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## ABSTRACT

This letter studies the Sp(2) covariant quantisation of gauge theories. The geometrical interpretation of gauge theories in terms of quasi principal fibre bundles  $Q(M_S, G_S)$  is reviewed. It is then described the Sp(2) algebra of ordinary Yang-Mills theory. A consistent formulation of covariant lagrangian quantisation for general gauge theories based on Sp(2) BRST symmetry is established. The original  $N = 1$ , ten dimensional superparticle is considered as an example of infinitely reducible gauge algebras, and given explicitly its Sp(2) BRST invariant action.

**Key-words:** Gauge theories; BRST symmetry; Fibre bundles; Covariant quantisation

## 1. Gauge theories in terms of quasi-principal fibre bundles.

Gauge theories have a nice geometrical interpretation in terms of connections on a principal fibre bundle (pfb)  $P(M, G)$ , where  $M$  is the base space-time manifold and  $G$  is the gauge group [1,2, 3, 4]. However, quantisation of gauge theories requires the introduction of fields  $(c_m^a, \pi_m^a)$ . It would be then desirable to have a formalism where those extra fields fit into some representation of a larger group and all the fields are components of a superfield. This is a step in the direction of recovering a geometrical interpretation of quantum gauge theories. The main ingredients in the construction of geometrical quasi-principal fibre bundles (qpfb) are a space-time base manifold  $M$ , a gauge group  $G$ , an extended superspace manifold  $M_S$  which is obtained by adding two extra Grassmann variables  $\theta^a$  ( $a = 1, 2$ ) to  $M$ , in the case of  $Sp(2)$  symmetry, and a supergroup  $G_S$ . The construction is performed basically in three steps [1,2, 3]. It starts with a pfb  $P(M, G)$  and extend the gauge group  $G$  to a supergroup  $G_S$ . The composition of  $G$  with a Grassmann algebra  $B$  prolongs  $P(M, G)$  to a pfb  $P'(M, G_S)$ . The most general supergroup  $G_S$  can be represented in matrix form. In particular,  $OSp(N/M)$  groups are represented by block matrices of the form

$$\begin{pmatrix} A & E \\ C & D \end{pmatrix} \quad (1.1)$$

where  $A, D$  are  $(N \times N)$  and  $(M \times M)$  matrices whose elements are taken from the even part of the Grassmann algebra  $B$  constructed over a complex vector space  $W$ , whilst  $E, C$  are  $(N \times M)$  and  $(N \times M)$  rectangular matrices whose elements belong to the odd part of  $B$ . Next, it is enlarged the base space manifold  $M$  to a superspace  $M_S$  in  $P'(M, G_S)$  by adding Grassmann variables. At this stage, a pfb  $P''(M_S, G_S)$  is obtained. Finally, the pfb  $P''(M_S, G_S)$  is transform into a quasi-principal fibre bundle  $Q(M_S, G_S)$ . For instance, given a one-form valued function  $\alpha(x) = A_\mu dx^\mu$  on  $M$  this induces a connection  $\omega$  on the pfb  $P(M, G)$ . The one-form valued function  $\alpha'$  on  $M_S$  is found by

$$\alpha'(x, \theta^a) = g^{-1} A_\mu dx^\mu g + g^{-1} dg \quad (1.2)$$

where  $g = g(x^\mu, \theta^a)$  ( $a = 1, 2, \dots$ ) which induces a connection  $\omega'$  on the qpfb  $Q(M_S, G_S)$  [1,2,3,4].

## 2. The $Sp(2)$ BRST Algebra of Yang-Mills Theory.

It has been realized for some time [5,6,7,8] that a geometrical construction can be useful for the discussion of BRST and anti-BRST symmetry. The idea is to use a superspace with coordinates  $Z^M = (x^\mu, \theta^a)$ , where ( $a = 1, 2$ ) and  $\theta^a$  is an anti-commuting scalar coordinate and the BRST generators  $s^a$  are realized as differential operators on superspace,  $s^a = \frac{\partial}{\partial \theta^a}$ , so that  $s^a s^b + s^b s^a = 0$  holds automatically<sup>1</sup>. For example, in Yang-Mills theory the gauge potential  $A_\mu^i$  and the Faddeev-Popov ghost  $(c^a)^i$  (where  $i$  is an adjoint group

