[ D^\mu \Sigma^\dagger D_\mu \Sigma \right]. \quad (1)$$

In our notation  $W_\mu$  and  $B_\mu$  are the  $SU(2)_L$  and  $U(1)_Y$  gauge fields with  $W_\mu \equiv W_\mu^i \tau_i$ . The matrix  $\Sigma \equiv \exp(i\vec{\omega} \cdot \vec{\tau}/v)$ , contains the Goldstone bosons  $\omega_i$  that give the  $W$  and  $Z$  their masses via the Higgs mechanism and the  $SU(2)_L \times U(1)_Y$  covariant derivative is given by:

$$D_\mu \Sigma = \partial_\mu \Sigma + \frac{i}{2} g W_\mu^i \tau^i \Sigma - \frac{i}{2} g' B_\mu \Sigma \tau_3. \quad (2)$$

This non-renormalizable Lagrangian is interpreted as an effective field theory, valid below some scale  $\Lambda \leq 3$  TeV. The lowest order interactions between the gauge bosons and fermions, as well as the kinetic energy terms for all fields, are the same as those in the minimal standard model.

For LEP observables, the operators that can appear at tree-level are those that modify the gauge boson self-energies. To order  $\mathcal{O}(1/\Lambda^2)$  there are only three:[3, 4]

$$\mathcal{L}^{(2GB)} = \beta_1 \frac{v^2}{4} \left( \text{Tr} \left[ \tau_3 \Sigma^\dagger D_\mu \Sigma \right] \right)^2 + \alpha_8 g^2 \left( \text{Tr} \left[ \Sigma \tau_3 \Sigma^\dagger W_{\mu\nu} \right] \right)^2 + g g' \frac{v^2}{\Lambda^2} L_{10} \text{Tr} \left[ \Sigma B^{\mu\nu} \Sigma^\dagger W_{\mu\nu} \right], \quad (3)$$

which contribute respectively to  $T$ ,  $U$  and  $S$ .

In this paper we will consider operators that affect the  $Z$  partial widths at the one-loop level. We will restrict our study to only those operators that appear at order  $\mathcal{O}(1/\Lambda^2)$  in the gauge-boson sector and that respect the custodial symmetry in the limit  $g' \rightarrow 0$ . We will also consider an additional operator which is parity violating, but CP conserving. Because of the parity violating nature of this term it can lead to observable signals at LEP II via  $e^+ e^- \rightarrow W^+ W^-$ . [5] The Lagrangian we consider is thus:

$$\begin{aligned} \mathcal{L}^{(4)} = & \frac{v^2}{\Lambda^2} \left\{ L_1 \left( \text{Tr} \left[ D^\mu \Sigma^\dagger D_\mu \Sigma \right] \right)^2 + L_2 \left( \text{Tr} \left[ D_\mu \Sigma^\dagger D_\nu \Sigma \right] \right)^2 \right. \\ & - i g L_{9L} \text{Tr} \left[ W^{\mu\nu} D_\mu \Sigma D_\nu \Sigma^\dagger \right] - i g' L_{9R} \text{Tr} \left[ B^{\mu\nu} D_\mu \Sigma^\dagger D_\nu \Sigma \right] \\ & \left. + g \hat{\alpha} \epsilon^{\alpha\beta\mu\nu} \text{Tr} \left( \tau_3 \Sigma^\dagger D_\mu \Sigma \right) \text{Tr} \left( W_{\alpha\beta} D_\nu \Sigma \Sigma^\dagger \right) \right\}, \quad (4) \end{aligned}$$

where the field strength tensors are given by:

$$\begin{aligned} W_{\mu\nu} &= \frac{1}{2} \left( \partial_\mu W_\nu - \partial_\nu W_\mu + \frac{i}{2} g [W_\mu, W_\nu] \right) \\ B_{\mu\nu} &= \frac{1}{2} \left( \partial_\mu B_\nu - \partial_\nu B_\mu \right) \tau_3. \end{aligned} \quad (5)$$

The operators in Eq. 3 and Eq. 4 would arise when considering the effects of those in Eq. 1 at the one-loop level or from the new physics responsible for symmetry breaking at a scale  $\Lambda$  at order  $1/\Lambda^2$ . We therefore explicitly introduce the factor  $v^2/\Lambda^2$  in our definition of  $\mathcal{L}^{(4)}$  so that the coefficients  $L_i$  are naturally of  $\mathcal{O}(1)$ . In the present paper we will compute the contribution from the operators of Eqs. 3 and 4 to the  $Z$  partial widths that are measured at LEP.

