

Field Flattening in Superconducting Beam Transport Magnets*

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Abstract—Dipoles in which the beam traverses the midplane well away from the magnet axis may benefit from flattening of the vertical field on the midplane. A procedure is described for doing so, making use of Chebyshev polynomials. In the case of the large aperture "DX" magnets located immediately on each side of the six intersection regions of the Relativistic Heavy Ion Collider (RHIC), a comparison is made of the field of coils optimized in this way and of coils optimized in the more common way by minimizing the leading coefficients of the Fourier expansion about the magnet axis. The comparison is of the integrated Fourier coefficients of the field expanded locally along the beam trajectory.

1. INTRODUCTION

The 4.28 tesla, 4.0 m long, superconducting-dipole magnets "DX" on each side of the six intersections in the Relativistic Heavy Ion Collider (RHIC) at Brookhaven National Laboratory (BNL) require a large aperture because both particle beams go through them and diverge horizontally at the end away from the intersection. The circular aperture, "cosine-theta approximation" dipoles commonly used to produce fields above two tesla, have increasingly large field deviations as the distance from the centerline exceeds about 2/3 the aperture radius. The cosine-theta approximation consists of turns arranged in blocks of decreasing size and spaces between them of increasing size from the midplane towards the pole. It was thought that since the two beams remain on the horizontal midplane, and do not require as much vertical space as horizontal, the field quality might be increased at large radii on the midplane, perhaps at the expense of field quality well above or below the midplane. By "field quality" is meant the terms above the dipole in the expansion of the field in Fourier coefficients integrated along the beam trajectory.

Three new algorithms had to be programmed for this study: first, a means of computing Fourier expansion coefficients for off-axis points due to an extended conductor inside a circular iron aperture. It is not sufficient to compute on-axis expansion coefficients and transform them to the new position, because in principle, infinitely many on-axis coefficients must be computed. In practice, of course, the higher order terms may be sufficiently small to be ignorable, but at points close to the coil, many higher order terms are likely to be needed. Second, an algorithm for transforming on-axis harmonics to off-axis points is needed anyway,

because coils are designed with built-in, on-axis harmonics to compensate for harmonics which arise during magnet excitation. These must be subtracted off the off-axis expansion coefficients of the coil to find the net effect on the particle beam. Third, a means for reducing field variations on the midplane. The method selected is to minimize the leading coefficients in the expansion of the field in Chebyshev polynomials. The rationale for this is the property of these polynomials that their extrema over the range of the expansion are either plus or minus one. If the coefficients of the expansion are decreasing rapidly, which is the usual case, then minimizing the leading terms by rearranging conductor block size and position will result in the least deviation of field over the range of expansion.

Finally, for comparing coils designed by the two methods (minimized on-axis Fourier expansion coefficients vs minimized on-axis Chebyshev coefficients), the off-axis Fourier expansion coefficients are integrated along a likely trajectory. RHIC can be run with either the same or unlike species of heavy ion in the two beams with zero crossing angle. The DX magnets can be moved transversely to center the beams as nearly as possible in the magnet aperture; having done so, the exit radius of either beam is about the same whatever the species. A typical case is gold-proton collisions, with an entrance angle (α') of 3.85 mrad and radius equal to zero at the end nearest the collision region. At the other end of DX, the beam radius is a maximum, about ± 37 mm for non-symmetric beams; the beam fringing is about ± 21 mm (6σ). In DX, the particle beams are contained within a relatively small "warm" beam tube which is thermally insulated from the larger cold tube containing the coil and helium coolant, in order to reduce refrigerator load, since the intersection region is at room temperature. At the beginning of this study, the coil inner radius was 100 mm, the cold tube had about a 200 mm od and the warm tube od was about 175 mm. The coil radius has since been reduced to 90 mm, and the warm bore tube od to 140 mm.

