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WEAK MATRIX ELEMENTS EFFORTS ON THE LATTICE: STATUS & PROSPECTS*

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WEAK MATRIX ELEMENTS EFFORTS ON THE LATTICE: STATUS & PROSPECTS*

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Abstract

Lattice approach to weak matrix elements is reviewed. Recent progress in treating heavy quarks on the lattice is briefly discussed. Illustrative sample of results obtained so far is given. Among them I elaborate on B_K, f_B and $B \to K^*\gamma$. Experimental implications especially with regard to constraints on the Standard Model (i.e. Wolfenstein) parameters, V_{td} measurements and expectations for B_s-$\bar{B}_s$ oscillations are briefly discussed.

Following is the outline of this talk:

1. Overview and Perspective

2. Theory: Brief Remarks

3. Theoretical developments on and off the Lattice for B-Physics

4. Samples of Results

5. Special Focus

- **5.1.** B_K: The Kaon B-Parameter
- **5.2.** f_B, f_{Bs}/f_{Bd} etc.
- **5.3.** $B \to K^*\gamma$

6. Implications for Phenomenology

- **6.1.** The Noose: Constraining the Standard Model
- **6.2.** Implication for Extraction of V_{td} from $B \to \rho + \gamma$

7. Summary and Outlook

*Plenary talk at the International Symposium on B-94, Nagoya, Japan, Oct. 1994.

1 Overview and Perspective

Despite innumerable triumphs, the Standard Model (SM) has many (irreducible) parameters, e.g. quark masses, gauge couplings, CKM angles etc. These parameters can be divided into two sets K and U; K being the known set and U the still unknown. The goal of many of the experiments is determination of the still unknown parameters (U). But experimental information rarely gives U directly; determination of U from experimental observations usually requires knowledge of non-trivial functions of the known set (K). These are non-trivial functions as they involve non-perturbative interactions. So that knowledge of K does not give us these functions in a simple fashion. To the extent that strong interactions, due to their non-perturbative nature are not soluble (at least at present), these functions themselves are treated as phenomenological hadronic parameters. A simple example is f_B, the pseudoscalar decay constant for the $B^{\pm}$ meson. Presumably f_B is a function of the quark masses m_b, m_u and the scale parameter of QCD, $\Lambda_{\rm QCD}$. So even if m_b, m_u, and $\Lambda_{\rm QCD}$ are precisely known we do not know how to deduce f_B from these subset (of K). To the extent that we do not know f_B we cannot use the measured value of x_d (the B-$\bar{B}$ mixing parameter) to deduce V_{td}.

Another simple example is the form factors for exclusive semi-leptonic decays. One of the most important one of this class is for the reaction $B \to \pi(\rho)e\nu$ as it controls V_{ub}. Recall that the differential spectrum for (e.g.) $B \to \pi e\nu$ goes as

$$\frac{dN}{dq^2} \sim |V_{ub}|^2 |f_+(q^2)|^2 \tag{1}$$

Thus precision with which we can extract V_{ub} from the experimental data on dN/dq^2 is controlled by how well we know $f_+(q^2)$, the form factor. Note that even at present the dominant source of error on V_{ub} comes from phenomenological models of f_+ and not from lack of data (i.e. statistics or experimental systematic errors). Indeed the practice of using the spread amongst the models to assign systematic errors, on such a fundamental quantity as V_{ub}, is itself displeasing and extremely unreliable. After all what if only one model calculation of this form factor was available. In recognition of these problems, the weak matrix elements effort [1,2] on the lattice has been in progress for about a decade with the aim to develop a comprehensive framework for calculating $f(K)$, the hadronic matrix elements.

The discussion above was centered exclusively to the SM. However, since QCD is an integral part of any beyond the SM theory, confrontation of experiment with any such theory also requires knowledge of hadronic matrix elements. Consider, e.g., $p \to e\pi$ wherein non-trivial QCD dynamics control the hadronic matrix element $\langle p|qqq|\pi\rangle$ and consequently reliable calculational methods for such matrix elements can have important bearing for the underlying (GUT) theory [3].

There is by now about forty lattice-physicists across the world working on hadronic matrix elements. In the remainder of this talk I will review the current status of these computations and also indicate what improvements we can expect in the next few years.

2 Theory: Brief Remarks.

1) Chiral Symmetry on the Lattice.

The effective Hamiltonian obtained after integrating out the heavy degrees of freedom provides the calculational basis:

$$H_{\rm eff} = \sum_i C_i O_i^{\rm cont} \tag{2}$$

where C's are the usual Wilson coefficients and $O^{\rm cont}$ the quark operators of the continuum theory. To the extent that the lattice action does not respect the symmetries of the continuum theory, in general the operators undergo mixing under renormalization [4]:

$$O_i^{\rm cont} = Z_i O_i^{\rm latt} + \sum_j Z_{ij} O_j^{\rm latt} \tag{3}$$

A particularly important example is the case when one has to deal with 4-quark operators. In this case $O_i^{\rm cont}$ and $O_i^{\rm latt}$ are $(V-A)\times(V-A)$ 4-quark operators. However, since Wilson fermions do not respect chiral symmetry on the lattice, the $O_j^{\rm latt}$ are $S\times S$, $P\times P\ldots$ This loss of chiral symmetry on the lattice for Wilson fermions can make extraction of matrix elements relevant to K, π physics extremely challenging [5].