We will first perform a complete calculation to order  $\mathcal{O}(1/\Lambda^2)$ . That is, we will include the one-loop contributions from the operator in Eq. 1 (and gauge boson kinetic energies). The divergences generated in this calculation are absorbed by renormalization of the couplings in Eq. 3. This calculation will illustrate our method and as an example we use it to place bounds on  $L_{10}$ .

We will then place bounds on the couplings of Eq. 4 by considering their one-loop effects. The divergences generated in this one-loop calculation would be removed in general by renormalization of the couplings in the  $\mathcal{O}(1/\Lambda^4)$  Lagrangian of those operators that modify the gauge boson self-energies at tree-level and perhaps by additional renormalization of the couplings in Eq. 3. Interestingly, we find that we can obtain a completely finite result for the  $Z \rightarrow \bar{f}f$  partial widths using only the operators in Eq. 3 as counterterms when we consider the parity conserving operators. This is not the case, however, for the parity violating operator,  $\hat{\alpha}$ .

We first regularize the integrals in  $n$  space-time dimensions and remove all the poles in  $n - 4$  as well as the finite analytic terms by a suitable definition of the renormalized couplings. We base our analysis on the leading non-analytic terms proportional to  $L_i \log \mu$ . These terms determine the running of the  $1/\Lambda^4$  couplings and cannot be generated by tree-level terms at that order. With a carefully chosen renormalization scale  $\mu$  (such that the logarithm is of order one), these terms give us the correct order of magnitude for the size of the  $1/\Lambda^4$  coefficients.[8] Quadratic divergences will be absorbed by the new physics arising at  $\mathcal{O}(1/\Lambda^4)$  and cannot be used to limit the coefficients of the  $\mathcal{O}(1/\Lambda^2)$  Lagrangian.[6] Clearly, the LEP observables do not measure the couplings in Eq. 4 and it is only from naturalness arguments like the one above that we can place bounds on the anomalous gauge-boson couplings. From this perspective, it is clear that these bounds are not a substitute for direct measurements in future high energy machines.

We will perform our calculations in unitary gauge, so we set  $\Sigma = 1$  in Eqs. 1, 3 and 4. For the lowest order operators we use the conventional input parameters:  $G_F$  as measured in muon decay; the physical  $Z$  mass:  $M_Z$ ; and  $\alpha(M_Z) = 1/128.8$ . Other lowest order parameters are derived quantities and we adopt one of the usual

definitions for the mixing angle:

$$s_Z^2 c_Z^2 \equiv \frac{\pi \alpha (M_Z)}{\sqrt{2} G_F M_Z^2}. \quad (6)$$

We neglect the mass and momentum of the external fermions compared to the  $Z$  mass. The only fermion mass that is kept in our calculation is the mass of the top-quark when it appears as an intermediate state.

With this formalism we proceed to compute the  $Z \rightarrow f\bar{f}$  partial width from the following ingredients.

- The  $Z \rightarrow f\bar{f}$  vertex, which we write as:

$$i\Gamma_\mu = -i \frac{e}{4s_Z c_Z} \gamma_\mu \left[ (r_f + \delta r_f)(1 + \gamma_5) + (l_f + \delta l_f)(1 - \gamma_5) \right] \quad (7)$$

where  $r_f = -2Q_f s_Z^2$  and  $l_f = r_f + T_{3f}$ . The terms  $\delta l_f$  and  $\delta r_f$  occur at one-loop both at order  $1/\Lambda^2$  and at order  $1/\Lambda^4$  and can be found in Ref. [9].

- The renormalization of the lowest order input parameters. At order  $1/\Lambda^2$  it is induced by tree-level anomalous couplings and one-loop diagrams with lowest order vertices. At order  $1/\Lambda^4$  it is induced by one-loop diagrams with an anomalous coupling in one vertex.
- Tree-level and one-loop contributions to  $\gamma Z$  mixing.
- Wave function renormalization.

