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Kraków.

REPORT NR 1651/AP

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Kraków, 1993

WYDANO NAKŁADEM
INSTYTUTU FIZYKI JĄDROWEJ
IM. HENRYKA NIEWODNICZAŃSKIEGO
KRAKÓW, UL. RADZIKOWSKIEGO 152

Kopia offsetowa, druk i oprawa: DRUKARNIA IFJ
Współpraca wydawnicza: SEKCJA WYDAWNICTW
DZIAŁU INFORMACJI NAUKOWEJ IFJ

Wydanie I zam. 5/94 Nakład 75 egz.

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ABSTRACT

A method of the thermal neutron time decay constant measurement in small bounded media is presented. A 14 MeV pulsed neutron generator is the neutron source. The system of recording of a die-away curve of thermal neutrons consists of a ^{3}He detector and of a multichannel time analyzer based on analyzer Canberra 35+ with multiscaler module MCS 7880 (microsecond range). Optimum parameters for the measuring system are considered. Experimental verification of a dead time of the instrumentation system is made and a count-loss correction is incorporated into the data treatment. An attention is paid to evaluate with a high accuracy the fundamental mode decay constant of the registered decaying curve. A new procedure of the determination of the decay constant by a multiple recording of the die-away curve is presented and results of test measurements are shown.

1. INTRODUCTION.

A pulsed beam of fast neutrons produced by a pulsed neutron generator is the source of the decaying thermal neutron flux which can be observed in a medium of interest. Knowledge of the decay constant of the thermal neutron flux in a bounded medium gives an information on the thermal neutron transport and diffusion parameters of the medium. High values of the decay constant (which correspond to the thermal neutron time of life in the range of 20 μs to 50 μs) are of the interest in the present research. These values characterize the decay of the thermal neutron fields in small volumes of the investigated materials having a high absorption cross section and/or characterized by a high heterogeneity. In these cases effects of a deformation of the time and energy neutron spectra are of a great importance. Theories which describe the behavior of the time dependent thermal neutron field in bounded media (including problems of the cooling of the energy spectrum and the extrapolation distance both for the homogeneous and heterogeneous bounded media) can be verified by a measurement of the fundamental mode of the time decay constant in the medium of interest. The values of the decay constants should be measured with an accuracy better than 0.5 per cent and the measurement time should be reduced to a minimum. This requires a proper experimental system with precisely defined parameters.

The method of measurement of the thermal neutron time decay constant described here is based on the measuring and recording system which consists of a thermal neutron ^{3}He detector, pulse forming and amplifying electronics, and the multichannel time analyzer Canberra 35+. The main item of the system is a fast multiscaler (MCS 7880) and the accuracy of the measurement is strongly influenced by its performance and possibilities.

The methodology of the measurement has to be adjusted to the equipment used. An experimental verification of a dead time of the instrumentation system has to be done and a count-loss correction has to be introduced to the data collected by the time analyzer. Optimum parameters for the measurement and for the determination of the decay constant have to be adjusted to the instrumentation system being used. The operation system of the registration part of the Canberra 35+ analyzer and the multiscaler have forced us to develop an unconventional procedure of the die-away curve collection and of the determination of the decay constant. The procedure is presented including a detailed statistical description and results of test measurements.

The methodology described below can be applied to instrumentation systems developed for thermal neutron time decay constant measurement different from the particular one used here.

2. THERMAL NEUTRON TIME DECAY IN BOUNDED MEDIUM.

A fast neutron burst in a bounded medium creates, after some delay time, a thermal neutron flux $\phi(t)$ which decays in time in the medium. It is described by a sum of exponentials and continuum function of the decay constant (SJÖSTRAND, 1985; WOŹNICKA, 1991):

$$\phi(t) = \sum_{\lambda_k < \lambda_c} A_k \, e^{-\lambda_k t} + \int_{\lambda_c}^{\infty} B_\lambda \, e^{-\lambda t} \, d\lambda \quad , \tag{1}$$

where

A_k – amplitude of the k-th mode flux,
B_λ – function of the continuum spectrum,
λ_k – decay constant of the k-th mode thermal neutron flux,
λ_c – Corngold's limit.

Higher modes of the thermal neutron flux usually vanish very fast and in most experiments, when the fundamental decay constant λ_0 is of main interest, the formula (1) can be simplified for only a few modes:

$$\phi(t) = \sum_{k=0}^{K} A_k \, e^{-\lambda_k t} \quad , \qquad k = 0, 1, 2, ..., K. \tag{2}$$

The fundamental mode decay constant λ_0 can be evaluated from the measured decay curve of the thermal neutron flux using more or less complicated fitting procedures (cf. DROZDOWICZ *et al.*, 1993). An idea of the pulsed thermal neutron experiment is presented in Fig. 1. After the fast neutron burst of the width T_n the created thermal neutron flux decays in time. In certain experimental systems (especially in hydrogenous media) and in certain geometrical conditions, the higher modes of the thermal neutron flux can be considered as vanishing during the time interval t_{0s}. This offers a possibility of a quite easy separation of the fundamental decay constant. Then the die-away curve in the interval T_z can be described by a single exponential function with a background. The next fast neutron burst arrives after the time T_{rep} when all thermal neutrons produced by the previous burst have vanished.

The process of the thermal neutron decay is determined by a transport of neutrons in the medium. The diffusion theory uses a formula which joins the fundamental mode decay

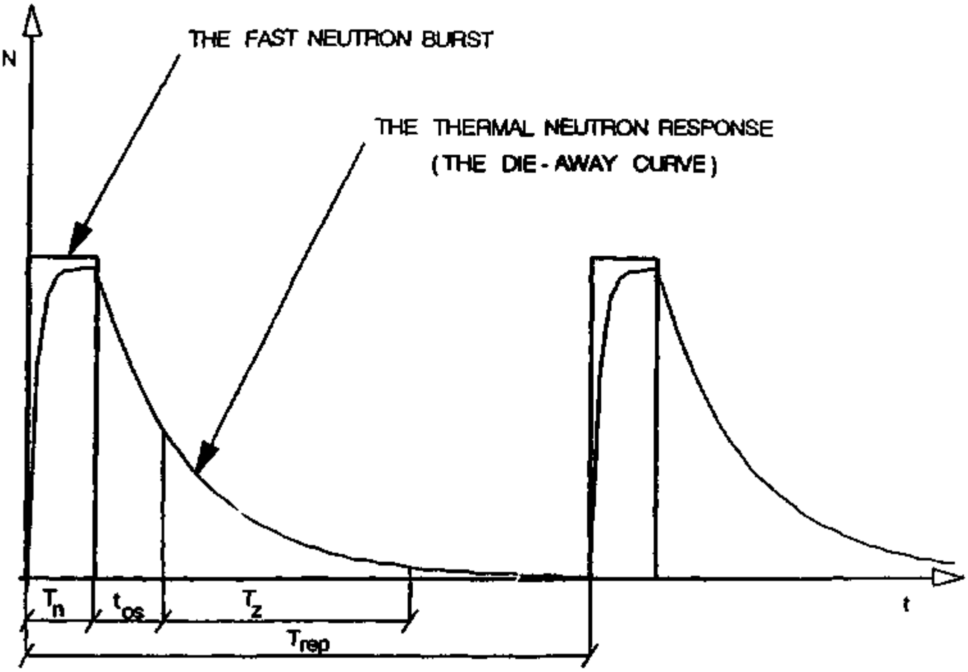


Fig. 1. Fast neutron burst and thermal neutron time decay.

constant and the thermal neutron parameters of the medium:

$$\lambda_0 = \langle \upsilon\Sigma_a \rangle + D_0 B_g^2 - C B_g^4 + \dots \, , \tag{3}$$

where:

λ_0 – decay constant of the fundamental mode thermal neutron flux,

υ – velocity of thermal neutrons,

Σ_a – thermal neutron macroscopic absorption cross section of the medium,

$\langle \upsilon\Sigma_a \rangle$ – flux averaged thermal neutron absorption rate (cf. WOŹNICKA, 1991),

D_0 – diffusion constant of the medium,

C – diffusion cooling coefficient of the medium,

B_g^2 – geometric buckling of the medium (*see e.g.* BECKURTS and WIRTZ, 1964; DROZDOWICZ, 1981). For the cylinder of radius R_g and high H_g the buckling is:

$$B_g^2 = \left(\frac{j_0}{R_g + d_R} \right)^2 + \left(\frac{\pi}{H_g + d_H} \right)^2 \quad , \tag{3a}$$

where

d_R, d_H – extrapolation distances linked to the radius and height, respectively,

j_0 – the first zero of the Bessel function of the first kind of the order zero.

A maximum discrete value of the λ_0 which can be experimentally observed is determined by the Corngold's limit (SJÖSTRAND, 1985):

$$\lambda_0 < \lambda_c = v\Sigma_t \quad , \tag{4}$$

i.e. the maximum value of the λ_0 is lower than the macroscopic total cross section Σ_t of the medium multiplied by the neutron velocity. In real experimental conditions, particularly for hydrogenous media, the value of the fundamental decay constant does not reach the Corngold's limit. *E.g.* for water the Corngold's limit is near 300 000 s^{-1} and the highest experimental values of the λ_0 observed in measurements are about 50 000 s^{-1}. That means that we are far from the continuum spectrum of the time distribution given in Eq. (1).

3. EXPERIMENTAL SET-UP.

The experimental set-up for the measurement of the die-away curve of thermal neutrons (Fig. 2) is situated on a moving platform closed to the tritium target of the pulsed neutron generator. The tritium target is on the axis of the irradiated sample which has a symmetric shape (a sphere, a cuboid, a regular cylinder). The neutron detector is placed at the bottom of the sample on the perpendicular symmetry axis. Such a position eliminates some higher modes of the thermal neutron flux. The sample as well as the neutron detector are covered by a cadmium layer to eliminate a disturbance from external thermal neutrons which are present in the experimental hall. Only a well-defined opening in the cadmium sheet between the bottom of the sample and the detector surface is made. Therefore, the thermal neutrons from the interior of the sample only are measured by the detector. An extra borated paraffin shield is used to reach better experimental conditions.

