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QUASISYMMETRY EQUATIONS FOR CONVENTIONAL STELLARATORS

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ABSTRACT

General quasisymmetry condition, which demands the independence of B^2 on one of the angular Boozer coordinates, is reduced to two equations containing only geometrical characteristics and helical field of a stellarator. The analysis is performed for conventional stellarators with a planar circular axis using standard stellarator expansion. As a basis, the invariant quasisymmetry condition is used. The quasisymmetry equations for stellarators are obtained from this condition also in an invariant form. Simplified analogs of these equations are given for the case when averaged magnetic surfaces are circular shifted torii. It is shown that quasisymmetry condition can be satisfied, in principle, in a conventional stellarator by a proper choice of two satellite harmonics of the helical field in addition to the main harmonic. Besides, there appears a restriction on the shift of magnetic surfaces. Thus, in general, the problem is closely related with that of self-consistent description of a configuration.

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Key words: quasisymmetry, stellarator, stellarator approximation

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1. INTRODUCTION

Stellarator optimization from the viewpoint of the transport remains until now an actual problem. If it is not resolved, the creation of reactor-stellarator with tolerable dimensions is hardly possible. The main reason of the enhanced transport in stellarators is the three-dimensional inhomogeneity of the magnetic field, which, one would think, is an inherent feature of the systems of such a type. However, even in the class of three-dimensional systems it is possible to find those which even at the absence of axial or helical symmetry possess some "hidden" symmetry, which is expected to reveal itself finally in the noticeable reduction of the transport coefficients [1-4]. Ideally, the condition, which is the foundation of quasisymmetry concept [1-4], demands [5] from the strength of the magnetic field **B** on the magnetic surface to be the one-dimensional function of angular variables in the Boozer coordinates [6] (a,θ_B,ζ_B):

$$\mathbf{B}^2 = \mathbf{B}^2(a,\theta_B - N\zeta_B) \; . \tag{1}$$

In three-dimensional configurations it can be satisfied only approximately, but in some cases with high enough accuracy and at quite acceptable geometrical restrictions [1-4]. These and other known results are related to stellarators with a spatial axis. But for the conventional stellarators with a planar circular axis this attractive possibility of optimization is not yet studied.

In conventional stellarators the type of symmetry and spatial dependence of toroidal and helical field are different, so it is impossible to demand the fulfillment of (1) in the whole volume of plasma confinement. But, in principle, as in a general case [3], a chance remains to satisfy the condition (1) at some single magnetic surface. Such a supposition with respect to conventional stellarators was expressed in Ref. [4]. To investigate this possibility, as a first step it is necessary to reduce Eq. (1) with account of the specific features of these systems. The present article is devoted to this very goal.

The concept of quasisymmetry was discussed in details in recently published excellent informative review-style articles [3,4]. It allows us to restrict ourselves by this brief characteristic of the problem only and to turn directly to solving the stated problem.

2. GENERAL RELATIONSHIPS

Outwardly simple equation (1) requires knowledge of the Boozer coordinates θ_B and ζ_B, therefore the full system of equations must include besides Eq. (1) two additional equations for these functions. However, the problem can be reformulated and reduced to single equation when the differential consequence of (1)

$$\left[\nabla a \ \nabla(\theta_B - N\zeta_B) \right] \cdot \nabla \mathbf{B}^2 = 0 \tag{2}$$

and two representations of the magnetic field in Boozer coordinates [6] (for details see [7])

$$2\pi\mathbf{B} = \left[\nabla\psi \ \nabla\zeta_B \right] + \left[\nabla\Phi \ \nabla\theta_B \right] , \tag{3}$$

$$2\pi \mathbf{B} = J \nabla_a \theta_B + F \nabla_a \zeta_B \qquad (4)$$

are used. Here, ψ and Φ are the poloidal and toroidal magnetic fluxes, J is the longitudinal current, F is the poloidal current external to the magnetic surface, ∇_a is the surface gradient:

$$\nabla_a = \nabla - \nabla a \frac{(\nabla a \cdot \nabla)}{|\nabla a|^2} . \qquad (5)$$