With all these ingredients we can find our final expression for the physical partial width. We find:

$$\Gamma(Z \rightarrow f\bar{f}) \equiv \Gamma_{SM}(Z \rightarrow f\bar{f}) \left( 1 + \frac{\delta\Gamma_f^{L_i}}{\Gamma_0(Z \rightarrow f\bar{f})} \right), \quad (8)$$

where  $\Gamma_0$  is the lowest order tree level result,

$$\Gamma_0(Z \rightarrow f\bar{f}) = N_{cf} (l_f^2 + r_f^2) \frac{G_F M_Z^3}{12\pi\sqrt{2}}. \quad (9)$$

The values for  $\delta L_f^{L_i}$  are given in the following sections. Using Eq. 8 we introduce additional terms proportional to products of standard model one-loop corrections and corrections due to anomalous couplings. These are small effects that do not affect our results.

We will not attempt to obtain a global fit to the parameters in our formalism from all possible observables. Instead we use the partial  $Z$  widths:[12]

$$\begin{aligned} \Gamma_e &= 83.98 \pm 0.18 \text{ MeV} \\ \Gamma_\nu &= 499.8 \pm 3.5 \text{ MeV} \\ \Gamma_Z &= 2497.4 \pm 3.8 \text{ MeV} \\ R_h &= 20.795 \pm 0.040 \end{aligned} \quad (10)$$

The bounds on new physics are obtained by subtracting the standard model predictions at one-loop from the measured partial widths as in Eq. 8. We use the numbers of Langacker [13] which use the global best fit values for  $M_t$  and  $\alpha_s$  with  $M_H$  in the range 60 – 1000 GeV. The first error is from the uncertainty in  $M_Z$  and  $\Delta r$ , the second is from  $M_t$  and  $M_H$ , and the one in brackets is from the uncertainty in  $\alpha_s$ . [13]

$$\begin{aligned}
\Gamma_e &= 83.87 \pm 0.02 \pm 0.10 \text{ MeV} \\
\Gamma_\nu &= 501.9 \pm 0.1 \pm 0.9 \text{ MeV} \\
\Gamma_Z &= 2496 \pm 1 \pm 3 \pm [3] \text{ MeV} \\
R_h &= 20.782 \pm 0.006 \pm 0.004 \pm [0.03]
\end{aligned}
\tag{11}$$

We add all errors in quadrature. <sup>2</sup>

### 3 Results

In this section we compute the corrections to the  $Z \rightarrow f\bar{f}$  partial widths from the couplings of Eq. 4, and compare them to recent values measured at LEP. We treat each coupling constant independently, and compute only its *lowest* order contribution to the decay widths.

#### 3.1 Bounds on $L_{10}$ at order $1/\Lambda^2$ .

The operators in Eq. 3 are the only ones that induce a tree-level correction to the gauge boson self-energies to order  $\mathcal{O}(1/\Lambda^2)$ . These corrections are of course well known and correspond, *at leading order*, to the new physics contributions to  $S$ ,  $T$ ,  $U$ . [1]

We have regularized our one-loop integrals in  $n$  dimensions and isolated the ultraviolet poles  $1/\epsilon = 2/(4 - n)$ . We find that we obtain a finite answer to order  $1/\Lambda^2$  if we adopt the following renormalization scheme:

$$\begin{aligned}
\frac{v^2}{\Lambda^2} L_{10}^r(\mu) &= \frac{v^2}{\Lambda^2} L_{10} - \frac{1}{16\pi^2} \frac{1}{12} \left( \frac{1}{\epsilon} + \log \frac{\mu^2}{M_Z^2} \right) \\
\beta_1^r(\mu) &= \beta_1 - \frac{e^2}{16\pi^2} \frac{3}{2c_Z^2} \left( \frac{1}{\epsilon} + \log \frac{\mu^2}{M_Z^2} \right).
\end{aligned}
\tag{12}$$

We thus replace the bare parameters  $L_{10}$  and  $\beta_1$  with the scale dependent ones above. As a check of our answer, it is interesting to note that we would also obtain a finite answer by adding to our result the one-loop contributions to the self-energies obtained in unitary gauge in the minimal standard model with one Higgs boson in the loop.