The 14 MeV fast neutrons are produced by a linear accelerator in the $^3H(d,n)^4He$ reaction. Solid tritium targets $^3H/Ti$ of activities from 70 GBq to 180 GBq (2 Ci to 5 Ci) are used. The accelerator works in a pulse regime by pulsing the extraction voltage of the ion source. The pulsing system is characterized by the following parameters:

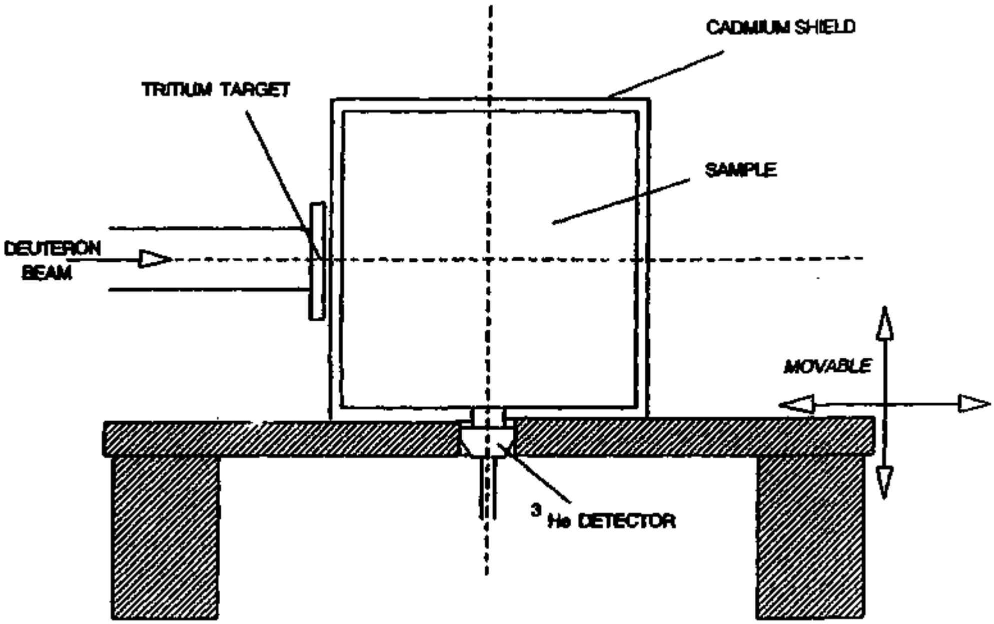


Fig. 2. Experimental set-up for the measurement of the thermal neutron time decay.

– width of the neutron burst: from 40 μs to 170 μs, adjustable,
– repetition time of neutron bursts: from 0.3 ms to 99 ms, adjustable,
– maximum neutron flux during the neutron burst: 5×10^9 n/(s 4π).

After each fast neutron burst the neutrons are slowed-down in the sample and thermal neutrons diffuse up to the final absorption in the sample or are scattered outside. The next neutron burst appears when the process of the diffusion is finished. The time parameters of the neutron generator are adjusted to the range which permits to record the neutron die-away curves characterized by the time decay constants λ_0 from 4000 s^{-1} to 50 000 s^{-1} (it corresponds to the thermal neutrons lifetime from 20 μs to 0.25 ms). For homogeneous hydrogenous media these decay constants are observed for the geometric buckling in the range from 0.03 cm^{-2} to 1.3 cm^{-2}.

4. INSTRUMENTATION SYSTEM FOR REGISTRATION OF THE THERMAL NEUTRON TIME DECAY CURVE.

A simplified block diagram of the instrumentation system for the registration of the thermal neutron time decay curve is shown in Fig. 3. The system consists of the following units:

I. Simple monitoring system with a BF_3 neutron detector surrounded with paraffin.

II. Main measuring system:

1. ^{3}He neutron detector.
2. Pulse forming and amplifying electronics.
3. Control system.
4. Multiscaler MCS 7880.
5. Multichannel time analyzer Canberra 35+ and IBM/PC.

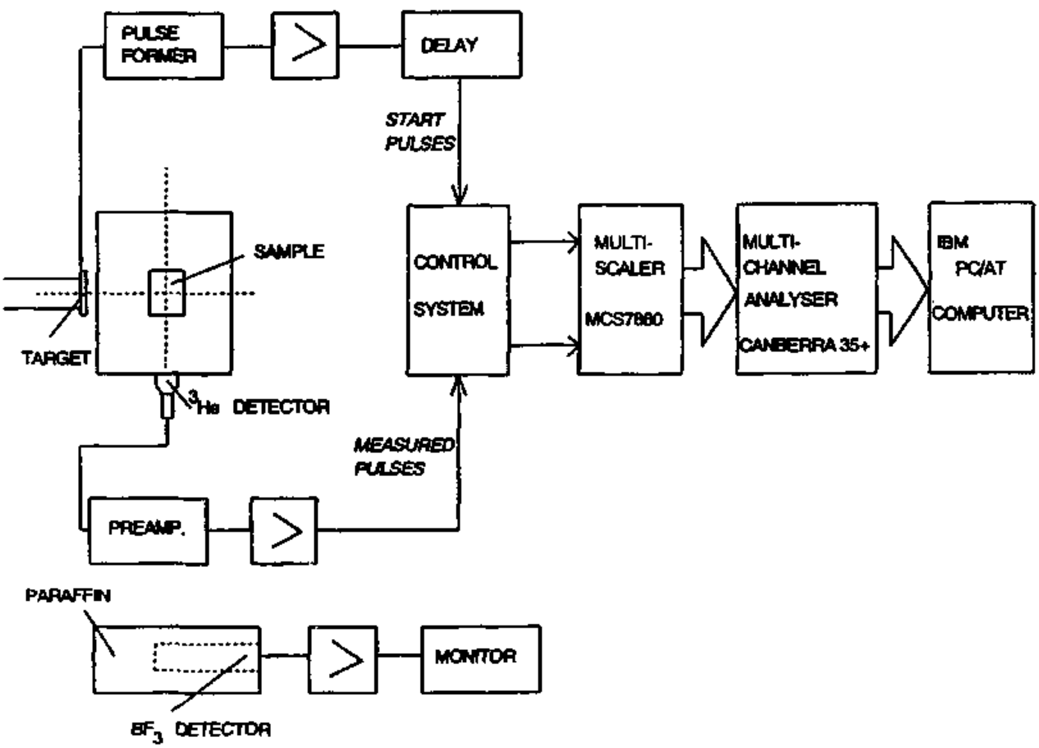


Fig. 3. Block diagram of the instrumentation system for the registration of the thermal neutron time decay curve.

ad 1) The thermal neutron detector is bell-shaped and filled with Helium-3 of the pressure of 0.3 MPa. Its sensitive volume is 56 mm in diameter and 56 mm in length. The operating voltage for the detector is from 2200 V to 2400 V.

ad 2) The start and measured pulses for the recording part of the system are produced by the pulse forming and amplifying electronics. These electronic systems consist of preamplifier, amplifiers, discriminators, delay units, H.V. power supplies, *etc.*

ad 3) The control system interrupts a collection of data if any disturbance in the work of the whole system occurs. The measurement should be done at conditions kept constant during the whole experiment. A constant average neutron flux, a proper counting statistics, a stable temperature, *etc.*, are required. Knowledge of the total number of the multiscaler runs and of the total number of measured pulses accumulated during a given die-away curve measurement is indispensable for further data treatment. In addition, the real time of the measurement and the total monitor counts are recorded. The main purpose of the control unit operation is to deliver proper start pulses to the multiscaler when the following conditions are fulfilled:

– the average intensity of measured pulses is contained in a given interval,
– the start pulses delivered by the start pulse former are correct in shape and frequency, which means that the ion source, the pulsed extraction voltage generator and the H.V. supply of the accelerator operate correctly,
– the background does not exceed assumed level,
– the vacuum pumps work correctly,
– the required constant temperature of the measured system is kept.

The control system was manufactured at the Institute of Nuclear Physics (BURDA *et al.*, 1982).

ad 4) The fast multiscaler, an essential part of the measuring system, is the MCS 7880 one manufactured as an external unit for the Canberra MCA 35+ analyzer. Its main parameters are following: dwell time from 0.2 μs to 1 sec in steps of 1, 2, 5; dead time between channels less than 5 ns.

ad 5) The multichannel analyzer, Canberra 35+, is equipped with an interface to the PC/XT or AT computers. The data collected in the analyzer are recorded on a diskette. The multiscaler, the analyzer, and the PC computer, are separated electrically from the other parts of the measuring system. All pulses required in this part of the measuring system are provided using the fiber optic links.