<sup>1</sup> It is usually defined a bosonic operator  $\sigma = \frac{1}{2} \epsilon_{ab} s^a s^b$  where  $\epsilon_{ab}$  is the symplectic invariant form of  $Sp(2)$ , so that  $\epsilon_{ab} = -\epsilon_{ba}$ ,  $\epsilon^{ab} \epsilon_{bc} = \delta_{ac}$  and  $\epsilon_{12} = 1$ . The generator  $\sigma$  is invariant

index) can be combined into a super-gauge field  $\mathcal{A}_M^i(Z)$  whose lowest order components are  $\mathcal{A}_M^i(Z)|_{\theta^a=0} = (\mathcal{A}_\mu^i, \mathcal{A}_{\theta^a}^i)|_{\theta^a=0} = (A_\mu^i, c^{a i})$ . Then the standard Yang-Mills BRST transformations arise from imposing the constraints  $\mathcal{F}_{\mu\theta^a}^i = 0$ ,  $\mathcal{F}_{\theta^a\theta^b}^i = 0$  on the superfield strength  $\mathcal{F}_{MN}^i$  [5,6]. This gives an elegant geometrical description of BRST and anti-BRST symmetry.

Let us review the construction of gauge theories in the superspace with coordinates  $Z^M = (x^\mu, \theta^a)$ , which gives a geometric formulation of  $Sp(2)$  BRST symmetry. We consider matter fields  $\Phi^i(x, \theta^a)$  and a gauge potential  $\mathcal{A}_M^i(x, \theta^a) = (\mathcal{A}_\mu^i(x, \theta^a), \mathcal{A}_{\theta^a}^i(x, \theta^a))$ . These can be used to define a covariant derivative

$$\mathcal{D}_M \Phi^i = \partial_M \Phi^i - (T^k)^i_j \mathcal{A}_M^k \Phi^j \quad (2.1)$$

and the field strength

$$\mathcal{F}_{MN}^i = \partial_M \mathcal{A}_N^i - (-1)^{MN} \partial_N \mathcal{A}_M^i + f^i_{jk} \mathcal{A}_M^j \mathcal{A}_N^k \quad (2.2)$$

where  $(-1)^{MN}$  is 1 unless both  $M$  and  $N$  are indices referring to anti-commuting coordinates, in which case it is  $-1$ . The gauge potential  $\mathcal{A}_M$  contains more component fields than the physical gauge and ghost fields and so, as in supersymmetric theories, constraints should be imposed on the field strength  $\mathcal{F}$ . Appropriate constraints are [5,6]

$$\mathcal{F}_{\theta^a\theta^b} = 0, \quad \mathcal{F}_{\mu\theta^a} = 0. \quad (2.3)$$

These can be written more explicitly as

$$\partial_\mu \mathcal{A}_{\theta^a} - \partial_{\theta^a} \mathcal{A}_\mu + [\mathcal{A}_\mu, \mathcal{A}_{\theta^a}] = 0 \quad (2.4)$$

$$\partial_{\theta^a} \mathcal{A}_{\theta^b} + \frac{1}{2} [\mathcal{A}_{\theta^a}, \mathcal{A}_{\theta^b}] = 0 \quad (2.5)$$

$$\epsilon^{ab} (\partial_{\theta^a} \mathcal{A}_{\theta^b} + [\mathcal{A}_{\theta^a}, \mathcal{A}_{\theta^b}]) = 0. \quad (2.6)$$

Defining the component expansions

$$\mathcal{A}_\mu(x, \theta^a) = A_\mu(x) + \theta^a \Lambda_{a\mu}(x) + \theta^a \theta^b \Omega_{ab\mu}(x) \quad (2.7)$$

$$\mathcal{A}_{\theta^a}(x, \theta^b) = c^b(x) + \theta^c \Upsilon_a^b(x) + \theta^c \theta^d \omega_{ac}^b(x), \quad (2.8)$$

however, it was found that if  $A_\mu$ ,  $c^a$ ,  $\pi$  are identified with the gauge, ghost (anti-ghost) and auxiliary fields respectively then the supergauge fields have the expansions \*

$$\mathcal{A}_\mu = A_\mu + \theta^a (s^a A_\mu) + \theta^a \theta^b (s^a s^b A_\mu) \quad (2.9)$$

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under  $Sp(2)$  and satisfies  $s^a s^b = 0$ . The  $Sp(2)$  generators  $\sigma^i$  ( $i = \pm, 0$ ) and the fermionic charges  $s^a$  together form an algebra which is a contraction of  $OSp(1,1/2)$  and denoted as  $ISp(2)$  [9].