Our result for  $L_{10}$  at order  $1/\Lambda^2$  is then:

$$\frac{\delta\Gamma_f^{L_{10}}}{\Gamma_0(Z \rightarrow f\bar{f})} = \frac{e^2}{c_Z^2 s_Z^2} L_{10}^r(\mu) \frac{v^2}{\Lambda^2} \frac{2r_f(l_f + r_f)}{l_f^2 + r_f^2} \frac{c_Z^2}{s_Z^2 - c_Z^2}
\tag{13}$$

<sup>2</sup>See also the contribution of A. Sopczak [14] to these proceedings for a discussion of extracting bounds on new physics from LEP measurements.

Once again we point out that, at this order, the contribution of  $L_{10}$  to the LEP observables occurs only through modifications to the self-energies that are proportional to  $q^2$ . At this order it is therefore possible to identify the effect of  $L_{10}$  with the oblique parameter  $S$ . If we were to compute the effects of  $L_{10}$  at one-loop (as we do for the  $L_{9L,9R}$ ) comparison with  $S$  would not be appropriate.

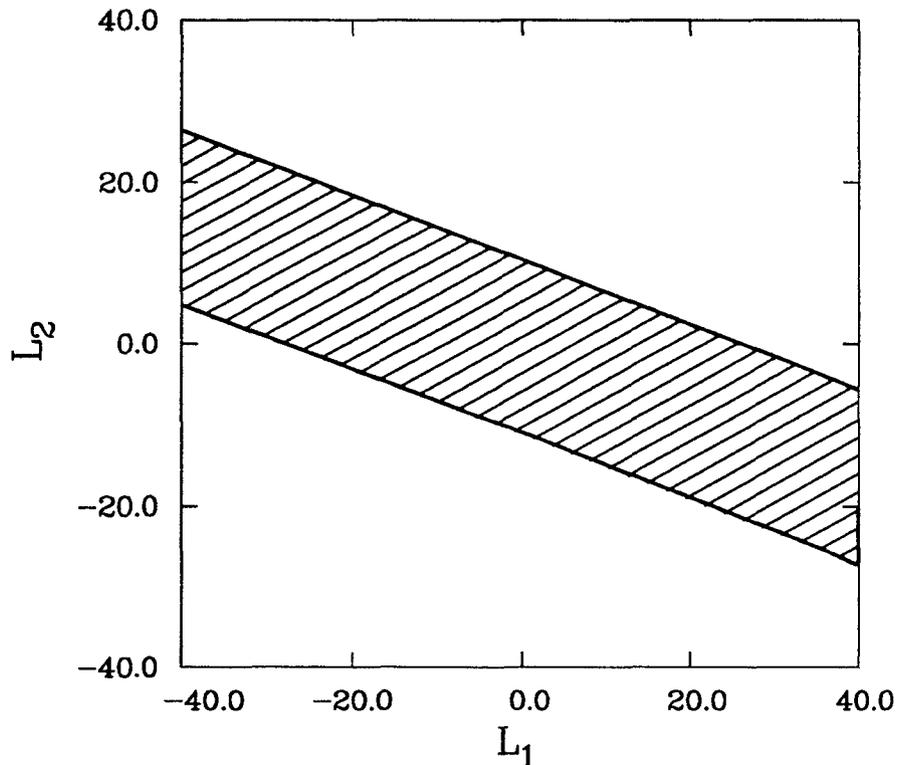


Figure 1: 90% confidence level bounds on  $L_{1,2}$  from the  $Z \rightarrow f\bar{f}$  partial widths, (Eq. 14). The allowed region is shaded.

Numerically we find the following 90% confidence level bounds on  $L_{10}$  when we take the scale  $\Lambda = 2$  TeV:

$$\begin{aligned}
 \Gamma_e &\rightarrow -1.7 \leq L'_{10}(M_Z)_{new} \leq 3.3 \\
 R_h &\rightarrow -1.5 \leq L'_{10}(M_Z)_{new} \leq 2.0 \\
 \Gamma_Z &\rightarrow -1.1 \leq L'_{10}(M_Z)_{new} \leq 1.5
 \end{aligned}
 \tag{14}$$

### 3.2 Bounds on $L_{1,2}$ at order $1/\Lambda^4$ .

The couplings  $L_{1,2}$  enter the one-loop calculation of the  $Z \rightarrow f\bar{f}$  width through the four gauge boson couplings. Our prescription calls for using only the leading non-

analytic contribution to the process  $Z \rightarrow f\bar{f}$ . This contribution can be extracted from the coefficient of the pole in  $n - 4$ . Care must be taken to isolate the poles of ultraviolet origin (which are the only ones that interest us) from those of infrared origin that appear in intermediate steps of the calculation but that cancel as usual when one includes real emission processes as well.