It follows from equations (3) and (4)

$$\left[\nabla a \; \nabla\theta_B \right] = \frac{2 \pi}{\langle \mathbf{B}^2 \rangle V'} \left[F\mathbf{B} + \left[\mathbf{B} \; \nabla\psi \right] \right] , \qquad (6)$$

$$\left[\nabla a \; \nabla\zeta_B \right] = - \frac{2 \pi}{\langle \mathbf{B}^2 \rangle V'} \left[J\mathbf{B} + \left[\mathbf{B} \; \nabla\Phi \right] \right] , \qquad (7)$$

where $\langle \mathbf{B}^2 \rangle V' = F'\Phi' - J'\psi'$. After substitution of these expressions into the initial equation (2) it is transformed into

$$\left\{ \left[F + NJ \right] \mathbf{B} + \left[\mathbf{B} \; \nabla \left[\psi + N\Phi \right] \right] \right\} \cdot \nabla \mathbf{B}^2 = 0 . \qquad (8)$$

We have assumed, as usual in the theory, that lines $\theta_B = const$ and $\zeta_B = const$ form, correspondingly, the toroidal and poloidal contours on a magnetic surface. If a new "poloidal" angle $\alpha_B = \theta_B - N\zeta_B$ is introduced, then the appearance of representations (3) and (4) at the replacement of θ_B on α_B will not change, but instead of F and ψ the other quantities

$$F_N = F + NJ , \qquad \psi_N = \psi + N\Phi , \qquad (9)$$

will stand there, and equation (2) in variables a, α_B, ζ_B will look as a requirement that $\mathbf{B}^2$ must be independent on ζ_B and will reduce to the

$$\Big[\, F_N \mathbf{B} \;+\; \big[\, \mathbf{B}\, \nabla\psi_N \,\big] \Big]\cdot\nabla \mathbf{B}^2 \;=\; 0\,. \qquad (10)$$

In such a form the quasisymmetry condition was derived first in Ref. [4] from the analysis of the drift equations of motion. It is clear that (8) and (10) are equivalent.

3. SIMPLIFICATION OF THE MAIN EQUATION

The method, which will be used here for simplification of the equation (8), is closely related with the traditional for stellarators approach based on "stellarator expansion" [8]. We will make all calculations in the cylindrical coordinates r, ζ, z, where ζ is the toroidal angle. The standard notation will be used:

$$\bar{f} \equiv \langle f \rangle_\zeta = \frac{1}{2\,\pi}\int_0^{2\pi} f\, d\zeta\,, \qquad \tilde{f} = f - \langle f \rangle_\zeta\,, \qquad \hat{\tilde{f}} = \int \tilde{f}\, d\zeta\,. \qquad (11)$$

The principles of the method and examples of its practical application are described in details in [7]. Here we will enumerate only briefly those matters which are the most essential for further analysis.

In conventional stellarators with a planar circular axis the magnetic field is the superposition of three components:

$$\mathbf{B} \;=\; \frac{F}{2\,\pi}\nabla\zeta \;+\; \bar{\mathbf{B}}_p \;+\; \tilde{\mathbf{B}} \;=\; B_t \left[\, \mathbf{e}_\zeta \;+\; \frac{\bar{\mathbf{B}}_p}{B_t} \;+\; \frac{\tilde{\mathbf{B}}}{B_t} \right]. \quad (12)$$

Subscripts t and p denote, respectively, the toroidal and poloidal components, $\tilde{\mathbf{B}}$ is the helical field of a stellarator, $\mathbf{e}_\zeta$ is the unit vector along the $\nabla\zeta$. The second and the third terms in the right-hand side of Eq. (12) are small. It can be used at substituting $\mathbf{B}$ in such a form into Eq. (8). Besides, we will need two known results [7,9] of stellarator theory obtained also with using the smallness of these two values: the expression for the derivative of Φ over the volume V inside the magnetic surface

$$\frac{d\Phi}{dV} = \frac{F}{4\pi^2 R^2} \left\langle \frac{R^2}{r^2} - \frac{\tilde{\mathbf{B}}^2}{B_0^2} \right\rangle_\zeta \cong \frac{F}{4\pi^2 R^2} , \qquad (13)$$

allowing to exclude Φ'/F from Eq. (8), and the expression for ψ

$$\psi = \bar{\psi} + \tilde{\psi} = \bar{\psi} - \delta\mathbf{r}\cdot\nabla\bar{\psi} \cong \bar{\psi}(\mathbf{r} - \delta\mathbf{r}) , \qquad (14)$$