The method of data collection after each neutron burst is explained on the timing diagram presented in Fig. 4. The start pulse for the multichannel time analyzer is produced after each neutron burst and releases a single run of the multiscaler. (The pulsed current of deuterons produces voltage pulses in a start pulse voltage former). Only few measured pulses

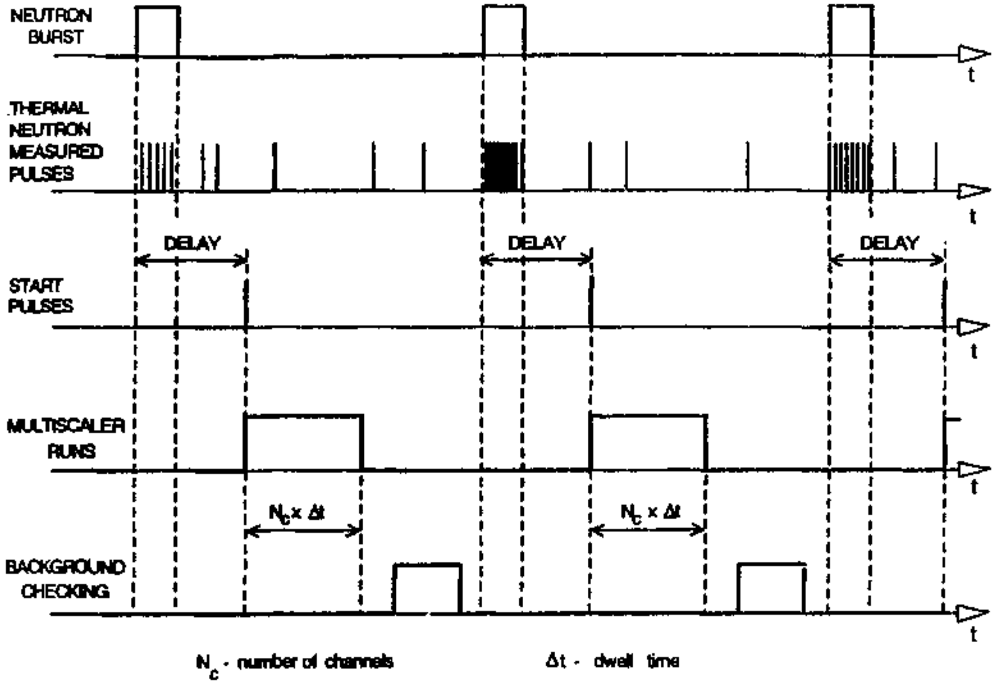


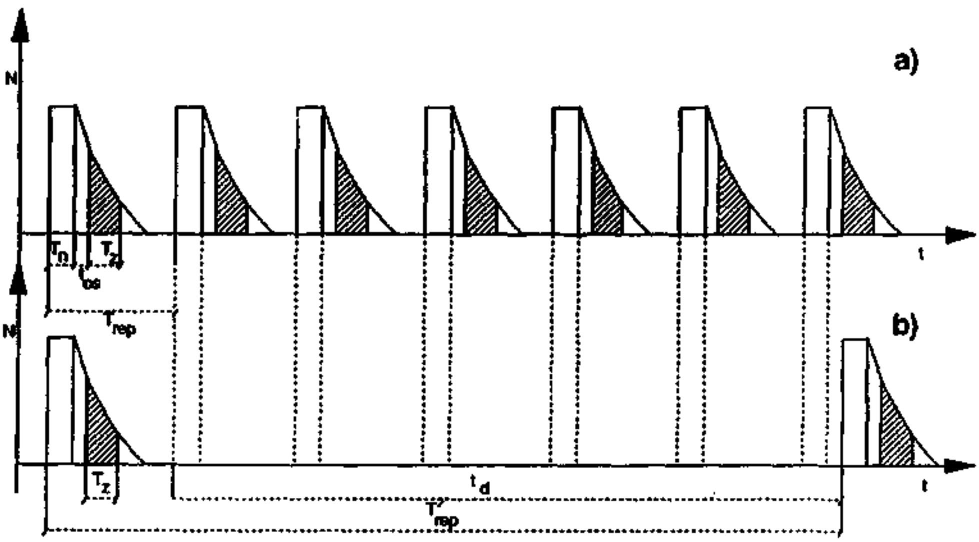
Fig. 4. Timing diagram of pulsed neutron measurement.

are registered in the analyzer memory during a single run. The channel number at which a given pulse is registered corresponds to the time between the start of the analyzer run and the moment of the appearance of the measured pulse in the input. The procedure is repeated after each neutron burst until a required counting statistics is reached. The start pulse can be delayed (adjustable). During the delay time the fast neutrons are slowed-down in the investigated system and higher order modes of the thermal neutrons flux vanish.

The background is checked in the following way: measured pulses are counted in the defined time interval which is placed just before the beginning of the next neutron burst. The number of these pulses collected during a fixed number of runs is a relative measure of the background.

The recording part of the system (the Canberra analyzer and multiscaler) has a big disadvantage: in certain cases it has to operate with a repetition time of neutron bursts T'_{rep} which is about ten times longer than the shortest possible time T_{rep} admitted by the physical phenomenon. This results from a long time necessary for the data transmission between the multiscaler and the analyzer after the one run is finished. This feature involves in most our experiments a very long total time of the measurement, which creates then problems to reach a sufficient accuracy of determination of high values of the decay constant (because of the problems with the long-term stability of all parts of the system). One can see in the timing diagram presented in Fig. 5, how the measurement is going in the most favorable conditions and in the case when the measurement system with the Canberra 35+ MCA and multiscaler MCS 7880 is used.

The thermal neutron decay curve accumulated in the analyzer memory is transmitted to the computer by the serial/parallel interface and an interpretation of the data is performed using semi-automatic computation programs and proper data-base procedures.



T_n — WIDTH OF THE FAST NEUTRON BURST,
t_{0s} — DELAY TIME OF THE START PULSE FOR THE ANALYZER,
T_z — DATA ACQUISITION TIME AFTER A SINGLE BURST,
T_{rep} — OPTIMUM REPETITION TIME,
T'_{rep} — REPETITION TIME FOR THE SYSTEM WITH CANBERRA 35+ MCA,
t_d — EXTRA DELAY CAUSED BY THE USE OF CANBERRA 35+ MCA (TIME OF THE DATA TRANSMISSION FROM THE MULTISCALER TO MCA AND TIME OF SUMMING THE DATA FROM THE CURRENT SCAN TO THE STORED SPECTRUM).

Fig. 5. Timing diagram of the pulsed neutron measurement:
a) optimum conditions; b) in the system with Canberra 35+ MCA.

5. OPTIMUM PARAMETERS FOR THE MEASUREMENT AND DETERMINATION OF THE TIME DECAY CONSTANT λ_0 FROM THE RECORDED CURVE.

The recorded die-away curve can always be fitted to a sum of exponentials. The curve contains also a constant component due to the background. The decay of higher mode terms is much faster than that of the fundamental one. A fitting procedure is usually complicated. In most cases it is possible to get such a decomposition of the curve which contains a constant term and two exponentials from which one corresponds to the fundamental mode and the second contains a contribution from higher modes. When the experiment itself is performed under particularly selected conditions it is possible to fix such a time interval on the recorded die-away curve that its representation by a single exponential with a constant term is possible. The conditions, however, depend on the fundamental decay constant λ_0 which is to be measured. For each measurement of the λ_0 the following parameters fixing the optimum conditions, both for the experiment and the linked interpretation procedure, should be defined:

T_{rep} – repetition time of neutron bursts,
T_n – width of the neutron burst,
t_{0s} – delay time of the start pulse for the analyzer,
T_z – time interval covered by the analyzer (*i.e.* data acquisition time after a single burst),
T_w – time interval for the determination of the λ_0 value, movable inside the interval T_z,
Δt – width of the time channel of the multiscaler (dwell time),
P – total counting statistics of measured pulses.

These parameters are marked in Figs 1 and 6.

We have obtained some relations to determine values of the above parameters when the measurements are performed with hydrogenous or partly hydrogenous samples. The relations result from an analysis of about 2000 measurements of the thermal neutron decay constants performed by our group at the Institute of Nuclear Physics (WOŹNICKA *et al.*, 1987). A scaling unit of time is the lifetime τ of thermal neutrons in a medium:

$$\tau = \frac{1}{\lambda_0} \quad . \tag{5}$$

Then the repetition time T_{rep} of neutron bursts is assumed to be:

$$T_{rep} = 20\,\tau \quad . \tag{6}$$

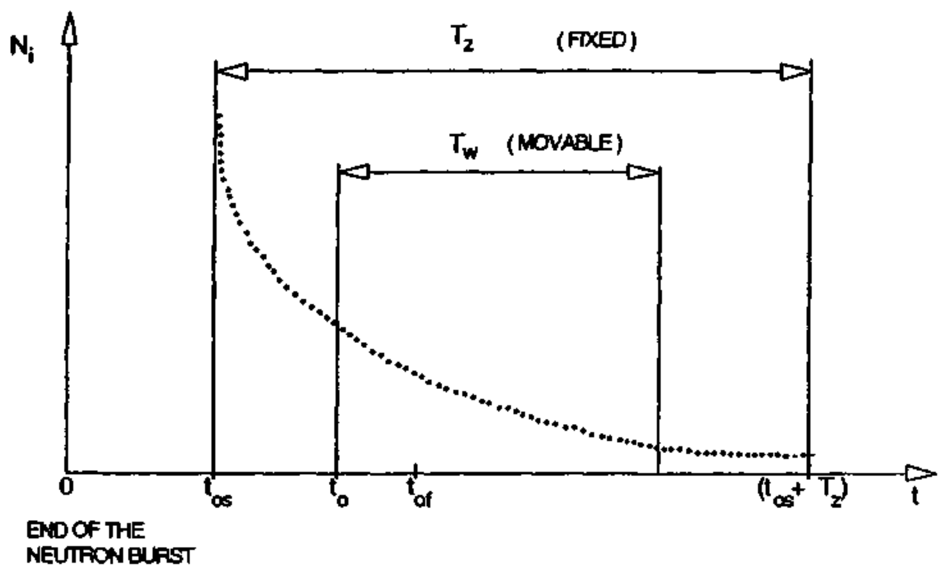
This is a time which allows to vanish all thermal neutrons coming from the present burst. A longer repetition time may be used but then the total measurement time is also longer, of course. The delay of the start pulse for the time analyzer after the end of the neutron burst is assumed to be

$$t_{0s} = 1.5\ \tau \tag{7}$$

and the time interval which should be covered by the time analyzer:

$$T_z = 6\ \tau\ . \tag{8}$$

The values of the parameters used in the experiment should not differ more than 10 per cent from the ones defined in Eqs (7) and (8).



t_{0s} — START OF THE ANALYZER RUN (FIXED DELAY TIME),
T_z — TOTAL TIME INTERVAL COVERED BY THE ANALYZER (FIXED),
T_w — TIME INTERVAL FOR THE DETERMINATION OF THE λ_0 VALUE (MOVABLE),
t_0 — CURRENT DELAY TIME (VARIABLE), $t_{0s} < t_0 < t_{0f}$,
t_{0f} — FINAL DELAY FOR INTERVAL T_w , $(t_{0f} + T_w) < (t_{0s} + T_z)$.