Consider, as an example, the K-$\bar{K}$ mixing matrix elements. Lowest order chiral perturbation theory predicts that the matrix element of the 4-quark $(V-A)\times(V-A)$ operators vanishes in the chiral limit i.e. it is proportional to (meson mass)^2. The non $(V-A)\times(V-A)$ operators in eqn. (3), on the other hand, approach a constant as $m^2 \to 0$. Thus, in the limit of small meson mass, the matrix elements of the extraneous operator (O_j) must have significant cancellations amongst themselves for the complete matrix element to exhibit the behavior predicted by CPTh. This technical problem has made even a seemingly simple calculation, such as B_K, the kaon B-parameter, extremely difficult on the lattice, with Wilson fermions. Kogut-Susskind (or staggered) fermions have significant advantage in this regard as they manifestly respect chiral symmetry; in that formulation of fermions it is the flavor symmetries that present significant difficulties. The staggered approach therefore has made impressive progress in calculating B_K[6].

2) Matrix Elements in the Continuum Limit.

Although many symmetries of the continuum theory are not respected by the lattice action, those symmetries are retrieved in the limit as the lattice spacing $a \to 0$. For that and many other reasons it is very important to explicitly check the dependence on a of the matrix elements. Calculations done for several different values of β (i.e. a) can then be used to fit the data as a function of a and thereby extrapolate the data to the continuum limit, $a \to 0$. Unless there are explicit theoretical reasons for $O(a)$ scale breaking effects to be absent one anticipates them to be there. Thus the results obtained from different values of a can be fitted linearly. In some special instances, e.g. with improved Wilson actions, or staggered fermion (calculation of B_K) the term linear in a can be shown to be absent. Then one expects only $O(a^2)$ scale breaking effects to be present which means scale breaking effects should be significantly smaller compared to the $O(a)$ case.

3 Theoretical developments on and off the Lattice for B-Physics.

In the past few years significant progress has been made towards an improved treatment of heavy quarks on the lattice. Here are the important points:

1) Kronfeld-MacKenzie Normalization [7].

Kronfeld and MacKenzie have suggested a very simple way to limit the $O(a)$ error due to quark mass approaching the cut-off value, $am \sim 1$. By comparing the lattice quark propagator with its continuum counterpart in the free field limit, they suggest the renormalization of fermion fields as [7]

$$\psi^{\text{cont}} = [2\tilde{\kappa}\exp(a\tilde{m})]^{1/2}\psi^{\text{latt}} \tag{4}$$

with

$$a\tilde{m} = \ell n \left[\frac{1}{2\tilde{\kappa}} - 3\right] \tag{5}$$

where $\tilde{\kappa} = \kappa/8\kappa_c$ (κ being the hopping parameter and κ_c its critical value.)

2) Lepage-MacKenzie Improved Perturbation Theory [8].

Lattice weak coupling perturbation theory (LWCPT) provides an important bridge between the lattice operators and their continuum counterparts (see eqn. (3)). The coefficients (Z's in eqn. (3)) are calculated in LWCPT. Lepage-MacKenzie have made important contributions in this regard in stressing that g_{bare} is a poor expansion parameter. Instead they suggest use of a gauge coupling defined in terms of some physical quantity, such as the heavy quark potential

$$V(q) = -\frac{16}{3}\pi\alpha_V(q^2)/q^2 \tag{6}$$

This boosts the effective expansion parameter considerably. Here, as an example, is the effect on some of the commonly used couplings:

β	$g^2_{\rm bare}$	g^2_V
5.7	1.05	1.95
6.0	1.00	1.77
6.3	.95	1.62

The use of the boosted coupling has the important effect that it improves the approach to the continuum. In other words Monte Carlo data obtained at relatively strong couplings such as $\beta(\equiv 6/g^2) = 5.7$–$6.0$, in conjunction with the use of g^2_V for calculating the corresponding perturbation corrections, seems to show scaling behavior.

3) Isgur-Wise (Heavy Quark) Symmetry [9].

The realization that QCD possesses additional flavor symmetries in the limit of large quark mass becomes very helpful in computing the hadronic matrix elements of heavy-light mesons. The point is that many (heavy-light) quantities of physical interest have a rigorous expansion in $1/m_Q$, e.g. $\sqrt{m_Q}f_B$; $\sqrt{m_Q}T_2^{B\to K^*\gamma}(q^2_{\rm max})$, $A_1^{B\to\rho}(q^2_{\rm max})\ldots$ Lattice data attained at several values of heavy quark mass, i.e. $m_Q \gtrsim a^{-1}$, can therefore be fitted to the functional form suggested by heavy quark symmetry (HQS). Thus e.g.

$$\sqrt{m_Q}T_2 = A_0 + A_1\frac{1}{m_Q} + A_2\frac{1}{m_Q^2} + \ldots \qquad (7)$$

$$\sqrt{m_Q}f_m = B_0 + B_1\frac{1}{m_Q} + B_2\frac{1}{m_Q^2} + \ldots \qquad (8)$$

The fitted coefficients can then be used to deduce that matrix element of interest at values of experimental interest, i.e. $m_b \sim 5$ GeV.

Heavy quark symmetry of QCD therefore has the important consequence that actual simulations need not necessarily be done with $m_Q \sim m_b \sim 5$ GeV. With the help of HQS coarser lattices can be used to deduce some results pertaining to B-physics.