which is valid in linear approximation in small parameter $|\tilde{\mathbf{B}}|/B_0$. Here $\bar{\psi} = \bar{\psi}(r,z)$ is the function of two variables,

$$\delta\mathbf{r} = \frac{r}{B_t} \hat{\tilde{\mathbf{B}}}_p . \qquad (15)$$

Here and in the following R is the radius of stellarator geometrical axis, B_0 is the toroidal magnetic field at this axis. At low β (which is the ratio of plasma pressure to the magnetic one) $rB_t \cong RB_0 = const$, which will be also used below. Finally, it is natural to assume for stellarators that $\mu b/R \ll 1$, $\ell\mu/m \ll 1$, where $\mu = -\psi'/\Phi'$ is the rotational transform, b is the minor radius of a configuration, ℓ is the multipolarity of the helical field, m is the number of its periods along the major circumference of the torus.

The presence of small parameters in the problem allows to represent the vector from Eq. (8) as

$$F_N \mathbf{B} + \left[\mathbf{B}\ \nabla\psi_N \right] = F B_t \left[\mathbf{q}_0 + \mathbf{q}_1 + \mathbf{q}_2 \right], \tag{16}$$

where $\mathbf{q}_0$ and $\mathbf{q}_1$ are the main terms of the expansion, and $\mathbf{q}_2$ includes all other smaller corrections of higher orders:

$$\mathbf{q}_0 = \mathbf{e}_\zeta + \frac{N}{4\pi^2 R^2} \left[\mathbf{e}_\zeta\ \nabla V_0 \right], \tag{17}$$

$$\mathbf{q}_1 = \frac{\tilde{\mathbf{B}}}{B_t} + \frac{N}{4\pi^2 R^2} \left[\frac{\tilde{\mathbf{B}}}{B_t}\ \nabla V_0 \right] + \frac{N}{4\pi^2 R^2} \left[\mathbf{e}_\zeta\ \nabla V_1 \right]. \tag{18}$$

Here $V_0 = V(\bar{\psi})$,

$$V_1 = V'(\bar{\psi})\tilde{\psi} = - \delta\mathbf{r}\cdot\nabla V_0 , \tag{19}$$

and $V(\psi)$, let us remain, is the volume inside the magnetic surface $\psi = const$. At this stage of calculations we use almost linear dependence (13) of Φ on V and take into account the three-dimensionality of a configuration explicitly by making use of Eq. (14).

The quantity $\mathbf{B}^2$ can be expressed also in the fashion similar to (16):

$$\mathbf{B}^2 = B_0^2 \left[1 + b_1 + b_2 \right]. \tag{20}$$

Here b_1 and b_2 are the small (as compared with unity) quantities describing the helical and toroidal inhomogeneity of $\mathbf{B}^2$:

$$b_1 = 2\frac{R}{r}\frac{\tilde{B}_\zeta}{B_0} + 2\frac{\bar{\mathbf{B}}_p\cdot\tilde{\mathbf{B}}}{B_0^2}\,, \tag{21}$$

$$b_2 = \frac{R^2}{r^2} - 1 + \frac{\tilde{\mathbf{B}}^2}{B_0^2} + \frac{\bar{\mathbf{B}}_p^2}{B_0^2}\,. \tag{22}$$

Now Eq. (8) can be reduced to the following system:

$$\mathbf{q}_0\cdot\nabla b_1 = 0\,, \tag{23}$$

$$\left\langle \mathbf{q}_0\cdot\nabla b_2 + \mathbf{q}_1\cdot\nabla b_1 \right\rangle_\zeta = 0\,. \tag{24}$$

The first equation corresponds to separation of the terms oscillating in ζ (in the main approximation in $|\tilde{\mathbf{B}}|/B_0$) in the right-hand side of Eq. (8). Taking into account that at low β the field $\tilde{\mathbf{B}}$ practically does not differ from the vacuum one, $\tilde{\mathbf{B}} = \nabla\tilde{\varphi}$, and disregarding toroidal corrections, we find that Eq. (23) is reduced to the equation for the vacuum helical field potential $\tilde{\varphi}$:

$$\left\{ \mathbf{e}_\zeta + \frac{N}{4\pi^2 R^2}\left[\, \mathbf{e}_\zeta\ \nabla V_0 \,\right] \right\}\cdot\nabla\tilde{\varphi} = 0\,. \tag{25}$$

The second equation, which appears at averaging Eq. (8), represents the condition of the mutual compensation of main nonoscillating terms in Eq. (8). This terms are related, first, with inhomogeneity of $\mathbf{B}^2$ over the poloidal azimuth due to toroidicity. In an expanded form Eq. (24) looks unwieldy, but, as it is shown below, it can be reduced to a simple and convenient form. For this it is necessary to reform the quantity $\mathbf{q}_1\cdot\nabla b_1$.