\* In Ref [6], it was obtained explicitly a geometrical formulation of BRST and anti-BRST symmetries and given the field content of  $\Lambda$ ,  $\Upsilon$ ,  $\Omega$  and  $\omega$ . The components in the expansion can also be read as conditions on the mapping of the coordinates  $\phi^i$  of the fibres over  $\{U_i\}$  (covering set of  $M_G$ ) and expressed as cocycle conditions.

$$\mathcal{A}_\alpha^b = c^b + \theta^\alpha (s^\alpha c^b) + \theta^\alpha \theta^\alpha (s^\alpha s^\alpha c^b). \quad (2.10)$$

The BRST and anti-BRST generators  $s^\alpha$  are then identified with the superspace differential operators  $\partial_{\theta^\alpha}$  [5,6], and the complete set of BRST and anti-BRST transformations are given by [10,11,12]

$$\begin{aligned} s^\alpha \phi^i &= R_\alpha^i c^{\alpha a}, & s^\alpha c^{\alpha b} &= \epsilon^{\alpha b} \pi^\alpha - \frac{1}{2} f_{\beta\gamma}^\alpha c^{\beta a} c^{\gamma b}, \\ s^\alpha \pi^\alpha &= \frac{1}{2} f_{\beta\gamma}^\alpha \pi^\beta c^{\gamma a} - \frac{1}{12} (f_{\beta\gamma}^\alpha f_{\delta\tau}^\beta + f_{\delta\tau, i}^\alpha R_\gamma^i) c^{\delta a} c^{\tau a} \epsilon_{abcd} c^{\gamma b}, \end{aligned} \quad (2.11)$$

where the generators  $R_\alpha^i$  for the gauge field  $A_\mu^i$  read off from  $s^\alpha A_\mu^i = (D_\mu c^\alpha)^i$ , and  $\pi^\alpha$  is an auxiliary field which connects ghost and antighost sectors.

For the matter fields  $\Phi^i(x, \theta^\alpha)$ , we impose the constraint

$$\mathcal{D}_{\theta^\alpha} \Phi^i = \partial_{\theta^\alpha} \Phi^i - (T^k)^i_j \mathcal{A}_{\theta^\alpha}^k \Phi^j = 0 \quad (2.12)$$

which implies

$$\Phi^i = \psi^i + \theta^\alpha (s^\alpha \psi^i), \quad (2.13)$$

and the BRST and anti-BRST transformations again corresponds to translations in the  $\theta^\alpha$  direction with  $\frac{\partial}{\partial \theta^\alpha}$  realized as differential operators on the extended space manifold  $M_S$ .

### 3. Sp(2) formalism for General gauge theories.

Consider a general gauge theory with classical fields  $A^i(x^\mu)$  ( $i = 1, 2, \dots, n$ ) and classical action  $S_0(A^i)$ . The action is invariant under gauge transformations

$$\delta A^i = R_\alpha^i \xi^\alpha, \quad (3.1)$$

where  $\xi^\alpha$  is the local gauge parameter. The Noether equations are given by

$$S_{0, i} R_\alpha^i = 0, \quad \alpha = 1, 2, \dots, m \quad (0 \leq m \leq n), \quad (3.2)$$

and the generators of the gauge transformations satisfy

$$R_{\alpha, j}^i R_\beta^j - (-)^{\alpha\beta} R_{\beta, j}^i R_\alpha^j = -R_{\gamma}^i f_{\alpha\beta}^\gamma - S_{0, j} M_{\alpha\beta}^{ij}. \quad (3.3)$$

To construct a covariant lagrangian formalism for general gauge theories either with open ( $M_{\alpha\beta}^{ij} \neq 0$ ) or closed algebras, and based on Sp(2) BRST symmetry, it is needed to enlarge the base manifold  $M$  to  $M_S$ . It is then defined a superspace  $M_S$  to include classical fields  $\Phi^A$  and Sp(2) doublets of anti-fields  $\Phi_\Lambda^*$ ,  $\Phi_\Lambda^{**}$  [13,14,15]. The properties of these fields and anti-fields are  $\epsilon(\Phi^A) = \epsilon_A$ ,  $\epsilon(\Phi_\Lambda^*) = \epsilon_A + 1$ ,  $\epsilon(\Phi_\Lambda^{**}) = \epsilon_A$ ,  $gh(\Phi_\Lambda^*) = (-)^{\epsilon_A} - gh(\Phi^A)$  and  $gh(\Phi_\Lambda^{**}) = -gh(\Phi^A)$ . An extended Poisson superbracket is defined by

$$(F, G) = \frac{\delta_r}{\delta \Phi^A} F \frac{\delta_l}{\delta \Phi_\Lambda^*} G - (-)^{\epsilon_F \epsilon_G} \frac{\delta_r}{\delta \Phi^A} G \frac{\delta_l}{\delta \Phi_\Lambda^*} F, \quad (3.4)$$

