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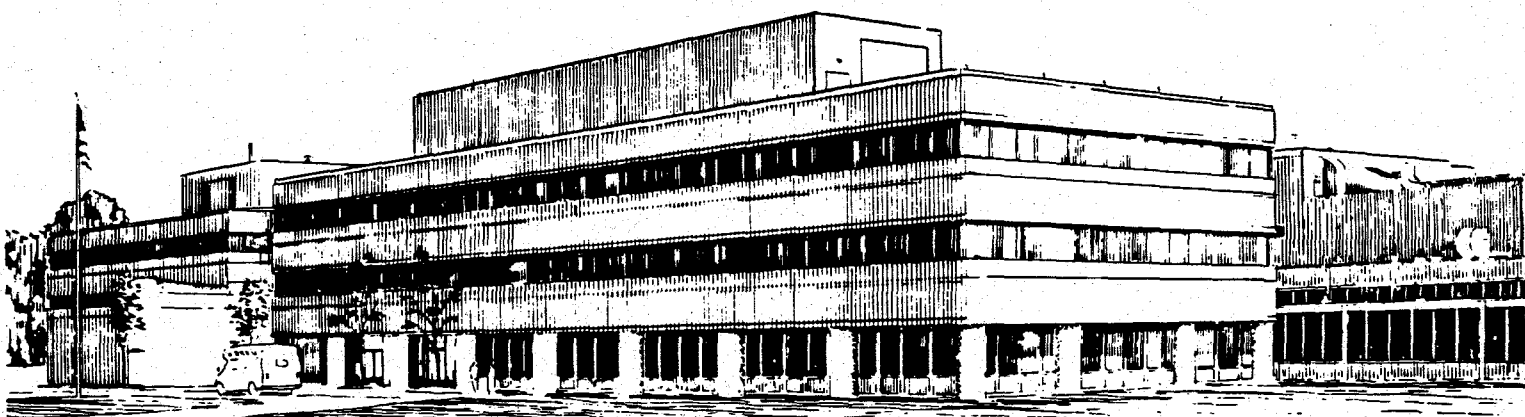
THREE-DIMENSIONAL MAGNETOSPHERIC EQUILIBRIUM  
WITH ISOTROPIC PRESSURE

BY

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# Three-Dimensional Magnetospheric Equilibrium with Isotropic Pressure

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## Abstract

In the absence of the toroidal flux, two coupled quasi two-dimensional elliptic equilibrium equations have been derived to describe self-consistent three-dimensional static magnetospheric equilibria with isotropic pressure in an optimal  $(\psi, \alpha, \chi)$  flux coordinate system, where  $\psi$  is the magnetic flux function,  $\chi$  is a generalized poloidal angle,  $\alpha = \phi - \delta(\psi, \phi, \chi)$ ,  $\phi$  is the toroidal angle,  $\delta(\psi, \phi, \chi)$  is periodic in  $\phi$ , and the magnetic field is represented as  $\vec{B} = \nabla\psi \times \nabla\alpha$ . A three-dimensional magnetospheric equilibrium code, the MAG-3D code, has been developed by employing an iterative metric method. The main difference between the three-dimensional and the two-dimensional axisymmetric solutions is that the field-aligned current and the toroidal magnetic field are finite for the three-dimensional case, but vanish for the two-dimensional axisymmetric case. With the same boundary flux surface shape, the two-dimensional axisymmetric results [Cheng, 1992] are similar to the three-dimensional magnetosphere at each local time cross section.

## Introduction

The problem of computing self-consistent high- $\beta$  ( $\beta = 2P/B^2$ , where  $P$  is the plasma pressure and  $B$  is the magnetic field intensity), static magnetospheric equilibria has attracted much attention from the early days of space physics research [Akasofu and Chapman, 1961]. Numerical as well as analytical solutions of general high- $\beta$  ( $\beta \geq 1$ ) axisymmetric magnetospheric equilibria with anisotropic pressures has been performed [Cheng, 1992] by integrating the Grad-Shafranov equation with prescribed boundary flux shapes. Numerical studies have been performed to examine the effects of plasma  $\beta$ , pressure anisotropy, and boundary condition on the axisymmetric magnetospheric equilibrium. The results agree very well qualitatively with satellite observations of the ring current distribution and the magnetic field intensity within about  $10 R_E$ . This successful method is now extended to study three-dimensional high- $\beta$  magnetospheric equilibria.