4 Samples of Results

Table 1 gives a sample of the results obtained so far for hadronic matrix elements in quenched lattice QCD [10]. In a few cases (B_K and f_K/f_π) some indication of the errors due to quenching has also become recently available either because of direct simulations or by comparison with experiment. For each physical quantity first the results from individual lattice groups are reported and compared and then a summary of the current status of that computation is given. In the numbers in the summary for each quantity the 90% CL errors that are given include statistical and systematic errors, based partly on the actual numbers from various groups; however, in part,

Table 1: SAMPLE OF RESULTS FOR HADRON MATRIX ELEMENTS FROM QUENCHED LATTICE QCD

QUANTITY	VALUE	AUTHORS (REMARKS)
The "B" Parameters		
$\hat{B}_K$	$.825 \pm .027 \pm .023$	Gupta, Kilcup, Sharpe (Staggered) [6]
	$.85 \pm .20$	ELC (Wilson) [11]
		Bernard, Soni (Wilson) [12]
$\hat{B}_K$	$.82 \pm .10$	Most likely $\equiv$ 90% CL [13] (inc. statistical and systematic errors)
$\hat{B}_B$	$1.3 \pm .2$	Bernard, *et al.* [14]
	$1.16 \pm .07$	ELC [15]
$\hat{B}_B \simeq \hat{B}_{B_S}$	$1.2 \pm .2$	Most Likely (90% CL) [13]
The Decay Constants		
f_K/f_π	$1.08 \pm .03 \pm .08$	Bernard, Labrenz, Soni [16]
f_D (MeV)	$174 \pm 26 \pm 46$	Bernard, *et al.* [14]
	190 ± 33	Degrand, Loft [17]
	210 ± 40	ELC [15]
	185^{+4+42}_{-3-7}	UKQCD [18]
	$208 \pm 9 \pm 32$	Bernard, Labrenz, Soni [16]
f_D (MeV)	197 ± 25	Most Likely (90% CL) [13]
f_{D_S} (MeV)	222 ± 16	Degrand, Loft [17]
	$234 \pm 46 \pm 55$	Bernard, *et al.* [14]
	230 ± 50	ELC [15]
	$212 \pm 4^{+46}_{-7}$	UKQCD [18]
	$230 \pm 7 \pm 35$	Bernard, Labrenz, Soni [16]
f_{D_S} (MeV)	221 ± 30	Most Likely (90% CL) [13]
f_B (MeV)	205 ± 40	ELC [20]
	$160 \pm 6^{+53}_{-19}$	UKQCD [18]
	$187 \pm 10 \pm 37$	Bernard, Labrenz, Soni [16]
	$188 \pm 23 \pm 15^{+27}_{-0} \pm 14$	FSG [20]
f_B (MeV)	173 ± 40	Most Likely (90% CL) [13]
f_{B_S} (MeV)	194^{+6+62}_{-5-9}	UKQCD [18]
	$207 \pm 9 \pm 40$	Bernard, Labrenz, Soni [16]
f_{B_S} (MeV)	201 ± 40	Most Likely (90% CL) [13]
f_{B_S}/f_B	$1.22^{+.04}_{-.03}$	UKQCD [18]
	$1.11 \pm .02 \pm .05$	Bernard, Labrenz, Soni [16]
	$1.22 \pm .04 \pm .02$	FSG [20]
f_{B_S}/f_B	$1.16 \pm .10$	Most Likely (90% CL) [13]
Radiative B Decays		
$R_{K^*} \equiv \frac{\Gamma(B\to\gamma K^*)}{\Gamma(b\to\gamma s)}$	$= 6.0 \pm 1.2 \pm 3.4\%$	Bernard, Hsieh, Soni [21]
	$8.8^{+2.8}_{-2.5} \pm 3.0 \pm 1.0$	UKQCD [22]

those 90% CL errors given in the summary for each item are subjective, it should be noted.

Included in this sample of results are the "B" parameters for kaons and B-mesons, the light-light and heavy-light pseudoscalar decay constants and the radiative form factors for B-mesons. In most instances the accuracy is $\sim 20\%$. For B_K the accuracy is remarkable $\sim 6\%$. Note that the calculated value of $f_K/f_\pi = 1.08 \pm .03 \pm .08$ differs from the experiment (1.21) by about 1.5 sigmas i.e. $\lesssim 15\%$. This difference may well be physical and could be the effect due to quenching but this is far from established given the size of the errors.

5 Special Focus

In the following I will elaborate on three important quantities selected from the above sample:

1. B_K
2. f_B
3. $B \to K^*\gamma$

5.1 B_K: The Kaon B-Parameter.

Recall that B_K is defined to be the ratio of the 4-quark K-$\bar{K}$ mixing matrix element divided by its value in vacuum saturation:

$$B_K \equiv \frac{\langle K|\bar{s}\gamma_u(1-\gamma 5)d\bar{s}\gamma_u(1-\gamma 5)d|\bar{K}\rangle}{\frac{8}{3}|\langle 0|\bar{s}\gamma_u(1-\gamma 5)d|K\rangle|^2} \tag{9}$$

This was the first matrix element attempted by various lattice groups, over a decade ago. The reason was that it appeared computationally extremely simple. However, as alluded to briefly in the first section, it was soon realized that the bad chiral properties of Wilson fermions make it extremely difficult to deduce precise results.