where  $\epsilon_F, \epsilon_G$  denotes the Grassmann parity of the  $F, G$  functions on  $M_S$ , and left (right) derivatives are understood with respect to anti-fields (fields) unless otherwise stated. The extended anti-bracket (3.4) satisfies

$$\epsilon((F, G)) = \epsilon(F) + \epsilon(G) + 1 \quad (3.5)$$

$$gh((F, G)^a) = -(-)^a + gh(F) + gh(G), \quad a = 1, 2 \quad (3.6)$$

$$(F, G) = -(-)^{\epsilon_F \epsilon_G} (G, F) \quad (3.7)$$

and

$$(-)^{\epsilon_F} \epsilon_G((F, G), H) + [\text{cycl. perm } (F, G, H)] = 0. \quad (3.8)$$

A bosonic action functional  $S = S(\Phi^A, \Phi_A^*, \Phi_A^{**})$  is constructed on  $M_S$ . This action satisfy the following generating equation

$$\bar{\Delta}^a \exp \frac{i}{\hbar} S(\Phi^A, \Phi_A^*, \Phi_A^{**}) = 0, \quad (a = 1, 2), \quad (3.9)$$

together with the boundary condition

$$S(\Phi^A, \Phi_A^*, \Phi_A^{**}) \Big|_{\Phi_A^* = \Phi_A^{**} = 0} = S(\Phi_A). \quad (3.10)$$

The operator  $\bar{\Delta}^a$  is defined by

$$\bar{\Delta}^a = \Delta^a + (i/\hbar)V^a, \quad \Delta^a = (-)^{\epsilon_A} \frac{\delta}{\delta \Phi^A} \frac{\delta_l}{\delta \Phi_A^*} \quad V^a = \epsilon^{ab} \Phi_A^* \frac{\delta_r}{\delta \Phi_A^{**}}. \quad (3.11)$$

The algebra of operators (3.11) satisfy the important property <sup>2</sup>

$$\bar{\Delta}^a \bar{\Delta}^b = 0. \quad (3.12)$$

The solution to the generating equation (3.9) is given as a power series of the Planck constant

$$S(\Phi^A, \Phi_A^*, \Phi_A^{**}) = \sum_{n=0}^{+\infty} \hbar^n S_{(n)}, \quad (3.13)$$

where the classical approximation  $S_{(0)}$  satisfies

$$\frac{1}{2}(S_{(0)}, S_{(0)})^a + V^a S_{(0)} = 0. \quad (3.14)$$

For a theory in which the  $Sp(2)$  algebra closes off-shell the classical solution  $S_{(0)}$  takes the form

$$S_{(0)} = S_0 + \Phi_{Aa}^* s^a \Phi^A + \frac{1}{2} \Phi_A^{**} \epsilon_{ab} s^a s^b \Phi^A + F_{AB} \epsilon_{ab} s^a \Phi^A s^b \Phi^B. \quad (3.15)$$

For more complicated theories like superparticles or superstrings,  $S_{(0)}$  has terms of higher order in the fields  $\Phi_A^*, \Phi_A^{**}$  to compensate those terms which makes the  $Sp(2)$  algebra to close on-shell. The classical solution to the generating equation is  $Sp(2)$  BRST invariant under modified BRST generators  $\bar{s}^a$  which satisfy  $\bar{s}^a \bar{s}^b + \bar{s}^b \bar{s}^a = 0$ .

<sup>2</sup> A supercommutative, associative algebra  $\mathcal{A}$  equipped with an extended Poisson anti-bracket structure plus a nilpotent property it is known as a BV-algebra, or coboundary Gerstehaber algebra (CGA) [16].

#### 4. Orthosymplectic Structure of the Original BSC Superparticle.

The original BSC superparticle  $S_{BSC}$  and further models are known to yield the same spectrum as that of  $D = 10$ ,  $N = 1$  super-Yang-Mills theory [17]. It is used here as an illustrative example to construct its  $Sp(2)$  covariant lagrangian, since the model has an infinitely reducible algebra. The BSC superparticle action is given by [18]

$$S_0 = \int d\tau [p_\mu \dot{x}^\mu - i\theta \dot{\theta} - \frac{1}{2} e p^2]. \quad (4.1)$$