An intrinsic feature of the three-dimensional magnetospheric structure is the existence of Birkeland currents, which are field-aligned currents linking the Earth's polar ionosphere with more distant magnetospheric plasma. Observations [Iijima and Potemra, 1976a,b] indicate that near Earth they flow in broad sheets, roughly aligned with the aurora oval. Those sheets form two large current systems, region 1 entering on the morning side of the polar cap and flowing out on the afternoonside, and region 2 further equatorward but with opposite polarities. At noon and midnight the current systems overlap in complicated ways, and during substorms region 1 on the nightside is reenforced by a "substorm wedge", which covers a limited sector in longitude. It is now generally considered that the region 2 currents originate from particle drift motion in the closed field line region [Schindler and Birn, 1978] where the plasma convective flow is slow in comparison with the thermal speeds. The sources of the region 1 currents are less clear and are still being actively debated: on the dayside they come from open field lines, and on the nightside they could come from the plasma sheet. Mathematical expressions for obtaining the field-aligned currents have been derived [Vasyliunas, 1970; Heinemann and Pontius, 1990; Birmingham, 1992]. However, quantitative studies of the self-consistent field-aligned currents have not been performed due to the lack of self-consistent three-dimensional magnetospheric equilibrium solutions.

The purpose of this paper is to provide numerical equilibrium solutions of the self-consistent three-dimensional magnetic field and current structure of the magnetosphere with isotropic pressure. By assuming that the toroidal flux is zero (or, equivalently, no average toroidal magnetic field over the toroidal direction), the three-dimensional magnetospheric equilibrium equations can be reduced to two coupled quasi two-dimensional equations in an optimal flux

coordinate system. A three-dimensional magnetospheric equilibrium code, the MAG-3D code, has been developed by employing an iterative metric method subject to boundary conditions of  $\alpha$  and  $\psi$  on the computational domain surrounded by the specified shapes of the inner and outer flux surfaces [Cheng, 1992]. Equations describing the current system are presented. The main difference between the three-dimensional and the two-dimensional axisymmetric solutions is that the field-aligned current and the toroidal magnetic field are finite for the three-dimensional case, but vanish for the two-dimensional axisymmetric case. The flux surfaces of the three-dimensional magnetosphere at each local time cross section are similar to the previous two-dimensional axisymmetric results [Cheng, 1992] with the same boundary flux surface shapes.

### Three-Dimensional Magnetospheric Equilibrium Equations and Current System

If the plasma convection in the magnetosphere is small, the static magnetospheric equilibrium with isotropic pressure is described in the rationalized EMU unit by the system of equations: the force balance equation  $\vec{\mathbf{j}} \times \vec{\mathbf{B}} = \nabla P$ ,  $\nabla \times \vec{\mathbf{B}} = \vec{\mathbf{j}}$ , and  $\nabla \cdot \vec{\mathbf{B}} = 0$ , where  $P$  is the plasma pressure. If the three-dimensional magnetospheric equilibrium has nested magnetic surfaces, the magnetic field can be expressed in a straight field line  $(\psi, \alpha, \chi)$  flux coordinate as  $\vec{\mathbf{B}} = \nabla \psi \times \nabla \alpha$ , where  $\psi$  is the magnetic flux function,  $\alpha = \zeta - q(\psi)\chi$ ,  $\zeta = \phi - \delta(\psi, \phi, \chi)$ ,  $\chi$  is a generalized poloidal angle,  $\phi$  is the toroidal angle in the cylindrical  $(R, \phi, Z)$  coordinate, and  $\delta(\psi, \phi, \chi)$  is periodic in both  $\chi$  and  $\phi$  so that  $\vec{\mathbf{B}} \cdot \nabla \zeta / \vec{\mathbf{B}} \cdot \nabla \chi = q(\psi)$ . The intersection of surfaces of constant  $\psi$  and  $\alpha$  defines the magnetic field line. The Jacobian is given by  $\mathbf{J} = (\nabla \psi \times \nabla \zeta \cdot \nabla \chi)^{-1}$ . The flux coordinate system is in general not orthogonal because  $\nabla \psi \cdot \nabla \chi \neq 0$ ,  $\nabla \psi \cdot \nabla \zeta \neq 0$ , and  $\nabla \zeta \cdot \nabla \chi \neq 0$ . Within a magnetic surface the poloidal flux is  $\Psi = (1/2\pi) \int d^3x \vec{\mathbf{B}} \cdot \nabla \chi = 2\pi\psi$ , and the toroidal flux is  $\Phi = (1/2\pi) \int d^3x \vec{\mathbf{B}} \cdot \nabla \zeta$ . We will choose  $q(\psi) = 0$  so that  $\alpha = \zeta$  and  $\Phi = 0$ . Then,  $\alpha$  is a cyclic function with a period of  $2\pi$  for all constant  $\psi$  surfaces. This property allows to reduce the general three-dimensional equilibrium equations to quasi two-dimensional equations in the flux coordinate system, and thus greatly simplifies the computational complexity.