Since in unitary gauge  $L_{1,2}$  modify only the four-gauge boson couplings at the one-loop level, they enter the calculation of the  $Z$  partial widths only through the self-energy corrections and the renormalization of the lowest order couplings. These operators induce a non-zero value for  $\Delta\rho \equiv \Pi_{WW}(0)/M_W^2 - \Pi_{ZZ}(0)/M_Z^2$ . For the observables we are discussing, this is the *only* effect of  $L_{1,2}$ . We do not place bounds on them from global fits of the oblique parameter  $T$ , because we have not shown that this is the only effect of  $L_{1,2}$  for the other observables that enter the global fits. The calculation to  $\mathcal{O}(1/\Lambda^4)$  can be made finite with the following renormalization of  $\beta_1$ :

$$\beta_1^r(\mu) = \beta_1 + \frac{3\alpha^2(1+c_Z^2)}{4s_Z^2c_Z^4} \left( L_1 + \frac{5}{2}L_2 \right) \frac{v^2}{\Lambda^2} \left( \frac{1}{\epsilon} + \log \frac{\mu^2}{M_Z^2} \right). \quad (15)$$

We obtain for the  $Z$  partial widths:

$$\frac{\delta\Gamma_f^{L_{1,2}}}{\Gamma_0(Z \rightarrow f\bar{f})} = -\frac{3\alpha^2(1+c_Z^2)}{2s_Z^2c_Z^4} \left( L_1 + \frac{5}{2}L_2 \right) \frac{v^2}{\Lambda^2} \log \left( \frac{\mu^2}{M_Z^2} \right) \left( 1 + \frac{2r_f(l_f+r_f)}{l_f^2+r_f^2} \frac{c_Z^2}{s_Z^2-c_Z^2} \right). \quad (16)$$

Using  $\Lambda = 2$  TeV, and  $\mu = 1$  TeV we find 90% confidence level bounds:

$$\begin{aligned} \Gamma_e &\rightarrow -50 \leq L_1 + \frac{5}{2}L_2 \leq 26 \\ \Gamma_\nu &\rightarrow -28 \leq L_1 + \frac{5}{2}L_2 \leq 59 \\ R_h &\rightarrow -190 \leq L_1 + \frac{5}{2}L_2 \leq 130 \\ \Gamma_Z &\rightarrow -36 \leq L_1 + \frac{5}{2}L_2 \leq 27 \end{aligned} \quad (17)$$

Combined, they yield the result:

$$-28 \leq L_1 + \frac{5}{2}L_2 \leq 26 \quad (18)$$

shown in Figure 1.

As mentioned before, the effect of  $L_{1,2}$  in other observables is very different from that of  $\beta_1$ . It is only for the  $Z$  partial widths that we can make the  $\mathcal{O}(1/\Lambda^4)$  calculation finite with Eq. 15.

### 3.3 Bounds on $L_{9L,9R}$ at order $1/\Lambda^4$ .

The calculation of the effects of the  $L_{9L}$  and  $L_{9R}$  operators can be carried out simultaneously with the one-loop effects of the lowest order effective Lagrangian, Eq. 1,

because the form of the three and four gauge boson vertices induced by these two couplings is the same as that arising from Eq. 1. Performing the calculation in this way, we obtain a result that contains terms of order  $1/\Lambda^2$  (those independent of  $L_{9L,9R}$ ), terms of order  $1/\Lambda^4$  proportional to  $L_{9L,9R}$  and terms of order  $1/\Lambda^4$  proportional to  $L_{10}$  and  $\beta_1$ .