4. TRANSFORMATION OF EQUATION (24)

The goal of the transformation is clear: it is necessary to pick up those terms in $\mathbf{q}_1\cdot\nabla b_1$ which give a nonzero contribution after averaging in ζ and to express them, finally, through $\tilde{\mathbf{B}}$.

The vector $\mathbf{q}_1$ is represented in (18) as a sum of three terms. Let us consider separately the corresponding contributions from $\mathbf{q}_1\cdot\nabla b_1$ into the left-hand side of (24).

To transform the first term we use the identity

$$\tilde{\mathbf{B}}\cdot\nabla b_1 = \frac{\partial}{\partial\zeta}\, div\, b_1\hat{\tilde{\mathbf{B}}} - div\, \frac{\partial b_1}{\partial\zeta}\hat{\tilde{\mathbf{B}}}\,. \tag{26}$$

If we put here the vector $\hat{\tilde{\mathbf{B}}}$ in the form

$$\hat{\tilde{\mathbf{B}}} = \frac{B_t}{r}\,\delta\mathbf{r} + \hat{\tilde{B}}_\zeta\mathbf{e}_\zeta\,, \tag{27}$$

where $\delta\mathbf{r}$ is the vector (15), we get as a result

$$\frac{\tilde{\mathbf{B}}}{B_t}\cdot\nabla b_1 = \frac{1}{B_t}\frac{\partial}{\partial\zeta}\, div\, b_1\hat{\tilde{\mathbf{B}}} - \frac{1}{B_t}\, div\, \hat{\tilde{B}}_\zeta\frac{\partial b_1}{\partial\zeta}\,\mathbf{e}_\zeta - r\, div\left[\mathbf{e}_\zeta\cdot\nabla b_1\right]\frac{\delta\mathbf{r}}{r}. \tag{28}$$

In the second term

$$\left[\,\tilde{\mathbf{B}}\;\nabla V_0\,\right]\cdot\nabla b_1 = \left[\,\mathbf{e}_\zeta\;\nabla V_0\,\right]\cdot\left[\,\mathbf{e}_\zeta\left[\,\nabla b_1\;\tilde{\mathbf{B}}\,\right]\right] \tag{29}$$

because, by definition, $\mathbf{e}_\zeta\cdot\nabla V_0 = 0$.

In the third term

$$\left[\mathbf{e}_\zeta \; \nabla V_1 \right] \cdot \nabla b_1 \;=\; r \; div \; V_1 \left[\nabla b_1 \; \nabla \zeta \right] . \qquad (30)$$

When the explicit expression (19) for V_1 via $\delta\mathbf{r}$ and V_0 is substituted here, one can get:

$$\left[\mathbf{e}_\zeta \; \nabla V_1 \right] \cdot \nabla b_1 = \left[\mathbf{e}_\zeta \; \nabla V_0 \right] \cdot \nabla \left[\delta\mathbf{r} \cdot \nabla b_1 \right] - r \; div \left[\left[\mathbf{e}_\zeta \; \nabla V_0 \right] \cdot \nabla b_1 \right] \frac{\delta\mathbf{r}}{r} . \qquad (31)$$

The meaning of all these transformations becomes evident after combining all given expressions together in the left-hand side of Eq. (24):

$$\mathbf{q}_0 \cdot \nabla b_2 \;+\; \mathbf{q}_1 \cdot \nabla b_1 \;=\; -\, r \; div \left[\mathbf{q}_0 \cdot \nabla b_1 \right] \frac{\delta\mathbf{r}}{r} \;+\; \tilde{f} \;+ \qquad (32)$$

$$\frac{N}{4\pi^2 R^2} \left[\mathbf{e}_\zeta \; \nabla V_0 \right] \cdot \left\{ \nabla b_2 \;+\; \nabla \left[\delta\mathbf{r} \cdot \nabla b_1 \right] \;+\; \frac{1}{B_t} \left[\mathbf{e}_\zeta \left[\nabla b_1 \; \tilde{\mathbf{B}} \right] \right] \right\} .$$