II. THE ALGORITHMS

A. Off-axis Fourier expansion coefficients of a conductor

The expansion of the 2-D field in Fourier coefficients about the origin, $H = \sum C_n(Z/r_1)^n$, $n = 0, 1, \dots$ (complex quantities are capitalized), inside coils in an infinitely permeable circular cylinder of radius R is well-known. In this expansion, r_1 is a normalization radius, commonly about 2/3 the coil radius. Formulae for the expansion coefficients in the case of constant current density

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are available in closed form for specialized conductor geometries; in particular, for a quadrangle in which the four corners are specified. A conductor in the present magnets is a thin ribbon, approximately a trapezoid with the centerline close to radial and the thick edge on the outside. If the magnet has $2N$ pole symmetry, the coefficients C_n for a conductor in one pole are simply multiplied by $2N$, and are "allowed" if $n = N(2m+1)-1$, $m = 0, 1, \dots$ and are zero otherwise.

In contrast to expansion about the origin, if the field is expanded about Z_1 , i.e., $H = \sum D_n ((Z-Z_1)/r_2)^n$, $n = 0, 1, \dots$, then for arbitrary Z_1 there are no unallowed coefficients. However, for $Z_1 = x_1$ with midplane symmetry, the imaginary part of D_n is zero. Additionally, when iron is present, there does not seem to be a single closed form expression for all harmonics of a polygonal conductor; the contour integral of the n^{th} harmonic has $n+2$ terms.

The method chosen for computation of the expansion coefficients is numerical integration of the coefficients for a filament over the conductor cross section, with the conductor duplicated at positions required by symmetry. The expansion of a filament at Z_0 , with iron at radius r , is given by (1).

$$H = \frac{-I}{2\pi} \sum_{n=0}^{\infty} \left[\frac{1}{(Z_0 - Z_1)^{n+1}} + \frac{1}{[(r^2/\rho^2)Z_0 - Z_1]^{n+1}} \right] (Z - Z_1)^n, \quad (1)$$

where $\rho^2 = Z_0 Z_0^*$

For the present work, sufficient accuracy of integration over the ribbon-shaped conductor is obtained with a two-point Gaussian quadrature, with the points located on the center line of the ribbon.

B. Transforming on-axis harmonics to an off-axis point

It is easily shown [1] that the transformation is given by (2).

$$D_j = \frac{1}{j!} \left(\frac{r_2}{r_1}\right)^j \sum_{n=j}^m \frac{n!}{(n-j)!} C_n \left(\frac{Z_1}{r_1}\right)^{n-j} \quad (2)$$

For a dipole, $C_n = 10^{-4} B_0 (b'_n + i a'_n)$. The b'_n and a'_n defined this way are termed "units". Let $D_i = 10^{-4} B_0 (d'_i + i e'_i)$. On the midplane, $Z_1 = x_1$ and the real and imaginary parts of D are given by (3) and (4), where m may be from zero to infinity.

$$d'_j = \frac{1}{j!} \left(\frac{r_2}{r_1}\right)^j \sum_{n=j}^m \frac{n!}{(n-j)!} b'_n \left(\frac{x_1}{r_1}\right)^{n-j} \quad (3)$$

For convergence, $r_2 + |Z_1| < \text{the coil radius}$.

$$e'_j = \frac{1}{j!} \left(\frac{r_2}{r_1}\right)^j \sum_{n=j}^m \frac{n!}{(n-j)!} a'_n \left(\frac{x_1}{r_1}\right)^{n-j} \quad (4)$$

C. Expansion of the field in Chebyshev polynomials and optimization

Computer routines have been published for the computation of the coefficients ch_i of the expansion in Chebyshev polynomials of a function $f(x)$ over a finite interval in x [2]. For the polynomial of degree n , the value of $f(x)$ is computed at the n points on the interval where the polynomial is zero. In the present case of a dipole, $f(x)$ is the field H_y on the midplane from the origin to a maximum x less than the coil radius; since the field is symmetric about the vertical axis, H_x is zero on it. It is necessary that H be known to high accuracy; the routine used is an old one using complex algebra methods, recently refurbished [3] to improve accuracy near the origin, where the contribution to the field due to the iron has an apparent singularity.