With staggered fermions Sharpe and collaborators have made remarkable progress in the last four years to attain high precision [6]. In 1991 Sharpe *et al.* reported [23] (contrary to their earlier indications [24]) that B_K has a non-trivial dependence on the lattice spacing a. Their calculations at $\beta = 5.7$, 6.0, 6.2 and 6.4 showed convincingly that B_K decreases with an increase in β. The result for $B_K^{a=0}$ (i.e. in the continuum limit) therefore had to be obtained by fitting the data to a polynomial in a. Fitting the data to the form $A_0 + A_2 a^2$ or to the form $B_0 + B_1 a$ gave them the results $\hat{B}_K \equiv B_K \alpha_s^{-2/9} = .78(3)$ or $.66(6)$ respectively.

At LAT'94 Sharpe *et al.* [6] reported significant progress in nailing down B_K. They now have theoretical arguments to show that the only viable fit to their data with staggered fermions has to be of the form $A_0 + A_2a^2$; the fit linear in a is not relevant. As a result they reported their final result [6]:

$$\hat{B}_K = .825 \pm .027 \pm .023 \tag{10}$$

There also has been some work on B_K in full-QCD. Using gauge configuration generated by the Columbia machine ($\beta = 5.7$, $n_f = 2$, $am = 0.01$) Kilcup [25] has shown that the difference in B_K calculated in the "full" theory and quenched theory (i.e. $\beta = 6.0$ quenched) is at best only a few percent. That difference seemed to be less than the statistical errors ($\lesssim 7\%$).

A similar study with full QCD ($\beta = 5.7, n_f = 2, am = 0.01$) has also been reported by Ishizuka *et al.* [26] with very similar results—B_K calculated in "full" theory (at $\beta = 5.7$) is in very good agreement with the results of the quenched theory at $\beta = 6.0$. The difference in the two was again found to be well within their statistical errors, i.e. $\lesssim 5\%$.

Although these studies are "exploratory" their findings are important in confirming the expectation that for most quantities of physical interest a significant fraction of the difference can be accounted for by making a small shift in the gauge coupling. In the above two studies $\beta = 5.7$ in the full theory corresponds to a great degree to $\beta = 6.0$ in the quenched theory.

Lastly, let me mention that there has also been some progress in improving the precision for B_K with Wilson fermions [12]. By now several methods exist for non-perturbatively dealing with the chiral symmetry problem [27–29]. We have analyzed most of the data at $\beta = 5.7$, 6.0 and 6.3 that was available to us as of Summer of 1994. We also find that B_K shows appreciable a dependence. In Fig. (1) we compare our numbers with the staggered ones for various β's. Wilson fermion results have a tendency to be a bit higher. The value for $B_K^{a=0}$ is obtained by fitting our data linearly in a. Thus we find

$$B_K^{a=0} = .70 \pm .09 \tag{11}$$

where the errors include statistical and systematic errors. Our results though a bit high is within one sigma of the result of Sharpe *et al.*: $B_K = .616 \pm .020 \pm .017$.

Based on all this and other information available we conclude that at 90% CL the value of $\hat{B}_K$ is given by

$$\hat{B}_K = .82 \pm .10 \tag{12}$$

5.2 f_B, f_{Bs}/f_{Bd} etc.

Much attention is being given by the lattice community for calculating f_B [30]. Although the calculation could in principle be straight forward the heaviness of the