Here $\tilde{f}$ is the function oscillating in ζ which gives zero when averaged in ζ:

$$\tilde{f} \;=\; \frac{\partial}{\partial \zeta} \left[\frac{\tilde{\mathbf{B}}^2}{r B_0^2} \;+\; \frac{1}{B_t} \; div \; b_1 \hat{\tilde{\mathbf{B}}} \right] \;-\; \frac{1}{B_t} \; div \; \hat{\tilde{B}}_\zeta \; \frac{\partial b_1}{\partial \zeta} \; \mathbf{e}_\zeta \; .$$

It can be easily checked that for the last term in the right-hand side of (32) the next equality is valid:

$$\left[\mathbf{e}_\zeta \left[\nabla b_1 \; \tilde{\mathbf{B}} \right] \right] = \frac{1}{r} \left\{ \nabla \left[b_1 r \tilde{B}_\zeta \right] - \nabla x^i \frac{\partial}{\partial \zeta} \left[b_1 \tilde{B}_i \right] - b_1 r \left[\mathbf{e}_\zeta \; \tilde{\mathbf{j}} \right] \right\}, \qquad (33)$$

Here we used representation $\tilde{\mathbf{B}} = \tilde{B}_i \nabla x^i$ for the magnetic field with a summation over the repeated index; $\tilde{\mathbf{j}} = rot\ \tilde{\mathbf{B}}$ is the current density component oscillating in ζ. At $\beta \ll 1$ (the condition which is always satisfied in stellarators) the contribution due to the term with $\tilde{\mathbf{j}}$ is vanishingly small, hence in the following it will be disregarded.

If condition (23) is fulfilled, the first term in the right-hand side of Eq. (32) vanishes. Besides, when Eq. (32) is averaged over ζ, the function $\tilde{f}$ disappears also, and, as a result, Eq. (24) takes a simple form

$$\left[\mathbf{e}_\zeta \ \nabla V_0 \right] \cdot \nabla \left\{ \frac{R^2}{r^2} + \left\langle \frac{\tilde{\mathbf{B}}^2}{B_0^2} + \delta \mathbf{r} \cdot \nabla b_1 + \frac{b_1 \tilde{B}_\zeta}{B_t} \right\rangle_\zeta \right\} = 0 \ . \quad (34)$$

Here we used relationship (33) and definition (21) of b_2 and dropped out the small quantity $\bar{\mathbf{B}}_p^2 / B_0^2$.

For further simplification of this already more compact equation we represent $\delta \mathbf{r} \cdot \nabla b_1$ as

$$\delta \mathbf{r} \cdot \nabla b_1 = \frac{r}{B_t} \hat{\tilde{\mathbf{B}}}_p \cdot \nabla b_1 = \frac{\partial}{\partial \zeta} \left[\frac{r}{B_t} \hat{\tilde{\mathbf{B}}}_p \cdot \nabla \hat{b}_1 \right] - \frac{r}{B_t} \tilde{\mathbf{B}}_p \cdot \nabla \hat{b}_1 \ , \quad (35)$$

where, after substitution of the explicit expression (21) for b_1, the last term takes the form:

$$- \frac{r}{B_t} \tilde{\mathbf{B}}_p \cdot \nabla \hat{b}_1 = \frac{2}{B_0^2} \left[\tilde{B}_\zeta^2 - r^2 \tilde{\mathbf{B}} \cdot \nabla \frac{\hat{\tilde{B}}_\zeta}{r} \right] \cong \frac{2}{B_0^2} \left[\tilde{B}_\zeta^2 - \tilde{\mathbf{B}}^2 \right] . \quad (36)$$

Here we disregarded a small toroidal correction and replaced $\tilde{B}_\zeta$ by its vacuum value.