The complex field $H = H_y + i H_x$ at Z_0 in a cylin-

$$H = \frac{i\sigma}{4\pi} \left[\oint \frac{(Z-Z_0)^*}{Z-Z_0} dZ - \oint \frac{ZZ^*}{r^2 - Z_0 Z^*} dZ^* \right] \quad (5)$$

drical iron cavity of radius r , is given by a counterclockwise

$$\sum_{n=1}^N \alpha_n \log\left(\frac{Z_n - Z_0}{Z_{n-1} - Z_0}\right), \quad \text{where} \quad (6)$$

$$\alpha_n = (Z_n - Z_0)^* - \beta_n (Z_n - Z_0),$$

$$\beta_n = \Delta Z_n^* / \Delta Z_n, \quad \text{and}$$

$$\Delta Z_n = Z_n - Z_{n-1}.$$

integral (5) around the contour of the conductor having current density σ [4,5]. The first integral on the right of (5) was evaluated on a polygonal contour by Beth [4], who obtained (6). In (6), subscript n denotes the n^{th} side or the corner at the end of the n^{th} side. Following Beth's lead, the

$$- \sum_{n=1}^N \frac{r^2}{Z_0^2} \eta_n^* \log\left(\frac{r^2 - Z_0 Z_n^*}{r^2 - Z_0 Z_{n-1}^*}\right) + \frac{\Delta Z_n^*}{2Z_0} (Z_n + Z_{n-1}), \quad (7)$$

$$\text{where } \eta_n^* = Z_n - Z_n^* / \beta_n + r^2 / \beta_n Z_0$$

second integral in (5) can also be integrated around a polygonal conductor, obtaining (7). In (7), if $Z_0 \rightarrow 0$, there

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is an apparent singularity which is not present in the original integral. The problem is resolved by expanding (7) in powers of Z_0/r about the origin, giving (8), which is used in place of (7) for $|Z_0/r| < 0.11$ with $3 \leq j \leq 11$; at the transition, $\Delta H/H = 2 \times 10^{-12}$.

$$\oint \frac{ZZ^*}{r^2 - ZZ_0} dZ^* = \sum_{n=1}^N \frac{\beta_n^*}{r^2} \sum_{j=3}^{\infty} \left[\frac{1}{j(j-1)} \left(\frac{Z_0}{r^2} \right)^{j-3} (Z_n^{*j} - Z_{n-1}^{*j}) \right] \quad (8)$$

Since the field is recomputed for each degree of polynomial, the computation of the coefficients ch_i is relatively slow, about an order of magnitude slower than the computation of an equivalent number of Fourier coefficients about the origin.

The program PAR2DOPT¹ used for optimization of the conductor positions minimizes $\chi^2 = \sum w_i b_i^2$, where the w_i are weights chosen by experience, and the b_i are the leading Fourier coefficients; typically, $i = 2, 4, \dots, 14$. For the present purpose, a bypass was installed in PAR2DOPT to compute $\chi^2 = \sum ch_i^2$, i typically = 1, 2, ..., 9.

III. RESULTS

The first results obtained using the field flattening procedure were encouraging; Fig. 1 is a plot of the deviation in field from the value at the magnet center as a function of distance from the center for 2 coils, one optimized to minimize $\sum w_i b_i^2$, the other has the same block structure, but it is reoptimized to minimize $\sum ch_i^2$, $i = 1$ thru 9, with the region of optimization $0 \leq x \leq 7$ cm.