Fig. 6. Relation between positions of the time interval during the computation of the decay constant λ_{0i} as a function of the delay time t_0 .

The fast neutron burst in an ideal experiment should be the Dirac delta pulse. From physical reasons it can be approximated by the box shape function of the width T_n which should be narrow in comparison to the lifetime τ. This is easy to fulfill for low thermal neutron decay constants. For rapid decays one has to accept a compromise between this condition and a reasonable efficiency of the pulsed neutron source. In such a case we assume for a maximum width of the neutron burst:

$$T_{n\ max} < 2\ \tau\ . \qquad (9)$$

A basic method of the evaluation of the fundamental mode decay constant λ_0 from the registered curve with a high accuracy has been described by DROZDOWICZ *et al.* (1993). The counts in the time channels are corrected for the dead time of the instrumentation system (see the next paragraph). Cornell's method of fitting a sum of exponentials with background is applied. Our computation procedure permits to fit the experimental points by the sum of both one or two exponentials with a background. It permits also to check whether the interval T_z (where only a one-exponential decay exists) has been properly chosen. If the time interval, in which the one-exponential decay is expected, does not arrive, the λ_0 is separated from the two-exponential fit. Such a situation occurs when the experiment has not been performed at the optimum parameters or when the decay curve is measured at difficult experimental conditions (*e.g.* for some heterogeneous samples, for samples with a very high absorption of thermal neutrons when a very high decay constant is measured and, generally, when the higher mode decay constants have values close to the fundamental one).

The λ_0 value is evaluated in the time interval T_w which is shorter than the time interval T_z covered by the analyzer (see Fig. 6). We take the time T_w equal to

$$T_w = 3.5\ \tau\ . \qquad (10)$$

It gives a possibility to compute from the collected data the decay constant λ_0 as a function of the delay time t_0 by moving the interval T_w inside the interval T_z. If the λ_0 value is estimated correctly the plot of λ_0 *vs* t_0 should be statistically time-independent in a certain time interval. An example of such a curve is presented later in Fig. 11. The weighted mean λ_0 and the standard deviation $\sigma(\lambda_0)$ are calculated within the interval of the statistically constant value and they are recognized as the final result of the estimation of the fundamental mode decay constant λ_0 (cf. DROZDOWICZ *et al.*, 1993).

The above consideration on the determination of the ultimate average λ_0 by moving the interval T_w inside the interval T_z, and using the start pulse initial delay t_{0s}, is based on the assumption that counting statistics meets requirements of a sufficient accuracy of the result. The counting statistics may be estimated by the total number P' of measured pulses accumulated during a given die-away curve measurement. These pulses are counted by the time analyzer within the interval T_z during all runs. The P' number can be estimated by

$$P' = m\,\lambda_0 \quad , \tag{11}$$

where

$$160\ [s] < m < 200\ [s] \quad . \tag{12}$$

An optimum width Δt of the analyzer time channel is also dependent on the measured decay constant and on the assumed counting statistics P'. In the Canberra 35+MCA system for measurements in the range of $5\,000\ s^{-1} < \lambda_0 < 50\,000\ s^{-1}$, the channel widths Δt = (1, 2, or 5) μs can be chosen. The number of the multiscaler channels N_c used in the measurement has to fulfill the condition

$$N_c \geq \frac{T_z}{\Delta t} \quad . \tag{13}$$

If $N_c \Delta t > T_z$, the channels following after the end of the interval T_z are not recognized in the data treatment because they contain counts which come mainly from the background.

6. EXPERIMENTAL VERIFICATION OF A DEAD TIME OF THE INSTRUMENTATION SYSTEM

There are the following reasons of the dead time τ_d of the instrumentation system: dead time of the ^3He thermal neutron detector, dead time of the preamplifier, dead time of the linear amplifier and of the pulse forming system, and dead time of the multiscaler (negligible in the system considered). An effect of the dead time on the shape of the stored time decay curve is shown in Fig. 7. Not all neutron pulses are collected by the multiscaler due to the existence of the dead time. If the dead time of the measuring system is not taken into account the final result of the measurement, the fundamental mode decay constant, is underestimated. A count-loss correction is introduced to recalculate number of counts in each channel of the analyzer according to the procedure described in DROZDOWICZ *et al.* (1993).

A value of the dead time τ_d of the instrumentation system has to be known to be able to use a count-loss correction. The dead time of the entire described system had to be found. A measurement by the two-source method is inadequate in this case. In our measurements,

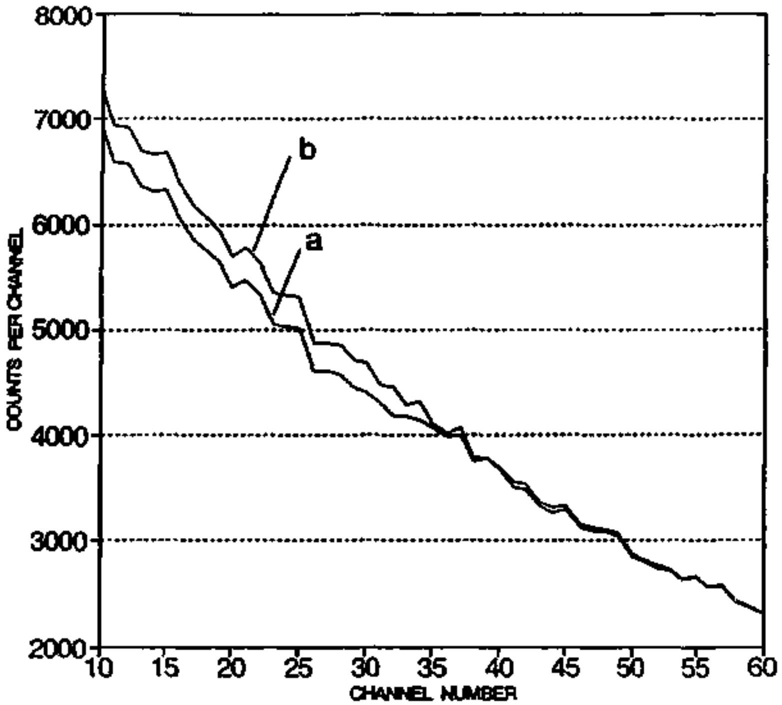


Fig. 7. Influence of the dead time on the shape of the die-away curve:
a) a curve deformed by the dead time effect; b) the corrected curve.

rapid dynamic processes are observed which influence the dead time itself (especially in gas detectors of neutrons) and a steady-state measurement gives wrong results. We have used an indirect method to determine the dead time of the system. A series of die-away experiments has been run with a few samples characterized by different decay constants λ_0. The measurement conditions have been carefully chosen to be sure that a one-exponential decay part of the registered die-away curve exists. The count-loss correction has been introduced using different values of the dead time. In each case the fundamental mode decay constant λ_0 has been obtained as a function of the delay time (as described in the previous paragraph). The dead time values has been used in the range from $\tau_d = 3\ \mu s$ to $\tau_d = 12\ \mu s$, within which the dead time of the electronic system used has been expected. The dead time effect influences stronger the beginning part of the registered curve because of a higher intensity of counts there than in the final part. This results in an apparent change of the slope of the curve, *i.e.* in a change of the determined λ_0 in respect to its true value. Then, the function $\lambda_0(t_0)$ instead to be statistically constant shows lower values for shorter delays t_0, if no correction is introduced. If the value of the dead time used in the count-loss correction

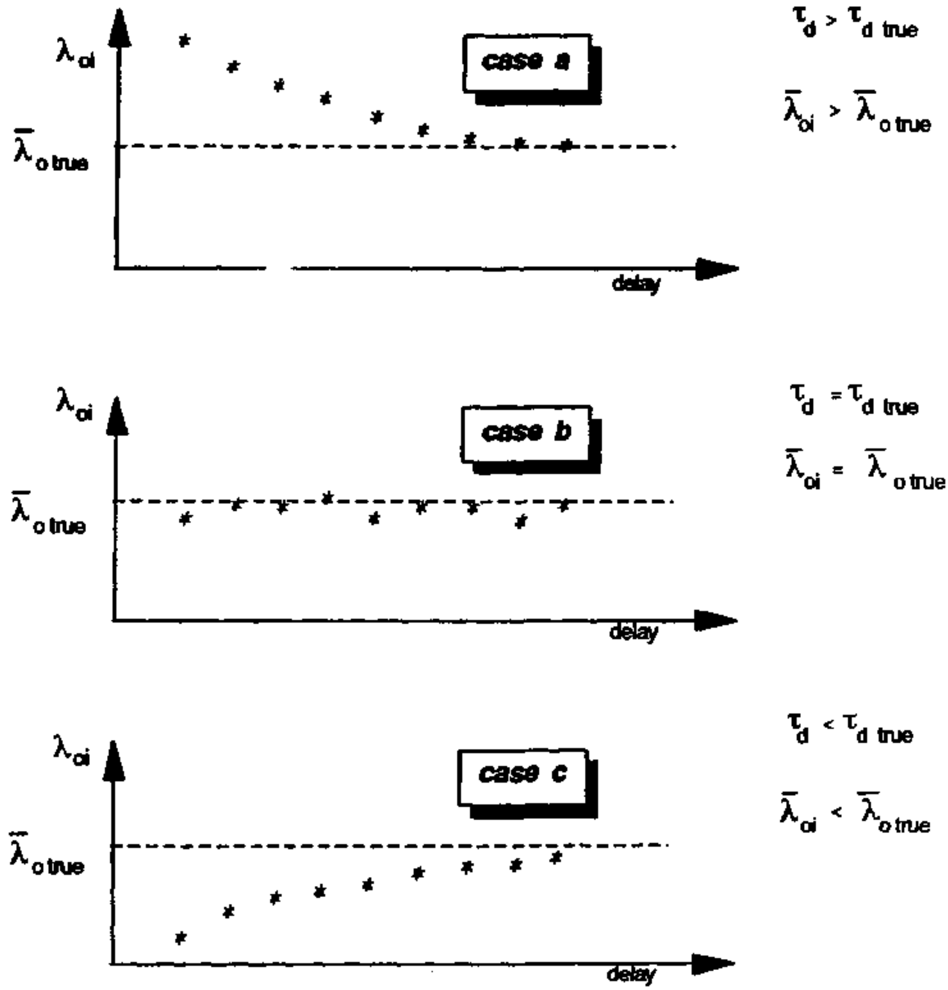


Fig. 8. Effect of too high (case a), too low (case c) and proper (case b) values of the dead time correction on the consecutive values of λ_{0i} as a function of the delay time and on the mean value $\bar{\lambda}_0$.