This action describes a particle with world-line parametrized by  $\tau$  moving through a ten-dimensional  $N = 1$  superspace with coordinates  $(x^\mu, \theta_A)$ . The superparticle action  $S_{BSC}$  is invariant under a 10 dimensional super-Poincaré symmetry

$$\delta\theta = \epsilon, \quad \delta x^\mu = i\epsilon \Gamma^\mu \theta, \quad (4.2)$$

together with world-line reparametrisations and a local fermionic symmetry

$$\begin{aligned} \delta\theta &= p\kappa, & \delta e &= 4i\kappa\dot{\theta} + \xi, \\ \delta x^\mu &= i\theta \Gamma^\mu p\kappa + \xi p_\mu. \end{aligned} \quad (4.3)$$

The Grassmann spinor  $\kappa_A$  parametrizes the local symmetry while  $\xi$  parametrizes a linear combination of world-line diffeomorphisms and a local *trivial* local symmetry. To construct a covariant  $Sp(2)$  orthosymplectic structure for this model, it is required the formalism of the previous section since the classical infinitely reducible gauge algebra  $\mathcal{A}$  closes on-shell. It is then defined a superspace  $M_S$  to include the classical fields  $\Phi^A = (x^\mu, p_\mu, e, \theta_A)$  and  $Sp(2)$  doublets of anti-fields  $\Phi_A^*$ ,  $\Phi_A^{**}$ . The classical approximation  $S_{(0)}$  which satisfies (3.14) is given by

$$S_{(0)} = S_{BSC} + S_1 + S_2 + S_3, \quad (4.4)$$

where  $S_{BSC}$  is the classical action of the original superparticle and  $S_1$ ,  $S_2$ , and  $S_3$  are

$$\begin{aligned} S_1 &= \int d\tau [\theta_a^* p \kappa_1^a + e_c^* (4i\kappa_1^a \dot{\theta} + \dot{c}^a) + \kappa_{nab}^* ((-)^n p) (f_c^{ab} \kappa_{n+1}^c + e^{ab} \pi_n) \\ &\quad + x_{\mu a}^* (i\theta \gamma^\mu p \kappa_1^a + p_\mu c) + c_{da}^* (-2i f_{rs}^{ad} \kappa_1^r p \kappa_1^s + e^{ad} \pi)], \end{aligned} \quad (4.5)$$

$$\begin{aligned} S_2 &= \int d\tau [\theta^{**} (-p^2 \epsilon_{ab} f_c^{ab} \kappa_2^c) + e^{**} (-4i \epsilon_{ab} f_c^{ab} \kappa_2^c p \dot{\theta} + 2i \epsilon_{ab} \kappa_1^a p \kappa_1^b) \\ &\quad + \kappa_{nr}^{**} (-p^2) (\epsilon_{ab} f_c^{br} f_s^{ac} \kappa_{n+2}^s + f_b^{br} \pi_{n+1}) \\ &\quad + x_\mu^{**} (ip^2 \epsilon_{ab}) (f_c^{ab} \theta \gamma^\mu \kappa_2^c - \kappa_1^a \gamma^\mu \kappa_1^b) \\ &\quad + c_e^{**} (-4ip^2) (\epsilon_{ab} f_{rs}^{be} f_c^{as} \kappa_1^r \kappa_2^c + f_{rb}^{be} \kappa_1^r \pi_1)], \end{aligned} \quad (4.6)$$

and

$$\begin{aligned} S_3 &= \int d\tau \frac{1}{2} e_a^* [\theta_b^* (-\kappa_2^c) (2f_c^{ab} + e^{ab} \epsilon_{rs} f_{rs}^c) \\ &\quad + x_{\mu b}^* (2i\kappa_1^a \gamma^\mu \kappa_1^b + i e^{ab} \epsilon_{rs} \kappa_1^r \gamma^\mu \kappa_1^s - i\theta \gamma^\mu \kappa_2^c (2f_c^{ab} + e^{ab} \epsilon_{rs} f_{rs}^c)) \\ &\quad + \kappa_{nAb}^* (-\kappa_{n+2}^c (e^{ab} \epsilon_{ps} f_q^{sa} f_{pq}^c + 2f_s^{Ab} f_c^{as}) - \pi_{n+1} (2e^{as} f_s^{Ab} + e^{ab} f_{pq}^A)) \\ &\quad - 4ic_{Ab}^* (\kappa_1^r \kappa_2^c (e^{ab} \epsilon_{ps} f_{rq}^{sa} f_{pq}^c + 2f_{rs}^{bA} f_c^{as}) + \kappa_1^r \pi_1 (e^{ab} f_{rs}^A + e^{ab} f_{qp}^A))]. \end{aligned} \quad (4.7)$$

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