Finally, after substitution of (36) into (34) and averaging over ζ we get:

$$\left[\mathbf{e}_\zeta\ \nabla V_0 \right] \cdot \nabla \left\{ \frac{R^2}{r^2} - \left\langle \frac{\tilde{\mathbf{B}}^2}{B_0^2} - 4 \frac{\tilde{B}_\zeta^2}{B_0^2} \right\rangle_\zeta \right\} = 0 . \qquad (37)$$

This equation must be solved together with (25). Both equations have been obtained here by the expansion method. They correspond to separation in the right-hand side of Eq. (2) of the dominating contributions from terms oscillating in ζ, which order of magnitude is determined by the value $2B_t\tilde{B}_\zeta$, and from the terms independent on ζ related with $\tilde{\mathbf{B}}^2$ and toroidal inhomogeneity of $\mathbf{B}^2$.

5. EQUATIONS OF QUASISYMMETRY IN THE MODEL OF CIRCULAR SHIFTED MAGNETIC SURFACES

To solve quasisymmetry equations, it is necessary to know the shape of magnetic surfaces determining the vector ∇V_0 entering Eqs. (25) and (37). The simplest model which is widely used in analytical theory of toroidal systems is the model of circular shifted magnetic surfaces. In this model the transition to flux coordinates is prescribed by the relationships

$$\begin{aligned} r &= R - \rho \cos u = R + \Delta - a \cos \theta, \\ z &= \rho \sin u = a \sin \theta . \end{aligned} \qquad (38)$$

Here ρ, u, ζ are quasicylindrical coordinates attached to the circular axis $r = R$ of the system. Outward shift corresponds to $\Delta > 0$. In our case (38) should be considered as a parameterization of averaged magnetic surfaces $\bar{\psi} = const.$

It follows from (38)

$$a\left[1 - \Delta' \cos\theta\right]\nabla a = \mathbf{e}_\rho\left[\rho + \Delta\cos u\right] - \mathbf{e}_u\,\Delta\sin u\,, \quad (39)$$

so vector $\left[\mathbf{e}_\zeta\,\nabla V_0\right]$ entering the quasisymmetry equation takes the form

$$\left[\mathbf{e}_\zeta\,\nabla V_0\right] = \frac{V_0'}{a\left[1 - \Delta'\cos\theta\right]}\left\{\mathbf{e}_u\left[\rho + \Delta\cos u\right] + \mathbf{e}_\rho\,\Delta\sin u\right\}. \quad (40)$$

Multiplier before the bracket is necessary only for the first equation. For circular torii

$$V_0 = 2\pi^2\left[R + \Delta\right]a^2, \qquad V_0' \cong 4\pi^2 Ra\,, \quad (41)$$

in V_0' toroidal corrections are dropped out. They could be essential at $|\Delta|/a \ll a/R$. It is desirable to satisfy quasisymmetry condition somewhere at the finite distance from the magnetic axis. There for estimates one can use the approximation $|\Delta|/a \ll 1$. In this case $\rho \cong a - \Delta\cos u$, and if only terms linear in $|\Delta|/a$ and Δ' are taken into account, the difference between a and ρ in them can be disregarded. At that, for example, in (40) $\Delta'\cos\theta \cong \Delta'(\rho)\cos u$.

At mentioned assumptions the first quasisymmetry equation is reduced to the

$$\frac{1}{N}\frac{\partial\tilde{\varphi}}{\partial\zeta} + \left[1 + \left[\frac{\Delta}{\rho} + \Delta'\right]\cos u\right]\frac{\partial\tilde{\varphi}}{\partial u} + \Delta\sin u\,\frac{\partial\tilde{\varphi}}{\partial\rho} = 0, \quad (42)$$

and the second one to

$$\left[\left(1 + \frac{\Delta}{\rho} \cos u \right) \frac{\partial}{\partial u} + \Delta \sin u \frac{\partial}{\partial \rho} \right] H = - 2 \frac{\rho}{R} \sin u , \quad (43)$$

where

$$H \equiv \left\langle \frac{\tilde{\mathbf{B}}^2}{B_0^2} - 4 \frac{\tilde{B}_\zeta^2}{B_0^2} \right\rangle_\zeta . \quad (44)$$