Since  $\vec{\mathbf{B}} \cdot \nabla P = 0$ , the pressure is constant along the field line. If the particle drift surfaces coincide with the constant  $\psi$  surfaces,  $P$  is a function of  $\psi$  only. The  $\vec{\mathbf{B}} \times \nabla \psi$  component of the force balance equation gives the radial current density,

$$\vec{\mathbf{j}} \cdot \nabla \psi = \nabla \cdot [(\nabla \psi)^2 \nabla \alpha - (\nabla \alpha \cdot \nabla \psi) \nabla \psi] = 0, \quad (1)$$

which is a two-dimensional elliptic equation on each constant  $\psi$  surface. Note that in the two-dimensional axisymmetric limit, Eq.(1) is trivially satisfied by  $\alpha = \phi$ . The  $\nabla\psi$  component of the momentum equation gives the ring current and the generalized Grad-Shafranov equation,

$$\vec{\mathbf{j}} \cdot \nabla \alpha = \nabla \cdot [(\nabla \alpha \cdot \nabla \psi) \nabla \alpha - (\nabla \alpha)^2 \nabla \psi] = dP/d\psi, \quad (2)$$

which is a two-dimensional elliptic equation on each constant  $\alpha$  surface. Note that in general Eqs.(1) and (2) are three-dimensional equations. However, by choosing the  $(\psi, \alpha, \chi)$  coordinate system we have reduced the dimensionality of Eqs. (1) and (2) from three-dimensional differential equations to quasi two-dimensional differential equations. Equations (1) and (2) form a coupled set of equations that determine  $\alpha$  and  $\psi$ , and can be solved by specifying  $\alpha$  and  $\psi$  on the computational boundary as well as the functional form of  $P(\psi)$ .

From the charge neutrality condition,  $\nabla \cdot \vec{\mathbf{j}} = 0$ , the field-aligned current density equation can be computed from

$$\vec{\mathbf{B}} \cdot \nabla (J_{\parallel}/B) = 2 \vec{\kappa} \times \vec{\mathbf{B}} \cdot \nabla P / B^2 = \nabla B^2 \times \vec{\mathbf{B}} \cdot \nabla P / B^4, \quad (3)$$

where  $J_{\parallel}$  is the field-aligned current density,  $\vec{\kappa} = \hat{\mathbf{b}} \cdot \nabla \hat{\mathbf{b}}$  is the magnetic field curvature, and  $\hat{\mathbf{b}}$  is a unit vector along a magnetic field line. The field-aligned current density can be obtained by integrating Eq. (3) along the field line. The right hand sides of Eq. (3) represent the source of the field-aligned current density which originates from the particle guiding-center  $\nabla B$  and curvature drifts across the pressure gradient. For axisymmetric case,  $\vec{\kappa} \times \vec{\mathbf{B}}$  is in the  $\nabla \phi$  direction, the right hand side of Eq. (3) is zero, and hence  $J_{\parallel} = 0$  everywhere.