is improper the function $\lambda_0(t_0)$ is underestimated or overestimated at the beginning. Let λ_{0i} denotes the λ_0 obtained for a consecutive step t_{0i} of the delay time t_0:

$$\lambda_{0i} = \lambda_0(t_{0i}) \quad . \tag{14}$$

For the delay time sequence:

$$t_{01} < t_{02} < t_{03} < \ldots \tag{15}$$

the dependence

$$\lambda_{01} > \lambda_{02} > \lambda_{03} > \ldots \tag{16a}$$

is observed when the dead time assumed in the calculation is higher than the true one and the dependence

$$\lambda_{01} < \lambda_{02} < \lambda_{03} < \ldots \tag{16b}$$

in the opposite case. The effect is schematically shown in diagrams in Fig. 8. Additionally, one will observe that the mean value $\bar{\lambda}_0$ calculated from the statistically constant part of the function $\lambda_0(t_0)$ is higher in the first case and lower in the second case in comparison to the true value λ_0. Testing the behavior of the functions $\lambda_0(t_0)$ obtained at different values τ_d allows us to find the true value of the dead time.

A helpful information desirable in this method is the accurate knowledge of the λ_0 for a given sample. We had this information from previous measurements performed using our former instrumentation system with another type of the multiscaler, AC-256 type. It had a long own dead time τ_{AC} which overlapped the dead times of all other parts of the system and which was accurately known, $\tau_{AC} = 9$ μs. Thus, for some samples measured now we know the true value of the fundamental decay constant. This offers a possibility to compare the new results (obtained while using different dead times in the count-loss correction) with these previously observed.

<u>An example.</u>

The Plexiglass cylindrical sample of height and diameter equal to $H = 2R = 17$ cm (*i.e.* of the geometrical buckling $B_g^2 = 0.10564$ cm^{-2}) has been used as a test sample to observe the effect of different values of the dead time taken to the data treatment procedure. The value of the fundamental mode of the decay constant for the reference sample of interest, obtained by our group with the previous experimental set-up, has been equal to:

$$\lambda_{0ref} = 8\ 023\ s^{-1} \quad ,$$
$$\sigma(\lambda_0)_{ref} = 35\ s^{-1} \quad . \tag{17}$$

That value has been obtained with a high reliability and repeatability. The result is also consistent with other experiments and calculations for Plexiglass (DROZDOWICZ and WOŹNICKA, 1986, 1987).

Results of the present test measurement are shown in Fig. 9. A number of the values λ_{0i} has been obtained as a function of the delay time t_0 in the interval:

$$202\ \mu s < t_0 < 342\ \mu s \quad . \tag{18}$$

The interval of the analysis of the die-away curve has been equal to $T_w = 360\ \mu s$. The channel width of the time analyzer has been equal to $\Delta t = 2\ \mu s$ and it has been possible to obtain $I = 36$ consecutive values of the λ_{0i} when the step has been equal to $4\ \mu s$. Case a in Fig. 9 presents the result obtained using in the data treatment the dead time value $\tau_d = 12\ \mu s$. Case b in Fig. 9 is for $\tau_d = 8\ \mu s$, and case c is for $\tau_d = 3\ \mu s$. The diagrams presented as cases a and c show the effect of a bad selection of the dead time value. An overestimation or an underestimation of the obtained λ_{0i} at the beginning of the function $\lambda_0(t_0)$ is visible, as explained above at relations (16). An interpretation of these results leads to a conclusion that the dead time is equal to $8\ \mu s$ which is also confirmed by the reference value λ_{0ref}.

A series of test measurements has been done for different sizes of Plexiglass cylinders (different values of the fundamental decay constant). Samples having the decay constants in the interval $5\ 000\ s^{-1} < \lambda_0 < 42\ 500\ s^{-1}$ have been measured. Data treatment has been performed for each sample using the dead time values in the range mentioned earlier, $\tau_d = 3, 5, 7, 8, 9, 12\ \mu s$. An analysis has been made as shown in the above example and the results have been compared also to the previous ones obtained with the multichannel time analyzer AC-256 (DROZDOWICZ and WOŹNICKA, 1986). Finally, the dead time of the instrumentation system

$$\tau_d = (8 \pm 0.5)\ \mu s \tag{19}$$

has been chosen as the best adjustment. The values obtained from the test measurements, and the references values λ_{0ref} for comparison, are collected in Table 1. The λ_{0ref} values measured with the AC-256 analyzer are cited after DROZDOWICZ and WOŹNICKA (1986). The agreement between the results is perfect, with the correlation coefficient $R = 0.99998$.

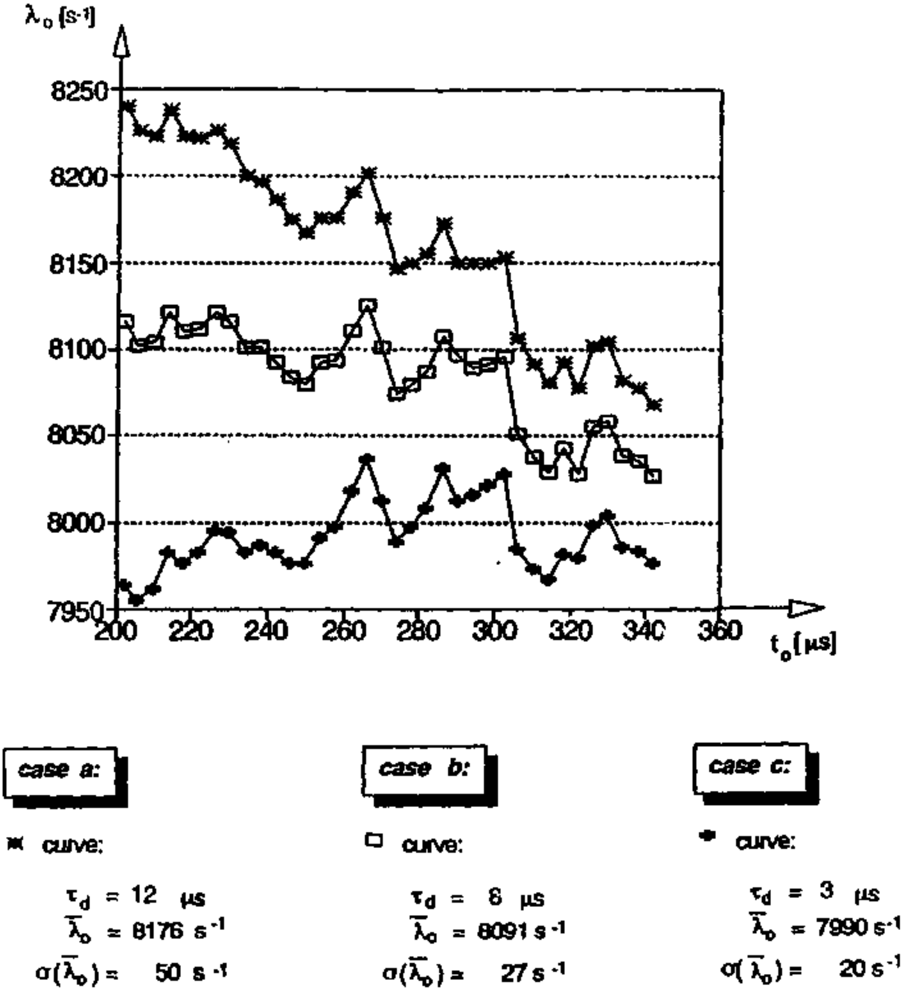


Fig. 9. Function $\lambda_0(t_0)$ for the Plexiglass cylindrical sample of the geometrical buckling $B_g^2 = 0.10564\ \mathrm{cm}^{-2}$ presented for three different cases (a, b, c) of the dead time correction used.

Case b presents a result of using the proper dead time correction. The λ_{0i} values are statistically constant up to the delay time $t_0 = 300\ \mu s$. The low values which are observed for the delay time longer than 300 μs result from the counting statistics being too low in the final channels of the time analyzer.

Table 1. Comparison between fundamental decay constants for cylindrical Plexiglass samples measured with two different registration systems: AC-256 analyzer and Canberra 35+ MCA.

B_g^2	AC-256 ANALYZER		CANBERRA 35+ MCA	
[cm^{-2}]	λ_{0ref} [s^{-1}]	$\sigma(\lambda_{0ref})$ [s^{-1}]	λ_0 [s^{-1}]	$\sigma(\lambda_0)$ [s^{-1}]
1.23403	42 357	517	42 165	699
1.03219	37 668	189	37 643	202
0.87054	32 814	311	32 926	149
0.74419	28 955	209	28 971	167
0.62587	25 296	122	25 391	120
0.56204	23 383	53	23 358	118
0.53373	22 465	85	22 467	166
0.48318	20 925	106	20 803	113
0.44985	19 709	53	19 625	67
0.41985	18 848	44	18 832	133
0.38431	17 595	50	17 638	67
0.35311	16 637	28	16 657	43
0.31298	15 179	47	15 145	52
0.28992	14 447	32	14 429	75
0.26932	13 664	42	13 730	31
0.25084	13 057	49	13 072	96
0.23420	12 482	58	12 488	78
0.21916	11 967	29	12 017	48
0.20553	11 486	30	11 551	60
0.19313	11 102	34	11 081	63
0.18182	10 710	32	10 693	65
0.17148	10 394	21	10 376	125
0.16200	10 048	51	10 038	42
0.15328	9 738	16	9 749	58
0.14525	9 440	34	9 446	52
0.13783	9 175	31	9 172	59
0.13097	8 955	28	8 925	62
0.11869	8 496	48	8 503	79
0.10564	8 023	35	8 035	55
0.03600	5 312	45	5 303	21

7. PROCEDURE OF A DETERMINATION OF THE DECAY CONSTANT BY A MULTIPLE RECORDING OF THE DIE-AWAY CURVE.

Any disturbance which appears during the measurement influences the accumulated die-away curve and causes its perturbation. If the perturbation is relevant within the time interval T_z, the fundamental decay constant λ_0 cannot be correctly determined from the stored data. This appears in a strange shape of the relationship $\lambda_0(t_0)$ which is then not statistically constant but is increasing, decreasing, convex, or broken in certain point(s). The method of the data collection and treatment can be improved in order to eliminate such disturbances which are mostly caused by the problems with the long-term stability of all elements of the measuring system. At the same time, a higher accuracy of the measurement result or the same accuracy at the counting statistics P' lower than defined in paragraph 5, can be obtained.