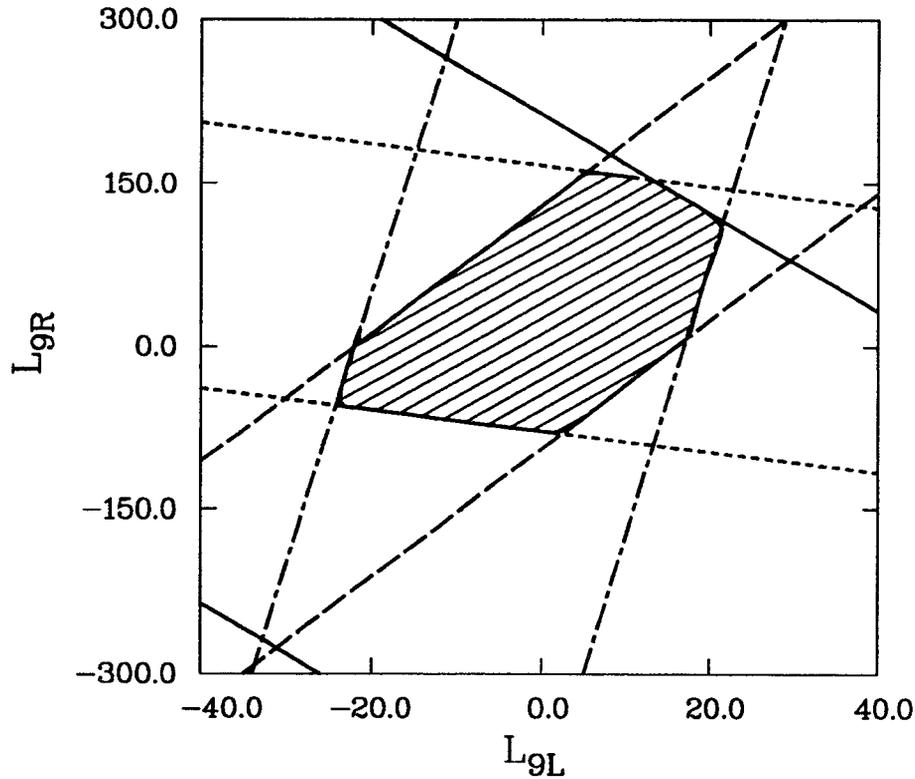


Figure 2: 90% confidence level bounds on  $L_{9L,9R}$  from the  $Z \rightarrow f\bar{f}$  partial widths, (Eq. 22). The allowed region is shaded. The solid, dotted, dashed, and dot-dashed lines are the bounds from  $\Gamma_e$ ,  $\Gamma_\nu$ ,  $R_h$ , and  $\Gamma_Z$ , respectively.

It is amusing to note that the divergences generated by the operators  $L_{9L,9R}$  in the one-loop (order  $1/\Lambda^4$ ) calculation of the  $Z \rightarrow f\bar{f}$  widths can all be removed by the following renormalization of the couplings in Eq. 3 (in the  $M_t = 0$  limit):

$$\beta_1^r(\mu) = \beta_1 - \frac{\alpha}{\pi} \frac{e^2}{96s_Z^4 c_Z^4} \frac{v^2}{\Lambda^2} \left[ c_Z^2 (1 - 20s_Z^2) L_{9L} + s_Z^2 (10 - 29c_Z^2) L_{9R} \right] \left( \frac{1}{\epsilon} + \log \frac{\mu^2}{M_Z^2} \right)$$

$$L_{10}^r(\mu) = L_{10} - \frac{\alpha}{\pi} \frac{1}{96s_Z^2 c_Z^2} \left[ (1 - 24c_Z^2)L_{9L} + (32c_Z^2 - 1)L_{9R} \right] \left( \frac{1}{\epsilon} + \log \frac{\mu^2}{M_Z^2} \right) \quad (19)$$

This proves our assertion that our calculation to order  $\mathcal{O}(1/\Lambda^4)$  can be made finite by suitable renormalizations of the parameters in Eq. 3. However, we do not expect this result to be true in general. That is, we expect that a calculation of the one-loop contributions of the operators in Eq. 4 to other observables will require counterterms of order  $1/\Lambda^4$ . Thus, Eq. 19 does *not* mean that we can place bounds on  $L_{9L,9R}$  from global fits to the parameters  $S$  and  $T$ . Without performing a complete analysis of the effective Lagrangian at order  $1/\Lambda^4$  it is not possible to identify the renormalized parameters of Eq. 19 with the ones corresponding to  $S$  and  $T$  that are used for global fits.