Alternatively, the current density can be expressed in a differential form [Shafranov, 1968; Heinemann and Pontius, 1990] and is given by

$$\vec{\mathbf{j}} = \nabla V \times \nabla P + g \vec{\mathbf{B}} = \nabla \psi \times [g \nabla \alpha - (dP/d\psi) \nabla V], \quad (4)$$

where  $g \vec{\mathbf{B}}$  is the force free current,  $V$  is periodic in  $\alpha$  and satisfies the magnetic differential equation,  $\vec{\mathbf{B}} \cdot \nabla V = 1$ , which is obtained by substituting Eq. (4) into Eq. (2). From  $\nabla \cdot \vec{\mathbf{j}} = 0$ ,  $\vec{\mathbf{B}} \cdot \nabla g = 0$  and  $g$  is a field line constant. Since  $\nabla \cdot \vec{\mathbf{j}} = 0$  and  $\vec{\mathbf{j}} \cdot \nabla \psi = 0$ , the current density can be written in the general form  $\vec{\mathbf{j}} = \nabla \psi \times [G(\psi) \nabla \alpha + T(\psi) \nabla \chi + \nabla \lambda(\psi, \alpha, \chi)]$  [Shafranov, 1968], where  $\lambda$  is periodic in both  $\alpha$  and  $\chi$ , and from Eq.(4) we show that  $g = g(\psi)$ . Note that  $g$  vanishes

if there is a north-south or east-west symmetry. In the  $(\psi, \alpha, \chi)$  flux coordinate system  $dV = ds/B = \mathbf{J}d\chi = d^3x/(d\psi d\alpha)$ , where  $ds$  is the element of arc length along the magnetic field line. Thus,  $V$  has the physical meaning of the magnetic flux tube volume per unit flux. The magnetic differential equation can be integrated to give  $V(\psi, \alpha, \chi) = V_s(\psi, \alpha, \chi) + V_o(\psi, \alpha)$ , where  $V_o(\psi, \alpha)$  is an integration constant along the field line, and  $V_s(\psi, \alpha, \chi) = \int \mathbf{J}d\chi$  is an indefinite integral along the field line.

From Eq. (4) the field-aligned current density is given by  $J_{||}/B = -(dP/d\psi)(\partial V/\partial\alpha) + (\mathbf{J}\vec{B} \cdot \nabla\chi \times \nabla P)/B^2 + g(\psi)$ . For any two points 1 and 2 along the field line we have  $(J_{||}/B)_2 - (J_{||}/B)_1 = -(\partial P/\partial\psi)(\partial U/\partial\alpha) + (\mathbf{J}\vec{B} \cdot \nabla\chi \times \nabla P)_2/B_2^2 - (\mathbf{J}\vec{B} \cdot \nabla\chi \times \nabla P)_1/B_1^2$ , where  $U(\psi, \alpha; 1, 2) = V_s(\psi, \alpha, \chi_2) - V_s(\psi, \alpha, \chi_1) = \int_{\chi_1}^{\chi_2} \mathbf{J}d\chi$ , and the subscripts 1 and 2 denote that the quantities are evaluated at  $\chi_1$  and  $\chi_2$ , respectively. For three-dimensional magnetospheric equilibria with north-south symmetry,  $J_{||} = \vec{B} \times \nabla\chi = 0$  at the equator. Thus, the field-aligned current density at the ionosphere can be expressed in terms of the equatorial quantities as [Vasyliunas, 1970; Birmingham, 1992]

$$J_{||i}/B_i = \vec{B}_e \times \nabla U(\psi, \alpha; e, i) \cdot \nabla P / B_e^2 + (\mathbf{J}\vec{B} \cdot \nabla\chi \times \nabla P)_i / B_i^2, \quad (5)$$

, where the subscripts e and i denote that the quantities are evaluated at the equator and the ionosphere, respectively, and the gradients,  $\nabla U$  and  $\nabla P$ , are taken at the equator. As pointed out by Birmingham [1992] the second term is missing in the expression given by Vasyliunas [1970], and it is much smaller than the first term by a factor of  $L^{-6}$  if we choose  $\chi = s$ . Note that if there is an east-west symmetry,  $J_{||}$  is zero in the noon-midnight meridian plane.

From Eq. (4) we have  $\mathbf{J}\vec{J} \cdot \nabla\chi = -(dP/d\psi)(\partial V/\partial\alpha)$  if there is no force free current. Since  $V$  is period in  $\alpha$ ,  $\int_0^{2\pi} d\alpha \mathbf{J}\vec{J} \cdot \nabla\chi = 0$  and the total net poloidal current across a constant  $\chi$  surface,  $I_p = (1/2\pi) \int d^3x \vec{J} \cdot \nabla\chi$ , is zero. Therefore, there is no net poloidal current into the planetary ionosphere in the absence of the force free current.