An idea of the proposed method which takes care of the disturbances is following. The die-away curve is accumulated until the total number of measured pulses within the interval T_z is equal to about $0.1\,P'$ only. Such a short measurement is repeated J times. Every j-th die-away curve is registered independently. Then, for each j-th curve registered within the time interval T_z, the fundamental decay constants $(\lambda_{0i})_j$ are found as a function of the delay time t_{0i} [cf. Eq.(14)]. Let λ_{0j} denotes the decay constant calculated as the weighted mean for a set of the delay times t_{0i}:

$$\lambda_{0j} = \overline{(\lambda_{0i})_j} \,, \qquad \begin{array}{l} i = 1, 2, \ldots, I \\ j = 1, 2, \ldots, J \end{array} \tag{20}$$

with the standard deviation $\sigma(\lambda_0)_j$ within the chosen interval of the delay time t_0 (cf. paragraph 5). If any serious perturbation appears in any short measurement result, then that measurement is to be rejected and additional one should be performed. This procedure requires from the experimenter a high experience because the data are usually scattered due to a poor counting statistics. The counting statistics in the single short measurement is, of course, insufficient to determine the decay constant with the final required accuracy. The standard deviation $\sigma(\lambda_0)_j$ obtained in a single short measurement may be even several times greater than the final required. The required accuracy can be achieved by a calculation of the mean decay constant from the results obtained for all accepted short measurements.

Let from J short measurements the fundamental decay constants λ_{0j} have been obtained ($j = 1, 2, \ldots, J$) together with their standard deviations $\sigma(\lambda_0)_j$. Then, the weighted mean $\bar{\lambda}_J$ (unbiased and of minimum variance) has the form (cf. BROWNLEE, 1965):

$$\bar{\lambda}_J = \frac{\sum w_j \lambda_{0j}}{W} \,, \tag{21}$$

where the weight w_j is defined as usually:

$$w_j = \frac{1}{\sigma^2(\lambda_0)_j} \quad , \tag{22}$$

W is equal to

$$W = \sum w_j \quad , \tag{23}$$

and the abbreviated summation mark stands for:

$$\sum = \sum_{j=1}^{J} \quad .$$

The calculated weighted mean $\bar{\lambda}_J$ is assumed as a final result of the decay constant λ_0 obtained from a given complete measurement consisting of J short partial measurements (*i.e.* $\lambda_0 = \bar{\lambda}_J$). The accuracy of the mean $\bar{\lambda}_J$ is given by its variance:

$$\sigma^2(\bar{\lambda}_J) = \sum \left[\frac{w_j}{W} \right]^2 \frac{1}{w_j} = \frac{1}{W} \quad . \tag{24}$$

The standard deviation $\sigma(\bar{\lambda}_J)$ is a measure of the dispersion of the λ_{0j} values around the calculated mean $\bar{\lambda}_J$ and it decreases at increasing number J of short measurements (cf. HIMMELBLAU, 1970). If the relative deviation $\sigma(\bar{\lambda}_J)/\bar{\lambda}_J$ is equal to or less then some level assumed in advance (*e.g.* 0.5 %), the measurement is completed. In the opposite case the next short measurement should be performed, and so on.

The standard deviation of the random variable λ_0 itself should also be found from the experiment. The unbiased estimator $s^2(\lambda_0)$ of the variance $\sigma^2(\lambda_0)$ of the decay constant λ_0 is expressed by

$$s^2(\lambda_0) = \frac{q}{q-1} \sum \frac{w_j}{W} (\lambda_{0j} - \bar{\lambda}_J)^2 \quad , \tag{25}$$

where

$$q = \frac{1}{\sum \left[\frac{w_j}{W} \right]^2} \quad , \tag{26}$$

which substituted into Eq. (25) yields:

$$s^2(\lambda_0) = \frac{W}{W^2 - \sum w_j^2} \sum w_j (\lambda_{0j} - \bar{\lambda}_J)^2 \quad . \tag{27}$$

The calculation performed to obtain the relations given in Eqs (25) and (26) is presented in details in the Appendix. It has been assumed in the evaluation of Eq.(25) i that the individual variances $\sigma^2(\lambda_0)_j$ have the same expected value.

In the case when the weights w_j are all equal, Eqs (27) and (24) give the well-known relations:

$$s^2(\lambda_0) = \frac{1}{J-1} \sum (\lambda_{0j} - \bar{\lambda}_J)^2 \quad , \tag{28}$$

$$\sigma^2(\bar{\lambda}_J) = \frac{1}{J} \sigma^2(\lambda_0) \approx \frac{1}{J(J-1)} \sum (\lambda_{0j} - \bar{\lambda}_J)^2 \quad . \tag{29}$$

An analysis of a series of measurements consisting of the short measurements indicates that a minimum number J should be from 4 to 6. The necessary number J is dependent on the scattering of the values $(\lambda_{0i})_j$ around the mean $\bar{\lambda}_J$ for every j-th short measurement and on the required value of the standard deviation $\sigma(\bar{\lambda}_J)$.

An example of the results λ_{0i} *vs* t_{0i} obtained during four short measurements (runs): $j = 1, 2, 3, 4$, is shown in Fig. 10. They are numbered from 4281a0 to 4281d0. An original numbering in the description of the presented examples is kept here in order to have an easy reference to the original data. The total number of pulses stored within the interval T_z is equal to about $0.1\,P'$ in every run [see Eq. (11)]. Patterns of the obtained functions $\lambda_0(t_0)$ are different. One can see that the patterns of runs 4281a0 and 4281d0 are regularly fluctuating around the mean value λ_{0j}. A high dispersion of the λ_{0i} values around the mean value is observed in run 4281d0. A plot of the function $\lambda_0(t_0)$ in run 4281b0 is increasing at the beginning and statistically constant later on. The λ_{0i} values of run 4281c0 can be considered as statistically constant around two different levels. Such patterns of the function $\lambda_0(t_0)$ are admissible taking into account a poor counting statistics of single runs. The decay constants λ_{0j} and the standard deviations $\sigma(\lambda_0)_j$ are collected in Table 2. The final results, $\bar{\lambda}_J$, $\sigma(\bar{\lambda}_J)$, and $\sigma(\lambda_0)$ are obtained according to Eqs (21), (24), and (27), and are labelled as "z0003". The standard deviation $\sigma(\bar{\lambda}_J)$ of the mean $\bar{\lambda}_J$ from No "z0003" is considerably lower than the individual $\sigma(\lambda_0)_j$ value from any particular run.

The presented four runs have been performed immediately one after another, without any change in the measurement parameters and without an interruption of the neutron generator operation. It means, that all values of the parameters of each run have been

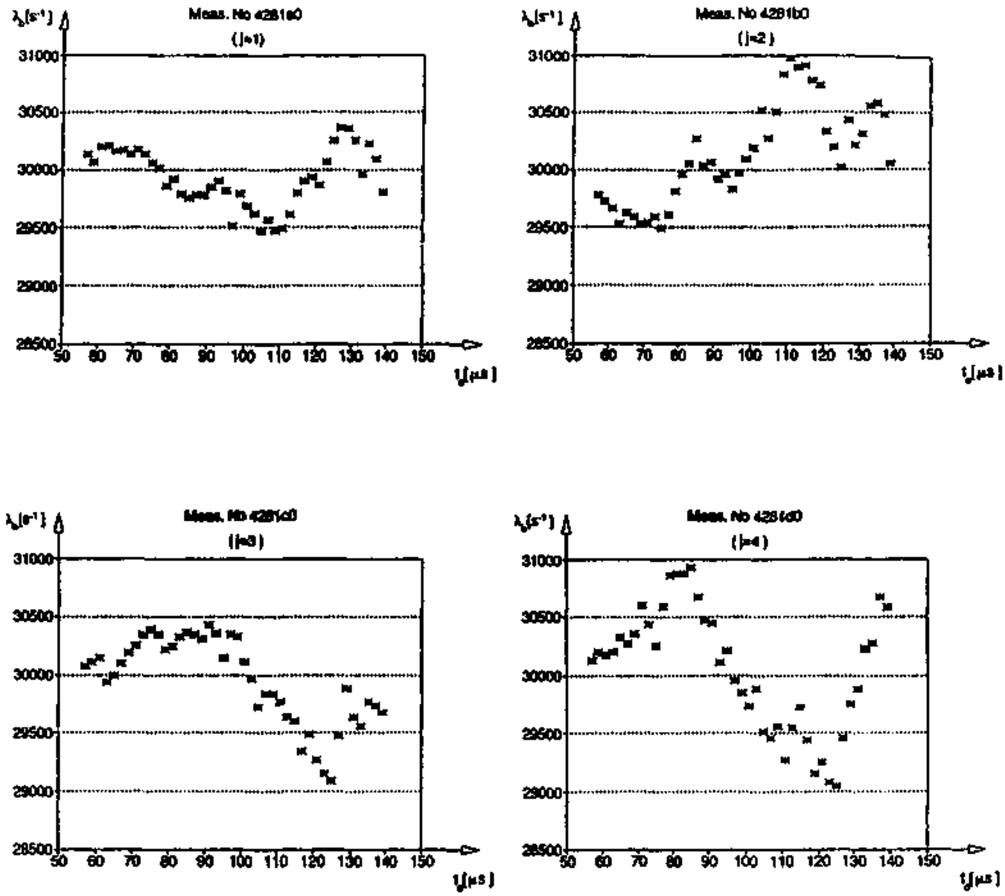


Fig. 10. Function $\lambda_0(t_0)$ in four consecutive runs (No 4281a0 to No 4281d0).

exactly the same, especially the delay time of the start pulse for the analyzer. Thus, the counts in respective channels of the analyzer, stored in consecutive runs, can be summed up to obtain the die-away curve which is a result from one long measurement (labelled as "z0003s0"). The function $\lambda_0(t_0)$ has been obtained from this measurement (Fig. 11) and the final decay constant λ_0 and the standard deviation $\sigma(\lambda_0)$ have been found and they are also included in Table 2. From comparison of results No "z0003" and No "z0003s0" it is visible that they are in an excellent agreement which means that the proposed method is adequate for the measurement.