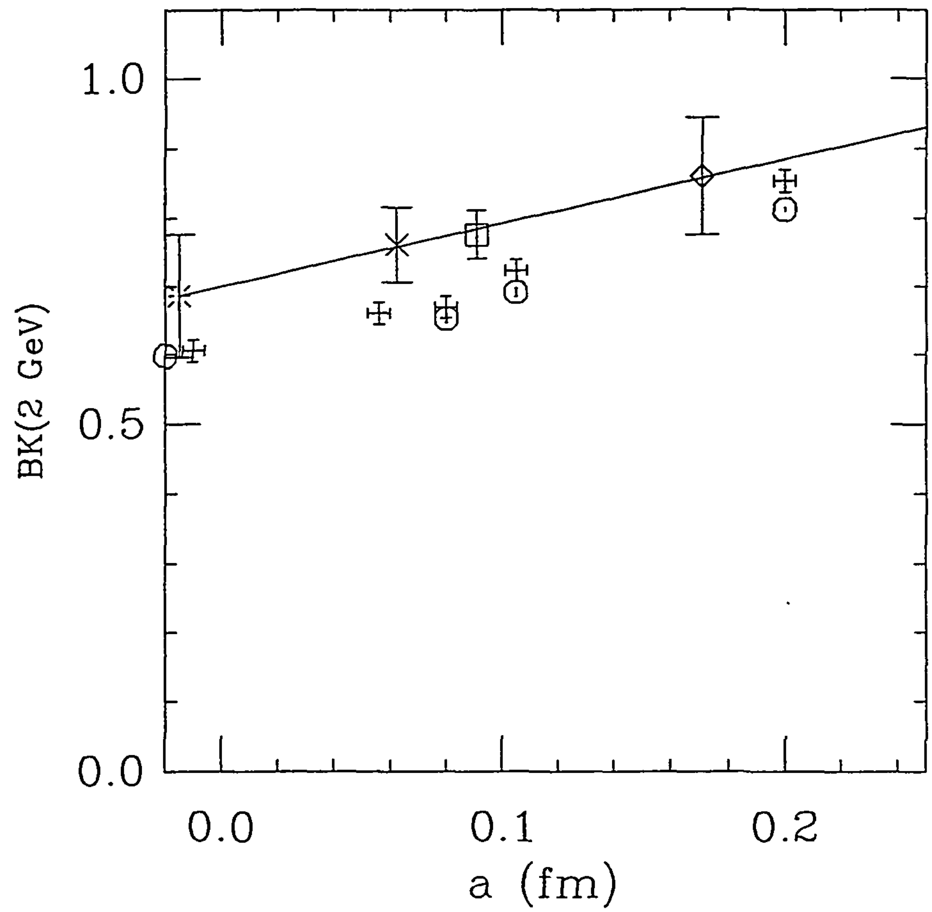

Figure 1: B_K versus the lattice spacing (a). Straight line is a fit to the Wilson data. Staggered method data tends to lie below the Wilson. Points shown on the vertical axis are the continuum values of B_K

b-quark, at present, presents problems due to the limitation of computational resources that are currently available. There are three methods that are being used for these calculations.

1. The propagating quark method [14]: Where quarks with mass $\sim$ cut-off are directly used with the hope that extrapolation to the physical mass of b-quark will be smooth. This method has become quite useful in conjunction with the Kronfeld-MacKenzie suggestion [7] for the normalization of the quark field and has been widely used.

2. The non-relativistic method [31]: Here the heavy quark propagator is used in the non-relativistic approximations. This method should start to produce interesting results soon.

3. The static quark method [32,30,20]: In this case the heavy mass is set to be infinite. Although in this limit the theory gains considerable simplicity the method is very challenging to implement numerically.

The earliest result on f_B obtained by using the propagating quark method gave a rather low value, $f_B \sim 105 \pm 17 \pm 35$ MeV [14]. There was a period of about two years $\sim$ 1989–1991 when there was considerable confusion as the static method tended to give very high values [33] ($\sim$ 350 MeV) in comparison to that old result obtained by us [14]. Though it must be noted that there always were problems of internal consistency in the analysis with the high static values [34,35]. Many groups have been using sophisticated sources for improving the signal/noise ratio which is a very serious problem with the static method [35–37,20]. With the use of these the static method no longer gives very high values for f_B^{static}. Indeed, the "FNAL static group" has recently completed their extensive study with the static method and find $f_B^{static} \sim 188 \pm 23 \pm 15^{+27}_{-0} \pm 14$ MeV[20]. In our second attempt at f_B, which formed the basis of Jim Labrenz's PhD thesis [36], the propagating quark method (with the use of the KM norm [7]) and the static method [32] were both used. As we have stressed since LAT'91 [35] we did not see any big discrepancy between the two methods [16]. Indeed our final result ($f_B = 187 \pm 10 \pm 37$ MeV) is obtained through interpolation between the two methods. This result is quite consistent with the UKQCD result ($f_B = 160 \pm 6^{+53}_{-19}$ MeV) [18]. In reporting the 90% level final answer ($f_B = 173 \pm 40$ MeV) given in Table 1, I have been influenced mostly by these two works.

Note that the use of the KM renormalization for the heavy quarks (which was not used in our old paper) would raise our old [14] published ($105 \pm 17 \pm 34$ MeV) result by about a factor of 1.5 and would make it quite consistent with modern numbers.

Most of the existing calculations for f_B were not dedicated ones i.e. f_B was obtained along with many physics results such as form factors etc. Consequently these calculations were not optimized for heavy light decay constants. In the last two years MILC collaboration has been making a dedicated effort to nail down f_B [38]. They are using a relatively large ensemble of gauge configuration at $\beta = 6.3$, 6.0 and 5.7. Preliminary results indicate $f_B \sim 150$ MeV. It is expected that the final result would have an accuracy of about 15% (including statistical and all systematic errors other then quenching).

Ratio of Decay Constants.

Table 2 gives various ratios of decay constants. These ratios test various flavor symmetries of QCD. For example f_K/f_π, f_{Bs}/f_B test the SU(3) symmetry of light quarks, f_{Ds}/f_{Bs} and f_D/f_B test the heavy quark symmetry.

f_{Bs}/f_B is extremely important phenomenologically. It can be used along with B-$\bar{B}$ mixing results and V_{td}/V_{ts} to predict Bs-$\bar{B}s$ oscillations. Conversely once Bs-$\bar{B}s$ mixing is experimentally measured f_{Bs}/f_B can be used to deduce V_{td}/V_{ts}. We will come back to this point later on. For now we want to draw attention to the three points regarding these ratios (Table 3).

1) There is about a two sigma difference in f_{Bs}/f_B between our group [16] ($1.11 \pm .02 \pm .05$) and the UKQCD [18] results ($1.22^{+.04}_{-.03}$).

2) In any case, for now, I am quoting a 90% CL number for this ratio as:

$$f_{Bs}/f_B = 1.16 \pm .10 \qquad (90\% \text{ CL}) \tag{13}$$

Table 2: Flavor Symmetries and Ratios of Decay Constants.

Group⇒ Ratio ⇓	FSG [20]	BLS [16]	APE [41]	UKQCD [18]
f_K/f_π		$1.08 \pm .03 \pm .08$		
f_D/f_B		$1.11 \pm .03 \pm .05$		
f_{D_s}/f_{B_s}		$1.11 \pm .02 \pm .05$		
f_{D_s}/f_D		$1.11 \pm .02 \pm .05$		$1.18 \pm .02$
f_{B_s}/f_B	$1.22 \pm .04 \pm .02$	$1.11 \pm .02 \pm .05$	$1.11 \pm .03$	$1.22^{+.04}_{-.03}$
f_{B_s}/f_B Summary: $1.16 \pm .10$ @ 90%CL				

Table 3: A sample compilation of the predictions for $R_{K^*} \equiv [\Gamma(B - K^*\gamma)/\Gamma(b \to s\gamma)]$. See Ref. [42].