## Numerical Results from the MAG-3D Code



Equations (1) and (2) can be cast into inverse equilibrium equations in terms of a flux coordinate  $(\psi, \alpha, \chi)$  system [Cheng, 1992]. A three-dimensional magnetospheric equilibrium code, the MAG-3D code, has been developed based on an iterative metric method for solving the coupled set of nonlinear inverse equilibrium equations. If we choose  $ds/d\chi = \mathbf{J} \cdot \mathbf{B} = F(\psi, \alpha)$ , we have an equal arc length coordinate system. The numerical grid on which finite differences are evaluated is tied to the equilibrium solution itself in such a way that grid points automatically accumulate in regions of steep gradients, thus yielding accurate solutions of high  $\beta$  magnetospheric equilibria. An iterative metric method is used to solve for the discrete rectangular coordinate  $[x(\psi, \alpha, \chi), y(\psi, \alpha, \chi), z(\psi, \alpha, \chi)]$  of constant  $\psi$  and  $\alpha$  surfaces such that the finite-differenced inverse equilibrium equations based on these points are satisfied to a small tolerance. The MAG-3D code is the first self-consistent three-dimensional magnetospheric equilibrium code.

In the paper we consider a fixed boundary problem. The computational domain is bounded by (1) an outer boundary with flux  $\psi = \psi_{\text{out}}$  and with its shape specified to take into account the effect of the solar wind and the interplanetary magnetic field, (2) an inner boundary with  $\psi = \psi_{\text{in}}$  which is mainly determined by the dipole magnetic field, and (3) the Earth's surfaces between  $\psi_{\text{in}}$  and  $\psi_{\text{out}}$  curves. The boundary condition on the Earth's surface is  $\alpha = \phi$ . In the computational domain, a  $(\rho, \alpha, \chi)$  flux coordinate is chosen with  $0 \leq \chi \leq \pi$ ,  $0 \leq \alpha \leq 2\pi$ , and  $0 \leq \rho \leq 1$ , where  $\psi = \psi(\rho)$  is chosen such that uniform  $\rho$  grids give optimal equatorial radial grids for the computational purpose. The equatorial dipole magnetic field intensity is  $B_D$  at  $R = R_o$ . The magnetic flux is chosen to be  $\psi_{\text{out}} \equiv -B_D R_o^3 / R_{\text{max}}$  at the outer magnetic surface and  $\psi_{\text{in}} \equiv -B_D R_o^3 / R_{\text{min}}$  at the inner magnetic surface. In a right-handed  $(r, \phi, \theta)$  spherical coordinate system with  $\theta = -\pi/2, 0, \pi/2$  corresponding to the south pole, the equator, and the north pole, respectively, the shape of the outer flux surface is specified by choosing  $\psi_{\text{out}} = -B_D R_o^3 \cos^2\theta / r + [B_C (r - R_E) \cos^2 2\theta + B_S (r - R_E)^2 \sin^2 3\theta] \cos \phi$ , and the inner flux surface shape is specified by choosing a dipole field surface,  $\psi_{\text{in}} = -B_D R_o^3 \cos^2\theta / r$ . Note that  $B_C$  and  $B_S$  determine the outer flux surface shape which has a significant effect on the magnetospheric equilibrium. For  $B_C > 0$  and  $B_S < 0$ , the outer flux boundary is compressed on the dayside due to solar wind pressure, and resembles a stretched tail-like surface on the nightside. The pressure profile is chosen to be  $P(\psi) = P_o [(\psi_2 - \psi)/\gamma]^\gamma [(\psi - \psi_1)/\nu]^\nu [(\gamma + \nu)/(\psi_2 - \psi_1)]^{\gamma+\nu}$ . The following numerical results are obtained with  $R_o = 6.6R_E$ ,  $R_{\text{min}} = 2R_E$ ,  $R_{\text{max}} = 10R_E$ ,  $\gamma = 2$ ,  $\nu = 2$ ,  $B_D = 1$ ,  $\psi_1 = \psi_{\text{in}}$ , and  $\psi_2 = \psi_{\text{out}}$ . These parameters are reasonable to model the basic features of the magnetospheric equilibria.