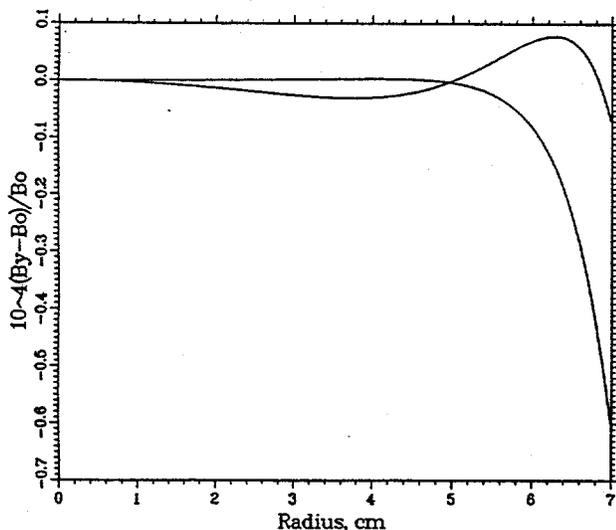


Fig. 1 OHFB83, Fourier & Chebyshev optimized over 7 cm.

Fig. 2 and Fig. 3 are similar plots of coils optimized to minimize $\sum ch_i^2$ over $0 \leq x \leq 8$ and over $0 \leq x \leq 9$ cm, respectively; the comparison curve in each case is that of the Fourier-optimized coil. In each case, the Chebyshev-optimized coil has maximum field deviations about an order of magnitude smaller than that of the Fourier-optimized coil, a six-block configuration, OHFB83, which was the best that could be found. As mentioned above, the field of the coil has intentional harmonics b_n^* for $n = 2, 4$ and 6 ; the field due to these is subtracted prior to Chebyshev optimization and prior to making these plots.

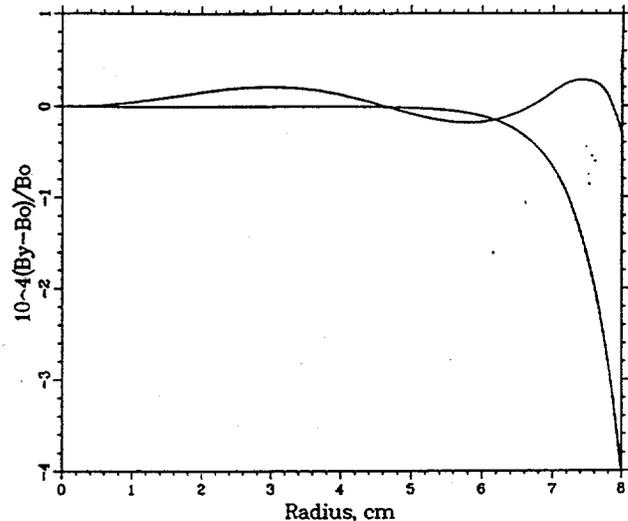


Fig. 2 OHFB83, Fourier & Chebyshev optimized over 8 cm.

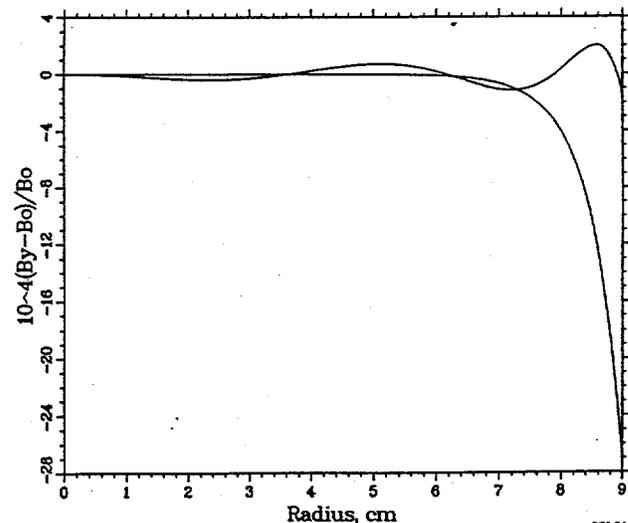


Fig. 3 OHFB83, Fourier & Chebyshev optimized over 9 cm.