Author(s)	R_{K^*}	Method
O'Donnell (1986)	97%	
Deshpande *et al.* (1988)	6%	Quark Model
Domingues *et al.* (1988)	$28 \pm 11\%$	QCD Sum Rules
Altomari (1988)	4.5%	Quark Model
Deshpande *et al.* (1989)	6–14%	"
Aliev *et al.* (1990)	39%	QCD Sum Rules
Ali *et al.* (1991)	28–40%	Extended HQS
Du *et al.* (1992)	69%	"
Faustov *et al.* (1992)	6.5%	
El-Hassan *et al.* (1992)	$\sim 0.7\% - 12\%$	
O'Donnell *et al.* (1993)	$\sim 10\%$	Quark Model
Ali *et al.* (1993)	$13 \pm 3\%$	QCD Sum Rules
Ball (1994)	$20 \pm 6\%$	"
Bernard *et al.* (1994)	$(6.0 \pm 1.2 \pm 3.4)\%$	Lattice
UKQCD (1994)	$8.8^{+2.8}_{-2.5} \pm 3.0 \pm 1.0$	"

for use in phenomenology. We anticipate that this ratio will be pinned down to 4% (1 sigma) accuracy in the next year or so.

3) I want to stress the importance of experimental measurements of f_{D_s} [39]. Lattice results which now give $f_{D_s}/f_{B_s} = 1.22 \pm .02 \pm .05$ could be used with a definitive experimental measurement of f_{D_s} for a prediction for f_{B_s} and f_B. Of course a definitive experimental measurement of f_{D_s} would provide an invaluable check on lattice calculation for f_{D_s}. Note that lattice measurements for f_{D_s} have existed in the literature for over 6 years and the numbers have been quite stable over the years. As given in Table 1, lattice computations give:

$$f_{D_s} = 221 \pm 30 \text{ MeV} \qquad (90\% \text{ CL}). \tag{14}$$

5.3 $B \to K^*\gamma$.

In recognition of the importance of the radiative decays of the b quark and the difficulty that continuum methods have in calculating the exclusive to inclusive ratios, the study of this reaction was initiated in 1990. Preliminary results were reported in Lattice '91 and '92 and the final results at Lattice '93 [40]. By now two lattice groups have published the results [21,22]. Results from a third group should be forthcoming soon [41].

On the lattice the calculation boils down to computing either T_1 or T_2 form factor. In practice both are done as many tests then become available. Calculation of $T_2(q^2\text{max})$ is very clean as it requires no momentum injection. Also heavy quark symmetry makes a prediction for $T_2(q^2\text{max})$:

$$\sqrt{m_M}T_2(q^2\text{max}) = A_0 + A_1\frac{1}{m_M} + \ldots \tag{15}$$

This can be used for reliable extrapolation to m_B. T_2 has the disadvantage that the actual physical reaction is at $q^2 = 0$, far from q^2max. We found that simple pole dominance works remarkably well ($\lesssim 13\%$ accuracy in the domain of momenta studied). Therefore we used pole-dominance to get our final result. Herein the largest source of systematic error is in fact due to the pole dominance ansatz.

UKQCD [22] has also studied $B \to K^*\gamma$ with both T_1 and T_2. Their latest result is indeed in even better agreement with us than their first number.

Note that in addition to studying the applicability of pole-dominance we also actually study the validity of HQS relation (eqn. (15)) and found the data in good agreement. Meson masses ranging from about 1.5 to 3.5 GeV (i.e. appreciably larger than charm mass) were used in this study.

Table 3 compares the completed lattice results with those obtained by use of other methods. Although in these first generation lattice calculations the errors are considerably large ($\sim$ 25% in amplitude) they are significantly better than other methods. Lattice finds the ratio to be about 6.5% with about 50% error whereas continuum methods ranged from a fraction of a percent to tens of percents. It is fair to say that even at this early stage lattice has made an important contribution on this issue. Of course the quality of the lattice results is expected to improve in the next few years.

6 Implications for Phenomenology.

In passing, I mention some phenomenological implications of the lattice results presented in the preceding pages.

6.1 The Noose: Constraining the Standard Model [43].

An important mission of the weak matrix element effort on the lattice, in conjunction with experimental results, is to deduce reliable constraints on the SM. In particular, much attention is focused on the Wolfenstein parameters ρ, η or equivalently the angles α and β of the unitarity triangle [44]. Using the lattice value of B_K ($\hat{B}_K = .82 \pm .10$) with the kaon CP violation parameter ϵ_K leads to one set of curves (see Fig. (2)) which enclose the allowed domain of the SM. Similarly, the B parameter for B-mesons ($\hat{B}_B = 1.2 \pm .2$) and f_B ($f_B = 173 \pm 40$ MeV) with the experimentally measured B-$\hat{B}$ mixing parameter, x_d, yields two more curves enclosing the allowed area for the SM by this set of considerations. Finally, the experimental observations of semi-leptonic B decays and the B-lifetime yield $\frac{V_{ub}}{V_{cb}}$ (we use $V_{ub}/V_{cb} = .08 \pm .03$ and $V_{cb} = .04 \pm .007$) leading to the two concentric circles on Fig. (2).