Numerical examples that illustrate qualitative equilibrium features of an isotropic pressure magnetosphere with north-south as well as east-west symmetries are given. Figures 1 shows the

flux surfaces in the noon-midnight meridian plane. The computation is performed for  $P_o = 0.6$  with 65 flux surfaces and 65 poloidal angle grid points and 65 toroidal angle grid points. The outer boundary flux shape is specified by  $B_C = 0.7$ ,  $B_S = -0.05$  so that the outer flux surfaces are compressed to  $R \approx 8.43R_E$  on the dayside and stretch out to  $R \approx 15.39R_E$  on the nightside. The solid lines correspond to the equilibrium constant  $\psi$  surfaces, and the dotted lines represent the corresponding dipole magnetic flux surfaces. In the low beta region with  $L < 4$  the flux surfaces are similar to the dipole surfaces, but for  $L > 4$  the flux surfaces are greatly distorted by the combined effects of finite  $\beta$  and the boundary flux shape. The effect of finite  $\beta$  is to expand the flux surface outward toward the lower magnetic field region. The effect of the outer flux shape is to compress the flux surfaces inward on the dayside and to stretch the flux surfaces outward on the nightside. Along the midnight equatorial line the pressure is peaked at  $R \approx 3.3R_E$ . But, the peak beta is at  $R \approx 11.2R_E$  and is about 2.25. These three-dimensional results, as well as the radial variation of the plasma pressure  $P$ , the plasma  $\beta$ , the toroidal ring current  $J_\phi$ , and the fractional difference between the self-consistent magnetic field and dipole field,  $(B-B_D)/B_D$ , in the equatorial plane, are similar to the two-dimensional axisymmetric results [Cheng, 1992] obtained with the same boundary flux surface shape at each local time cross section.

The main difference between the three-dimensional and two-dimensional axisymmetric results is that the field-aligned current and the toroidal magnetic field are finite for the three-dimensional case, but vanish for the two-dimensional axisymmetric case. Fig. 2 shows a quadrant of the outer ( $\psi = \psi_{out}$ ) flux surface with the magnetic field lines shown by the blue lines and the constant toroidal angle lines shown by the red lines. On the noon-midnight meridian plane as well as the equatorial plane the magnetic field does not have a toroidal component. Near the equatorial plane on the northern hemisphere the magnetic field has a toroidal component pointing toward the midnight direction on the nightside and pointing toward the noon direction on the dayside. From Eq.(3) the toroidal magnetic field provides a magnetic drift across the pressure gradient and gives rise to the field-aligned current. Figure 3 shows the constant field-aligned current density contours over the northern polar region. The solid contour curves represent  $J_{||} > 0$  and the dotted contour curves represent  $J_{||} < 0$ . On the noon-midnight and equatorial planes,  $J_{||} = 0$ . Thus, on the dusk side of the Earth's northern polar ionosphere, the field-aligned current density is positive (flowing into the ionosphere) for higher latitude flux surfaces where  $dP/d\psi < 0$ . The field-aligned current localizes in the region between  $60^\circ$  and  $70^\circ$  latitudes, and its density is peaked at about  $66^\circ$  in latitude and at 23:00 local time. The field-aligned current system is in the opposite direction on the dawn side of the Earth northern pole. These are similar to the observed region 2 current [Iijima and Potemra, 1976a,b].

## Discussion

The MAG-3D code results presented in the paper represent the first self-consistent solution of the three-dimensional magnetospheric equilibrium with isotropic pressure. A natural extension is to consider the pressure anisotropy effect. In order to provide a more conclusive test of the magnetospheric equilibrium calculations with satellite observations, detail information of ring current particle distribution (pressure anisotropy) and better boundary conditions may be required. For the information of particle distributions, issues related to the sources and composition of ring current particles as well as energization and injection processes associated with the storm time ring current formation need to be resolved. For the boundary conditions, we shall explore a more complicated form of the outer flux surface such as empirical shapes from the magnetic field models [Tsyganenko, 1987, 1989], which is obtained by considering all the major magnetospheric current systems outside the magnetopause. To determine the outer magnetic flux surface (magnetopause) self-consistently, the shape of the magnetopause will be determined iteratively as part of the equilibrium solution by a force balance between the magnetic field and a steady solar wind with the requirement that the normal component of the magnetic field vanish at the boundary.