Table 2. Decay constants obtained from consecutive runs and from the long measurement.

j	Run No	λ_{0j} [s^{-1}]	$\sigma(\lambda_0)_j$ [s^{-1}]	No	$\bar{\lambda}_j$, λ_0 [s^{-1}]	$\sigma(\bar{\lambda}_j)$ [s^{-1}]	$\sigma(\lambda_0)$ [s^{-1}]
1	4281a0	29 984	223				
2	4281b0	29 900	386	z0003	30 031	150	117
3	4281c0	30 097	284				
4	4281d0	30 226	436				
1÷4	—	—	—	z0003s0	30 046	—	136

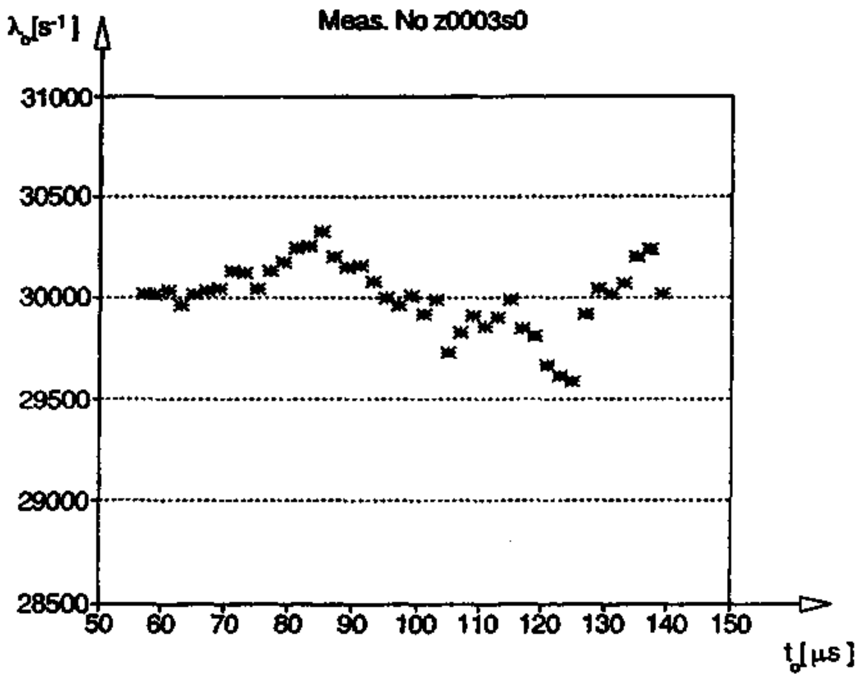


Fig. 11. Function $\lambda_0(t_0)$ from the long measurement (No "z0003s0").

Another example is presented in Fig. 12 and in Tables 3 and 4. There are collected results of five runs labelled as 4278a0, 4278b0, 4280a0, 4280b0, and 4280c0, and numbered in the Tables by j = 1, 2, 3, 4, 5, respectively. They have not been performed in one

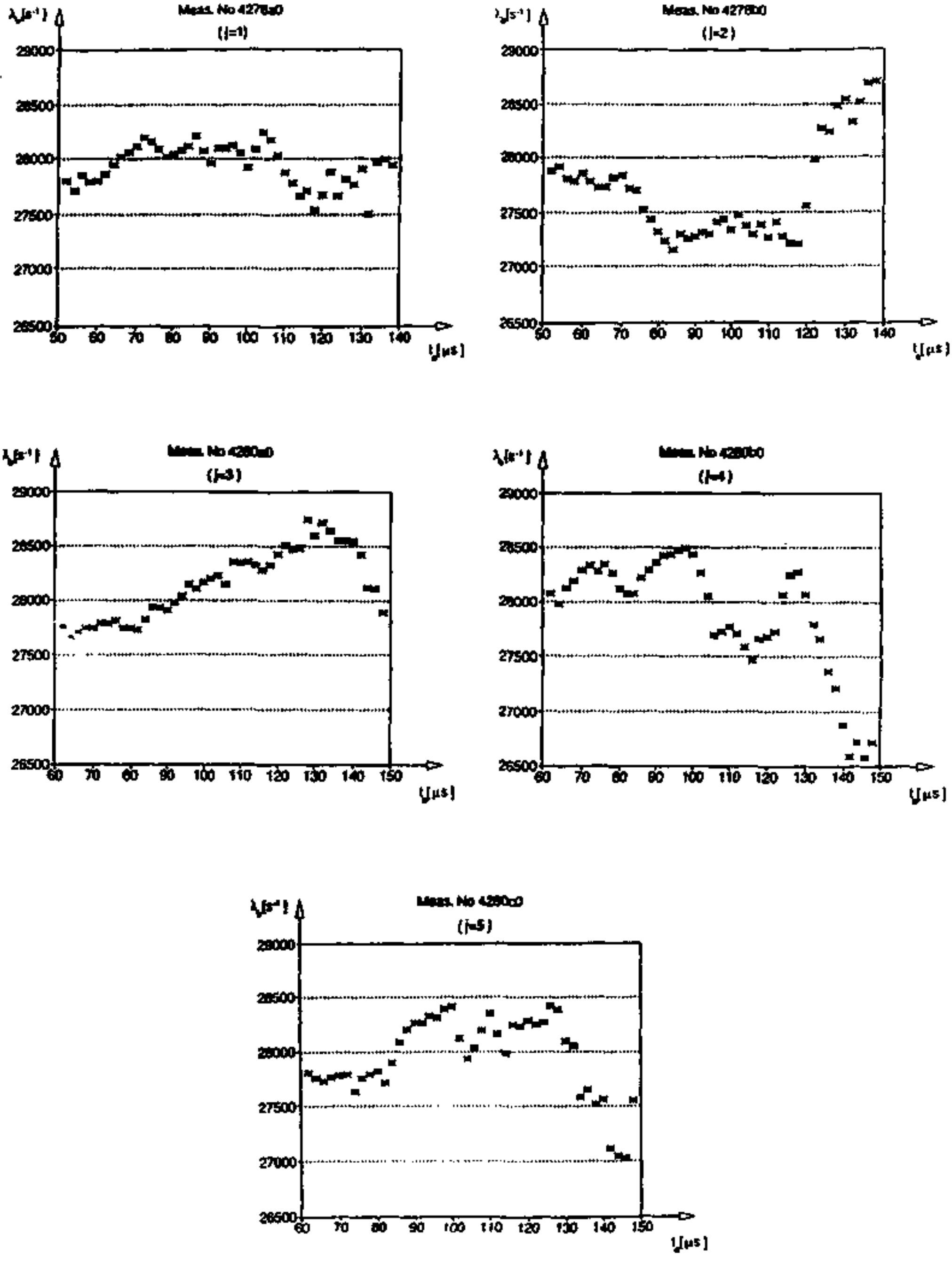


Fig. 12. Function $\lambda_0(t_0)$ in five runs from which one (No 4278b0) ought to be rejected.

Table 3. Decay constants λ_{0j} from the set of five runs.

j	Run No	λ_{0j} [s^{-1}]	$\sigma(\lambda_0)_j$ [s^{-1}]
1	4278a0	27 955	168
2	4278b0	27 649	315
3	4280a0	27 965	288
4	4280b0a	28 146	242
5	4280c0	27 937	278

Table 4. Average decay constant $\bar{\lambda}_J$ in three cases of elaboration of the set of five runs.

Case	Combination of j runs	No	$\bar{\lambda}_J$ [s^{-1}]	$\sigma(\bar{\lambda}_J)$ [s^{-1}]	$\sigma(\lambda_0)$ [s^{-1}]
a	1, 3, 4, 5	z0002	27 996	114	97
b	1, 2, 3, 4	—	27 959	116	176
c	1, 2, 3, 4, 5	—	27 956	107	155

measurement session and a summation of the stored data is not possible in this case. First, the four runs (4278a0, 4278b0, 4280a0, and 4280b0a) have been performed (j = 1, 2, 3, 4). Let us discuss the pattern of the function $\lambda_0(t_0)$ for each one. The values λ_{0i} *vs* t_{0i} are statistically constant in run 4278a0 (j = 1). Such a pattern as for run 4280a0 (j = 3) is acceptable, considering a low counting statistics of the run, although the λ_{0i} values are increasing. The plot from run 4280b0 (j = 4) is statistically constant except the seven last values λ_{0i} which are considerably lower than the average and they have not been taken into account for the final λ_{0j} calculation. These low values at the end of the plot are caused by a background which begins to dominate there, which is reflected by $\lambda_0 \rightarrow 0$. Run 4278b0 (j = 2) has to be rejected because the plot of the function $\lambda_0(t_0)$ is broken at t_0 equal to about 120 μs and the following high values have no explanation. It means that during the measurement some disturbance appeared which caused the observed perturbation. Therefore, an additional, fifth run (No 4280c0, j = 5) has been performed. Its pattern, shown also in Fig. 12, is admissible.