Note that amongst the experimental results providing the SM constraints, so far the lattice has not entered, in any serious way, the deduction of V_{ub}/V_{cb}. Of course much work is being done in this context too [1] but the accuracy of the lattice work has not yet reached the same degree of maturity as B_K or f_B.

The hatched area in Fig. (2) shows where the SM parameters can still lie [45]. Hopefully improvements in the lattice calculations along with experiments will soon eliminate any allowed domain for the SM in these plots!

Two other important quantities deducible, by use of the lattice results, are V_{td}/V_{ts} and x_s/x_d.

Using $f_B = 173 \pm 40$ MeV, $\hat{B}_B = 1.2 \pm .2$ along with the experimental result on B_d-$\hat{B}_d$ mixing (x_d) and the measured b-lifetime leads to:

$$\frac{V_{td}}{V_{ts}} = .22 \pm .08 \tag{16}$$

Next the relation between the mixing parameters for B_s-$\bar{B}_s$ and B_d-$\bar{B}_d$ can be quantified.

$$\frac{x_s}{x_d} \sim |f_{B_s}/f_{B_d}|^2 \frac{B_{B_s}}{B_{B_d}} \left|\frac{V_{ts}}{V_{td}}\right|^2 \tag{17}$$

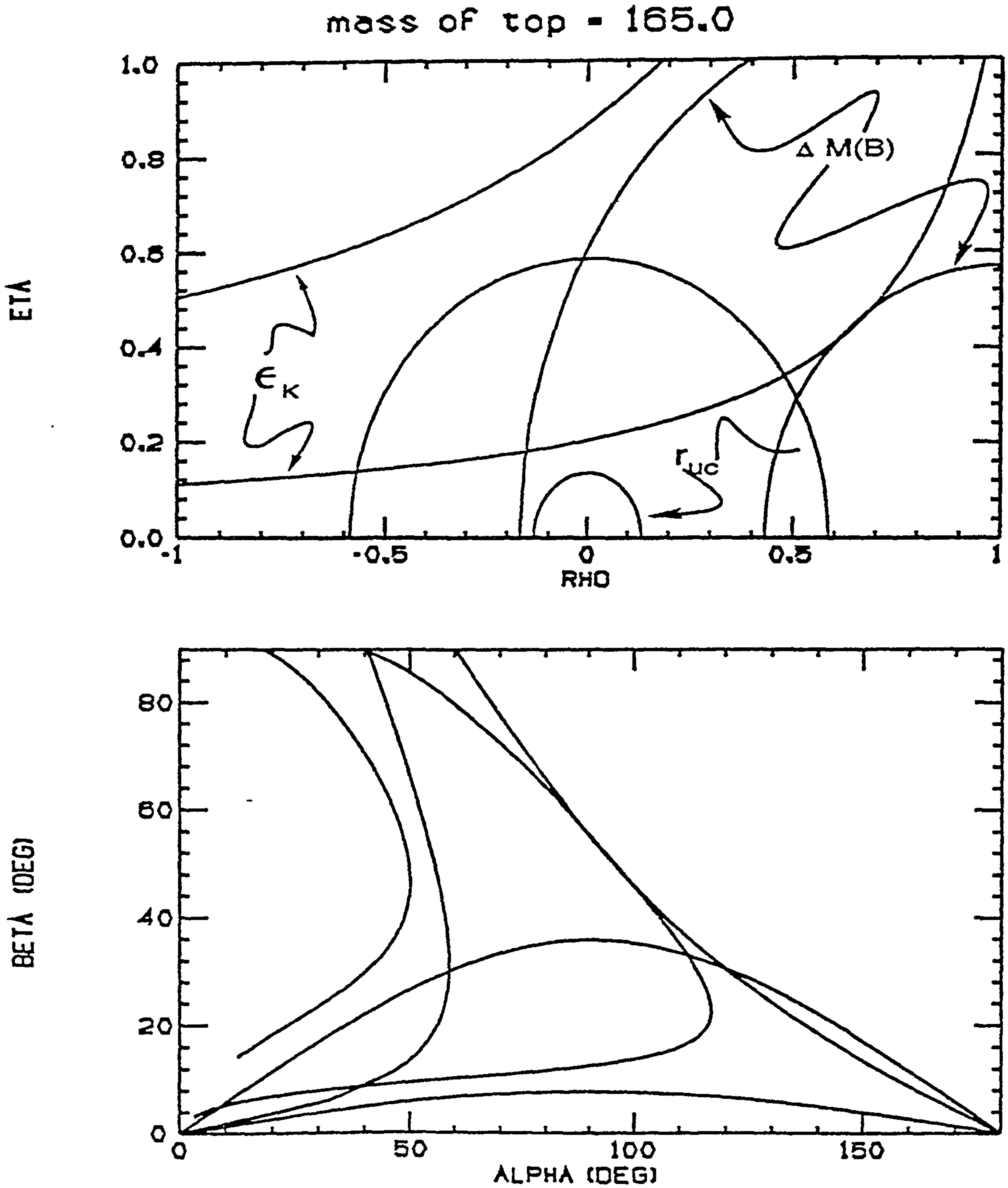

Figure 2: The "Noose": Constraints on the SM parameters (ρ, η or α, β) by using experiments and input from the lattice.