¹ Program maintained and mostly written by P.A. Thompson, BNL Magnet Division

The fields shown in Figures 1 thru 3 may be expanded in a Fourier series about the respective maximum x , 7, 8 and 9 cm. Table I is a comparison of the Fourier expansion coefficients over a 3 cm radius at $x = 7.0$ cm of the field of the Chebyshev-optimized and the Fourier-optimized coils.

Table I
 b_i VS ch_i EXPANDED AT $X = 7$ CM

i	1	2	3	4	5	6
b_i	-3.80	-10.0	-17.9	-25.1	-29.9	-30.6
ch_i	-2.09	-8.07	-16.9	-25.0	-30.2	-30.9
i	7	8	9	10	11	12
b_i	-26.6	-20.1	-14.5	-12.0	-11.9	-12.5
ch_i	-26.8	-20.1	-14.5	-11.9	-11.9	-12.5

As can be seen, the lower harmonics are slightly smaller for the Chebyshev optimized coil. The sum of the squares of the harmonics over the first 12 coefficients shown can be used as a figure of merit of the optimization; $\Sigma b_i^2 = 4661$ and $\Sigma ch_i^2 = 4619$. Similar comparisons can be made between the Fourier-optimized coil and coils Chebyshev-optimized over an 8 cm radius (with an expansion radius of 2 cm at $x = 8$ cm) and over a 9 cm radius (with an expansion radius of 1 cm at $x = 9$ cm); the 8 cm comparison is $\Sigma b_i^2 = 7211$ and $\Sigma ch_i^2 = 6747$ and the 9 cm comparison is $\Sigma b_i^2 = 13477$ and $\Sigma ch_i^2 = 10222$. The Chebyshev-optimized coils are always better at these large radii.

However, the beam in RHIC does not traverse the DX magnet at these large radii; a typical circular orbit has an initial $x' = 3.85$ mrad, initial $x = 0.0$ cm and exit $x = 3.68$ cm, as mentioned in the Introduction. Table II gives a comparison of harmonics integrated along this trajectory, in unit-meters, of the Fourier-optimized coil and of the Chebyshev-optimized coil optimized from $x = 0$ to $x = 9$ cm; the optimization (reference) radius is 6 cm.

Table II
HARMONICS b_i AND ch_i INTEGRATED ALONG A TRAJECTORY

i	1	2	3	4	5	6
b_i	-0.58	-2.11	-0.25	-0.62	-0.34	-0.33
ch_i	-0.57	-2.12	-0.20	-0.53	-0.25	-0.27
i	7	8	9	10	11	12
b_i	-0.14	-0.18	-0.17	-0.12	-0.10	-0.13
ch_i	-0.14	-0.18	-0.17	-0.12	-0.10	-0

For the data of Table II, the $\Sigma b_i^2 = 5.59$ and the $\Sigma ch_i^2 = 5.40$, only slightly lower. Two similar orbits are 2) (proton on gold), $x' = -3.85$ mrad and $x = 0.0$ cm at the entrance and $x = 3.68$ cm at the exit and 3) (like species) $x' = 0.0$ and $x = 0.0$ at the entrance and $x = 3.49$ cm at the exit. Case 2 gives $\Sigma b_i^2 = 5.27$ and $\Sigma ch_i^2 = 5.16$, and case 3) gives $\Sigma b_i^2 = 5.30$ and $\Sigma ch_i^2 = 5.18$. The Chebyshev-optimized coil results in slightly lower integrated harmonics in each case.

IV CONCLUSIONS

Expansion of the field on the midplane of a dipole in Chebyshev polynomials and minimization of the leading coefficients is an effective way to flatten the field. A magnet thus optimized has substantially lower Fourier expansion coefficients at points on the midplane more than about 2/3 the coil radius from the origin. Integrated Fourier expansion coefficients of the field of a Chebyshev-optimized coil, along a typical beam trajectory which is mostly within 1/3 the coil radius, are only slightly better than those of the best Fourier-optimized coil.

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