The decay constants λ_{0j} with the standard deviations $\sigma(\lambda_0)_j$ of the considered runs are given in Table 3. They have been further elaborated in three ways. The resulting values, $\bar{\lambda}_J$, $\sigma(\bar{\lambda}_J)$, and $\sigma(\lambda_0)$, obtained in the three cases are presented in Table 4. Case a takes into account four runs (j = 1, 3, 4, 5) while the disturbed 4278b0 run (j = 2) is rejected. The results of case a are accepted as the final results and the case is labelled as "z0002". For a comparison, two other cases are presented, when run 4278b0 has not been rejected. The results of case b are obtained from four consecutive runs, for j = 1, 2, 3, 4, as it would be done without any deeper analysis. In case c all available runs are included into the elaboration. From the comparison of the three presented cases it is visible that rejecting of the uncertain measurement decreases significantly the standard deviation $\sigma(\lambda_0)$, which confirms a suitability of the choice.

The example discussed above shows that the performing of several runs instead of one longer measurement allows to notice and to exclude a fragment of the measurement, when some disturbance has appeared. This leads to the rejection of a disturbed short measurement and, in consequence, a higher accuracy of the measurement result can be achieved.

8. CONCLUSIONS.

The methodology of the measurement of the thermal neutron die-away curves in bounded media, described in the report, has been directed to the experimental system with the Canberra 35+ analyzer. The main goal of the method has been to achieve the best possible accuracy of the experiment. The accuracy of the determination of the fundamental mode decay constant is not worse than 0.5 per cent even in the case of so short lifetimes as 20 μs, although in that case the measurement conditions are pushed at their limits. This experimental system with the Canberra 35+ MCA and MCS 7880 units is always suitable for measurements of lower decay constants (say, about 4 ms^{-1} to 10 ms^{-1}). In the case of higher decay constants (especially in a range about 25 ms^{-1} to 50 ms^{-1}) the system is relevant only for occasional laboratory reference measurements because of the disadvantages shown above. Another registration system ought to be applied when long series of such measurements are planned. Many solutions of the interpretation procedure presented in the paper have a general meaning and can be applied in other systems. The original method of the measurement which replaces one long-time measurement by a few short runs gives a possibility to eliminate effects of some electronic/apparatus instability or disturbances which, in other case, brake down a time consuming experiment.

APPENDIX

Evaluation of the unbiased estimator $s^2(x)$ of the variance $\sigma^2(x)$ of the random variable x is presented. J observations x_j with the same expectation ξ and with the individual variances $\sigma^2(x_j)$, *i.e.* with the individual weights, are considered.

From the definition (HIMMELBLAU, 1970), the variance $\sigma^2(x)$ of a discrete variable x is given by

$$\sigma^2(x) = \sum P(x_j)(x_j - E[x])^2 \quad , \tag{A1}$$

where $E[x]$ is the expectation of a random variable x:

$$E[x] = E[x_j] = \xi \quad , \tag{A2}$$

and $P(x_j)$ is a value of the probability $P(x)$ for a discrete variable x, appearing at $x = x_j$. The abbreviated summation mark stands for

$$\sum = \sum_{j=1}^{J} \tag{A3}$$

everywhere through the Appendix.

For the described case one can assume that the probability $P(x_j)$ is given by the expression:

$$P(x_j) = \frac{w_j}{W} \quad , \tag{A4}$$

where the weight w_j is defined as usually:

$$w_j = \frac{1}{\sigma^2(x_j)} \quad , \tag{A5}$$

and W is equal to

$$W = \sum w_j \quad . \tag{A6}$$

Inserting Eq.(A4) into Eq.(A1) gives for the variance $\sigma^2(x)$ the formula:

$$\sigma^2(x) = \sum \frac{w_j}{W} (x_j - E[x])^2 \quad . \tag{A7}$$

The estimator $s_0^2(x)$ of the variance $\sigma^2(x)$ is given by a sum:

$$s_0^2(x) = \sum \frac{w_j}{W} (x_j - \bar{x})^2 \quad , \tag{A8}$$

where the estimator $\bar{x}$ has been used instead of the expectation,

$$\bar{x} = \frac{\sum w_j x_j}{W} \quad . \tag{A9}$$

The $\bar{x}$ is an unbiased estimator of the expected value $E[x]$, which is shown below.

From the definition (HIMMELBLAU, 1970), an estimator $\hat{\theta}$ of a parameter θ is said to be unbiased if its expected value, $E[\hat{\theta}]$, is equal to the parameter value θ:

$$E[\hat{\theta}] = \theta \quad . \tag{A10}$$

For the mean $\bar{x}$ we obtain:

$$E[\bar{x}] = E\left[\frac{\sum w_j x_j}{W}\right] = \frac{\sum w_j}{W} E[x_j] = \xi \quad . \tag{A11}$$

i.e. the estimator is unbiased.

In the case when the individual variances $\sigma^2(x_j)$ are equal to each other the probability $P(x_j)$ given by Eq.(A4) becomes:

$$P(x_j) = \frac{1}{J} \quad . \tag{A12}$$

Thus, Eq.(A8) has the form:

$$s_0^2(x) = \sum \frac{1}{J} (x_j - \bar{x})^2 \quad , \tag{A13}$$

which is known as the biased estimator of the variance $\sigma^2(x)$ when the individual variances $\sigma^2(x_j)$ are equal to each other (HIMMELBLAU, 1970).

To find the unbiased estimator $s^2(x)$ of the variance $\sigma^2(x)$ let prove first that the estimator $s_0^2(x)$ given in Eq.(A8) is a biased one. Eq.(A8) has to be especially transformed. It is presented below.

$$s_0^2(x) = \sum \frac{w_j}{W}(x_j - \bar{x})^2 = \sum \frac{w_j}{W}[(x_j - \xi) - (\bar{x} - \xi)]^2 =$$

$$= \sum \frac{w_j}{W}(x_j - \xi)^2 - 2\sum \frac{w_j}{W}(x_j - \xi)(\bar{x} - \xi) + \sum \frac{w_j}{W}(\bar{x} - \xi)^2 =$$

$$= \sum \frac{w_j}{W}(x_j - \xi)^2 - 2(\bar{x} - \xi)\frac{1}{W}\sum w_j(x_j - \xi) + (\bar{x} - \xi)^2 \frac{1}{W}\sum w_j =$$

$$= \sum \frac{w_j}{W}(x_j - \xi)^2 - 2(\bar{x} - \xi)\frac{1}{W}\left(\sum w_j x_j - \xi \sum w_j\right) + (\bar{x} - \xi)^2 .$$

The second term of the last expression can be rearranged:

$$2(\bar{x} - \xi)\frac{1}{W}\left(\sum w_j x_j - \xi \sum w_j\right) = 2(\bar{x} - \xi)\left(\frac{\sum w_j x_j}{W} - \xi\right) = 2(\bar{x} - \xi)^2 .$$

Thus:

$$s_0^2(x) = \sum \frac{w_j}{W}(x_j - \xi)^2 - (\bar{x} - \xi)^2 . \qquad \text{(A14)}$$

Let us take the expected value of both sides of Eq.(A14):

$$E[s^2(x)] = E\left[\sum \frac{w_j}{W}(x_j - \xi)^2\right] - E\left[(\bar{x} - \xi)^2\right] =$$

$$= E\left[\sum \frac{w_j}{W}(x_j - E[x_j])^2\right] - E\left[(\bar{x} - E[\bar{x}])^2\right] . \qquad \text{(A15)}$$

The first term of Eq.(A15) is equal to $\sigma^2(x)$ according to Eq.(A7) and Eq.(A2). The second term is equal to $\sigma^2(\bar{x})$ from the definition of the variance (HIMMELBLAU, 1970). Thus, Eq.(A15) gives the relation:

$$E[s_0^2(x)] = \sigma^2(x) - \sigma^2(\bar{x}) \quad . \tag{A16}$$

The variance $\sigma^2(\bar{x})$ of the $\bar{x}$ is obtained from Eq.(A9):

$$\sigma^2(\bar{x}) = \sum \left[\frac{w_j}{W}\right]^2 \sigma^2(x_j) \quad . \tag{A17}$$

Assuming the same expectation of all individual variances $\sigma^2(x_j)$, *i.e.*:

$$E\,[\sigma^2(x_j)] = \sigma^2(x) \quad , \tag{A18}$$

we can substitute $\sigma^2(x_j)$ by $\sigma^2(x)$ in Eq.(A17). Then

$$\sigma^2(\bar{x}) = \frac{1}{q}\,\sigma^2(x) \quad , \tag{A19}$$

where

$$q = \frac{1}{\sum \left[\frac{w_j}{W}\right]^2} \quad . \tag{A20}$$

Inserting Eq.(A19) into Eq.(A16) one can obtain the expectation of the estimator $s_0^2(x)$ as:

$$E[s_0^2(x)] = \frac{q-1}{q}\,\sigma^2(x) \quad . \tag{A21}$$

It means that the estimator $s_0^2(x)$ is the biased estimator of the variance $\sigma^2(x)$. Therefore, the unbiased estimator $s^2(x)$ has to be expressed as

$$s^2(x) = \frac{q}{q-1}\,s_0^2(x) \quad . \tag{A22}$$

Introducing the expression for the estimator $s_0^2(x)$ given by Eq.(A8) we obtain:

$$s^2(x) = \frac{q}{q-1} \sum \frac{w_j}{W}\,(x_j - \bar{x})^2 \quad . \tag{A23}$$

Eq.(A23) defines the unbiased estimator $s^2(x)$ of the variance $\sigma^2(x)$ of a random variable x, when the individual weights w_j have been used.

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ACKNOWLEDGMENTS

We are indebted to Prof. Dr. Jan A. Czubek for helpful discussions and remarks on the manuscript. Our colleagues, Eng. Jacek Burda and Mr. Tadeusz Kowalik, are thanked for their assistance at the experiments, and M.Sc. Joanna Dąbrowska for her help in a preparation of the data base.

The work was sponsored by the State Committee for Scientific Research, Project No 2 0199 91 01.