With the ratio $f_{B_s}/f_{B_d} = 1.16 \pm .10$ given before, $B_{B_s}/B_{B_d} = 1.00 \pm .05$, and $\frac{V_{td}}{V_{ts}}$ from eqn. (16), we get

$$\frac{x_s}{x_d} \sim 18 \pm 14 \Rightarrow x_s/x_d < 50 \tag{18}$$

Although this ratio is far from precise the upper bound appears safe and already leads to the important experimental implication that B_s-$\bar{B}_s$ oscillations may be observable at LEP, SLC and at hadron machines such as HERA-B.

6.2 Implication for Extraction of V_{td} from $B \rightarrow \rho + \gamma$ [46].

Lattice calculations for $B \rightarrow K^*\gamma$ find the exclusive to inclusive ratio to be about $6.5 \pm 3\%$. Experimentally $BR(B \rightarrow K^* + \gamma)$ and $B(b \rightarrow s + \gamma)$ put the ratio to be about 21% with an error around 10%. The two results are certainly not inconsistent, given the size of the errors. However, if improved lattice calculations and improved experiments confirm their respective central values then the difference has the very important implication that experimental signal for $B \rightarrow K^* + \gamma$ implies contamination from long distance contributions. The crucial point is that lattice calculation only include the short-distance piece so the difference between the experiment and the lattice is a measure of the long distance contribution. The prime source for such a long distance contribution is $B \rightarrow \psi_{\text{virtual}} + K^*$ followed by $\psi_{\text{virtual}} \rightarrow \gamma$ as $B \rightarrow \psi + K^*$ has a hefty BR of around $\sim 10^{-3}$ [47]. Historically long distance contributions are notorious to calculate with any degree of reliability. The situation with regard to $B \rightarrow K^*\gamma$ is extremely interesting as the difference between the experiment and the lattice is a reliable quantitative measure of the long distance contribution.

If one makes a very strong assumption that long distance contributions to $B \rightarrow K^*\gamma$ and to $B \rightarrow \rho\gamma$ are negligible and the further assumption that SU(3) symmetry holds well, then V_{td} can be extracted from $B \rightarrow \rho + \gamma$ via [48]

$$\frac{B \rightarrow K^* + \gamma}{B \rightarrow \rho + \gamma} = \left|\frac{V_{td}}{V_{ts}}\right|^2 \tag{19}$$

This simple relation should not be used [46] for a deduction of $\frac{V_{td}}{V_{ts}}$ as it has serious problems:

1. Long distance pieces from $B \rightarrow \psi + K^*$ and $B \rightarrow \rho + K^*$ can contribute to the numerator and denominator of eqn. (19). As discussed above the indications at present are that these LD contributions are not negligible, possibly invalidating eqn. (19).

2. A second problem with eqn (19) is that processes such as $B^- \rightarrow \rho^- + \rho^0_{\text{virtual}}$ followed $\rho^0_{\text{virtual}} \rightarrow \gamma$ tend to invalidate eqn. (19). These processes mean that $B \rightarrow \rho + \gamma$ occurs even if $V_{td} = 0$; these contributions are driven, in fact, by V_{ub}. Thus eqn. (19) should be modified to:

$$\frac{BR(B \to \rho + \gamma)}{BR(B \to K^* + \gamma)} = A\frac{|V_{td}|^2}{|V_{ts}|} + B\frac{|V_{ub}|^2}{|V_{ts}|^2} \tag{20}$$

where A, B depend on SU(3) validity, phase space etc.

This class of LD contributions are expected to be significantly more serious for $B^\pm \to \rho^\pm+\gamma$ than for $B^0 \to \rho^0+\gamma$ [46]. Thus the branching ratios into $\rho+\gamma$ for charged and neutral B need not be equal. Indeed the difference in the $BR(B^\pm \to \rho^\pm + \gamma)$ and $BR(B^0 \to \rho^0 + \gamma)$ is an important measure of this type of LD contamination. Serious efforts to measure V_{td} via radiative B decays require separate measurements of charged and neutral B to $\rho + \gamma$

7 Summary and Outlook.

1. Lattice methodology is giving some numbers for use in phenomenology. Although at the moment, in most cases, the precision is limited, improvements in such a computational effort are bound to come with time. The kaon B_K parameter is a case in point in this regard. Indeed, it is this aspect (of improvements of results with time) that makes lattice methods completely distinct from phenomenological models.

2. Below are some examples. Given is the current status along with expectations for the near future and some remarks

Quantity	Current Status	Expectations (in $\lesssim 2$ years)	Remarks
1. $\hat{B}_K$	$.82 \pm .10$ (90%CL)	$\pm.10$	No improvement foreseen except in acquiring a better grip of quenching errors
2. $\hat{B}_B$	$1.2 \pm .2$ (90%CL)	$\pm.07$	
3. f_{D_s}	221 ± 30 MeV (90%CL)	±20	Challenge to experimentalists
4. f_B	173 ± 40 MeV (90%CL)	±20	
5. f_{B_s}/f_{B_d}	$1.16 \pm .10$ (90%CL)	$\pm.05$	
6. $\frac{B \to K^* + \gamma}{b \to s + \gamma}$	$6.0 \pm 1.2 \pm 3.4\%$	$\pm25\%$ (error)	

3. In the quenched approximations application to more physical processes is primarily (wo)manpower limited; less limited by computing power.

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