Keeping only terms linear in  $L_{9L,9R}$  we find,

$$\begin{aligned} \frac{\delta\Gamma^{L_9}}{\Gamma_0} &= \frac{\alpha^2}{24} \frac{1}{c_Z^4 s_Z^4} \frac{v^2}{\Lambda^2} \log\left(\frac{\mu^2}{M_Z^2}\right) \\ &\cdot \left\{ \left[ L_{9L}(1 - 24c_Z^2) + L_{9R}(-1 + 32c_Z^2) \right] \frac{2r_f(l_f + r_f)}{l_f^2 + r_f^2} \frac{c_Z^2}{s_Z^2 - c_Z^2} \right. \\ &+ 2 \left[ L_{9L}c_Z^2(1 - 20s_Z^2) + L_{9R}s_Z^2(10 - 29c_Z^2) \right] \left( 1 + \frac{2r_f(l_f + r_f)}{l_f^2 + r_f^2} \frac{c_Z^2}{s_Z^2 - c_Z^2} \right) \left. \right\} \\ &+ \frac{\alpha^2}{12} \log\left(\frac{\mu^2}{M_Z^2}\right) \frac{1 + 2c_Z^2}{c_Z^4 s_Z^4 (l_b^2 + r_b^2)} \frac{v^2}{\Lambda^2} \frac{M_t^2}{M_Z^2} \left[ L_{9R}s_Z^2 - 7L_{9L}c_Z^2 \right] \delta_{fb}. \end{aligned} \quad (20)$$

The last term in Eq. 20 corresponds to the non-universal corrections proportional to  $M_t^2$  that are relevant only for the decay  $Z \rightarrow \bar{b}b$ .

Using as before  $\Lambda = 2$  TeV and  $\mu = 1$  TeV we find 90% confidence level bounds:

$$\begin{aligned} \Gamma_e &\rightarrow -92 \leq L_{9L} + 0.22L_{9R} \leq 47 \\ \Gamma_\nu &\rightarrow -79 \leq L_{9L} + 1.02L_{9R} \leq 170 \\ R_h &\rightarrow -22 \leq L_{9L} - 0.17L_{9R} \leq 16 \\ \Gamma_Z &\rightarrow -22 \leq L_{9L} - 0.04L_{9R} \leq 17 \end{aligned} \quad (21)$$

We show these inequalities in Figure 2.

If we bound one coupling at a time we can read from Figure 2 that:

$$\begin{aligned} -22 &\leq L_{9L} \leq 16 \\ -77 &\leq L_{9R} \leq 94 \end{aligned} \quad (22)$$

In a vector like model with  $L_{9L} = L_{9R}$  we have the 90% confidence level bound:

$$-22 < L_{9L} = L_{9R} < 18. \quad (23)$$

The limits from  $\Gamma(Z \rightarrow \bar{b}b)$  are not competitive with those of Eq. 21 despite the enhancement of  $M_t^2/M_Z^2$ .

### 3.4 Bounds on $\hat{\alpha}$ at order $1/\Lambda^4$

The one loop contributions to  $Z \rightarrow f\bar{f}$  from the parity violating operator  $\hat{\alpha}$  cannot be made finite by a renormalization of the operators of Eq. 3. The divergence in this case will be absorbed by renormalization of the couplings appearing at  $\mathcal{O}(1/\Lambda^4)$ . The leading non-analytic contribution can be extracted without explicitly performing that renormalization. It is, [5]

$$\frac{\delta\Gamma_f^5}{\Gamma_f^0} = \frac{3\alpha}{2\pi} g_5^Z \log\left(\frac{\mu}{M_W}\right) \left[ \frac{2l_f}{l_f^2 + r_f^2} \frac{c_Z^2}{s_Z^2} + \left(1 + \frac{2r_f(l_f + r_f)}{l_f^2 + r_f^2} \frac{c_Z^2}{s_Z^2 - c_Z^2}\right) \right], \quad (24)$$

where

$$g_5^Z \equiv \frac{4M_Z^2 \hat{\alpha}}{\Lambda^2}. \quad (25)$$

We find the results for  $\Lambda = 2 \text{ TeV}$  and  $\mu = 1 \text{ TeV}$ ,

$$\begin{aligned} \Gamma_e &\rightarrow -0.1 \leq g_5^Z \leq 0.05 \\ \Gamma_\nu &\rightarrow -0.08 \leq g_5^Z \leq 0.04 \\ R_h &\rightarrow -0.07 \leq g_5^Z \leq 0.1 \\ \Gamma_Z &\rightarrow -0.9 \leq g_5^Z \leq 1.2 \end{aligned} \quad (26)$$

## 4 Conclusion

We can compare our results<sup>3</sup> with bounds that future colliders are expected to place on the anomalous couplings.