At small $|\Delta|/a$ the potential $\tilde{\varphi}$ satisfying equation (42) must look like

$$\tilde{\varphi} = \tilde{\varphi}_0 + \tilde{\varphi}_1 , \quad (45)$$

where $\tilde{\varphi}_0$ is the main component, and $\tilde{\varphi}_1$ is the "correction" of the order of Δ/a. Correspondingly

$$H = H_0(\rho) + H_1(\rho) \cos u . \quad (46)$$

For the function $\tilde{\varphi}_0$ it follows in this case from (42):

$$\frac{1}{N} \frac{\partial \tilde{\varphi}_0}{\partial \zeta} + \frac{\partial \tilde{\varphi}_0}{\partial u} = 0 . \quad (47)$$

As it could be expected, in the lowest approximation the quasisymmetry condition reduces just to the condition of helical symmetry: equation (47) can be satisfied by any function $\tilde{\varphi}_0(\rho, u - N\zeta)$. But in the next approximation in Δ/a we get a nontrivial condition which, at given main helical field, demands special choice of the additional one:

$$\frac{1}{N} \frac{\partial \tilde{\varphi}_1}{\partial \zeta} + \frac{\partial \tilde{\varphi}_1}{\partial u} = - \left(\frac{\Delta}{\rho} + \Delta' \right) \cos u \frac{\partial \tilde{\varphi}_0}{\partial u} - \Delta \sin u \frac{\partial \tilde{\varphi}_0}{\partial \rho} . \quad (48)$$

The main field can be often approximated by a single harmonic $\varphi_\ell(\rho)\sin(\ell u - m\zeta)$, which corresponds to $N = m/\ell$. Then $\tilde{\varphi}_1$ must be a superposition of two satellite harmonics $\varphi_{\ell\pm 1}(\rho)\sin[(\ell \pm 1)u - m\zeta]$, which amplitudes must be selected to satisfy simultaneously two conditions:

$$2\varphi_{\ell-1} = \varphi_\ell'\Delta + \ell\varphi_\ell \left[\frac{\Delta}{\rho} + \Delta' \right] , \qquad (49)$$

$$2\varphi_{\ell+1} = \varphi_\ell'\Delta - \ell\varphi_\ell \left[\frac{\Delta}{\rho} + \Delta' \right] . \qquad (50)$$

Fulfillment of (49) and (50) would mean that it is possible to preserve the dominating type of the symmetry (that of the helical field) even at the presence of significant shift of magnetic surfaces. Accompanying requirement of the absence of the toroidal asymmetry, Eq. (43), is satisfied if

$$H_1 - \Delta \frac{dH_0}{d\rho} = 2 \frac{\rho}{R} . \qquad (51)$$

At $H_0' > 0$ the contribution from the second term in the left-hand side of (51) is favorable if $\Delta < 0$, that is at the inward shift of magnetic surfaces, into the region of stronger toroidal field. But at such a shift the stability of a plasma can deteriorate. A hope remains to make the condition on Δ, which follows from (51), weaker due to the function H_1 depending itself implicitly on Δ.

The shift Δ in (49) - (51) is not a free parameter. In the general case it depends on the configuration of vacuum field and equilibrium distributions of plasma parameters. The problem of quasisymmetry becomes closely related, therefore, with equilibrium problem. And what's more the self-consistent noncontradictory solution of the problems requires an accuracy in imposing boundary conditions when amplitudes of satellite harmonics are varied. That is why even the analysis of vacuum configurations turns out into an independent problem.

6. CONCLUSION

Equations of quasisymmetry (25) and (37), derived in a general form for conventional stellarators, can be used without limitations for analysis of any configuration in this class of systems because nothing more than natural and reliable (which is generally acknowledged) stellarator expansion was used in deriving these equations. The equations are written in an invariant form, so there is no problem of their compatibility with any numerical code or analytical method for calculation of vacuum or equilibrium configurations. Equations (42) and (43), derived from the general ones under some simplifying assumptions, show that even in a simple model of circular shifted magnetic surfaces the analysis of quasisymmetry condition needs self-consistent calculation of magnetic configuration because for making $\mathbf{B}^2$ "quasisymmetrical" in the order, which is determined by toroidicity, the shift Δ is necessary. At $\Delta \neq 0$ to maintain the quasisymmetry in the main order in $|\tilde{\mathbf{B}}|/B_0$ is possible only if besides the main harmonic of helical field there are some "satellites". But they themselves produce the shift, so the problem becomes strongly nonlinear. A separate paper will be devoted to its analysis.

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