In Fig. 3, we compare our 95% confidence level bounds on  $L_{9L}$  and  $L_{9R}$  with those which can be obtained at LEP II with  $\sqrt{s} = 190 \text{ GeV}$  and an integrated luminosity of  $500 \text{ pb}^{-1}$ . [7, 11] We find that LEP and LEP II are sensitive to slightly different regions of the  $L_{9L}$  and  $L_{9R}$  parameter space, with the bounds from the two machines being of the same order of magnitude. We again emphasize our caveat that the bounds from LEP rely on naturalness arguments and are no substitute for measurements in future colliders.

The limits presented here on the four point couplings  $L_1$  and  $L_2$  are the first available for these couplings. They will be measured directly at the LHC.

Computing the leading contribution of each operator, and allowing only one non-zero coefficient at a time, our 90 % confidence level bounds are:

$$\begin{aligned} -1.1 &< L_{10}^r(M_Z)_{new} < 1.5 \\ -28 &< L_1 < 26 \\ -11 &< L_2 < 11 \end{aligned}$$

<sup>3</sup>Our normalization of the  $L_i$  is different from that of Ref. [7]. We have translated their results into our notation.

$$\begin{aligned}
-22 < L_{9L} < 16 \\
-77 < L_{9R} < 94 \\
-.07 < g_5^Z < .04.
\end{aligned}
\tag{27}$$

Two parameter bounds on  $(L_1, L_2)$  and  $(L_{9L}, L_{9R})$  are given in the text.

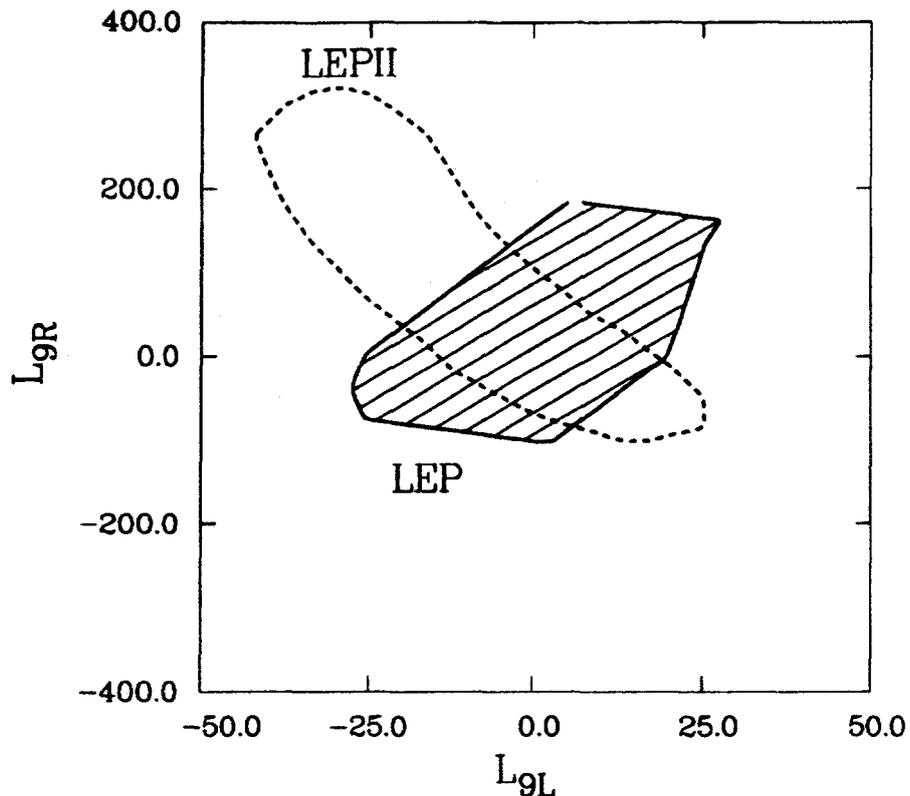


Figure 3: Comparison of the 95% confidence level bounds from the  $Z$  partial widths (shaded region) with that obtainable at LEP II with  $\sqrt{s} = 190 \text{ GeV}$  and  $\int \mathcal{L} = 500 \text{ pb}^{-1}$ , (dotted contour) [7].

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