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### Figure Captions

- Fig. 1 The constant  $\psi$  surfaces of a three-dimensional magnetospheric equilibrium with isotropic pressure with  $P_0 = 0.6$ ,  $B_C = 0.7$ ,  $B_S = -0.05$  in the noon-midnight meridian plane. The solid lines correspond to the magnetic field lines and the dotted lines correspond to the dipole field surfaces.
- Fig. 2 A quadrant of the outer ( $\psi = \psi_{out}$ ) flux surface with same parameters as in Fig. 1. The magnetic field lines (with the magnetic field pointing upward) are shown as blue lines and the constant toroidal angle lines are shown as red lines.
- Fig. 3 The field-aligned current density contours at the ionosphere boundary over the northern pole with same parameters as in Fig. 1. The solid (dotted) contour curves represent field aligned current density flowing into (out of) the ionosphere.

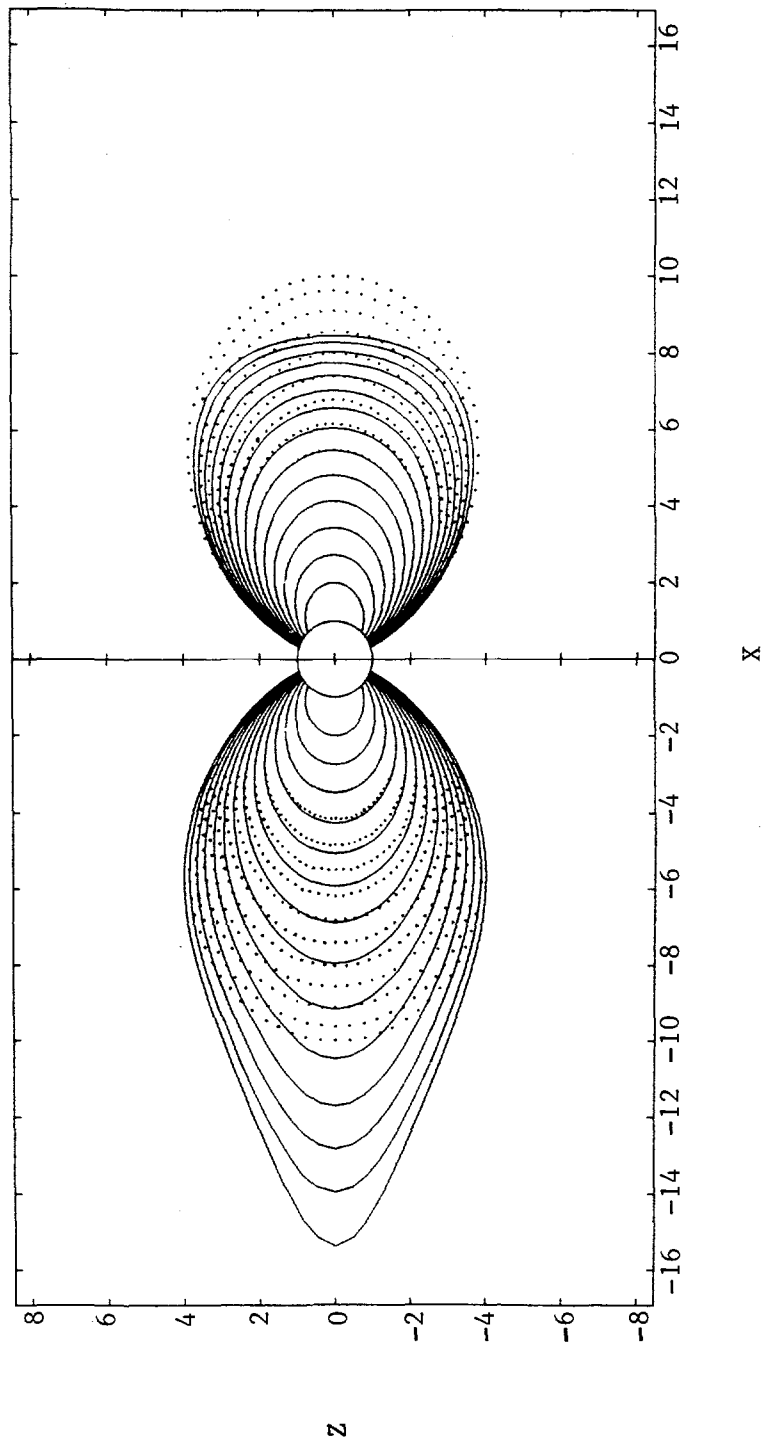


Fig. 1

# Magnetic Flux Surface and Field Lines

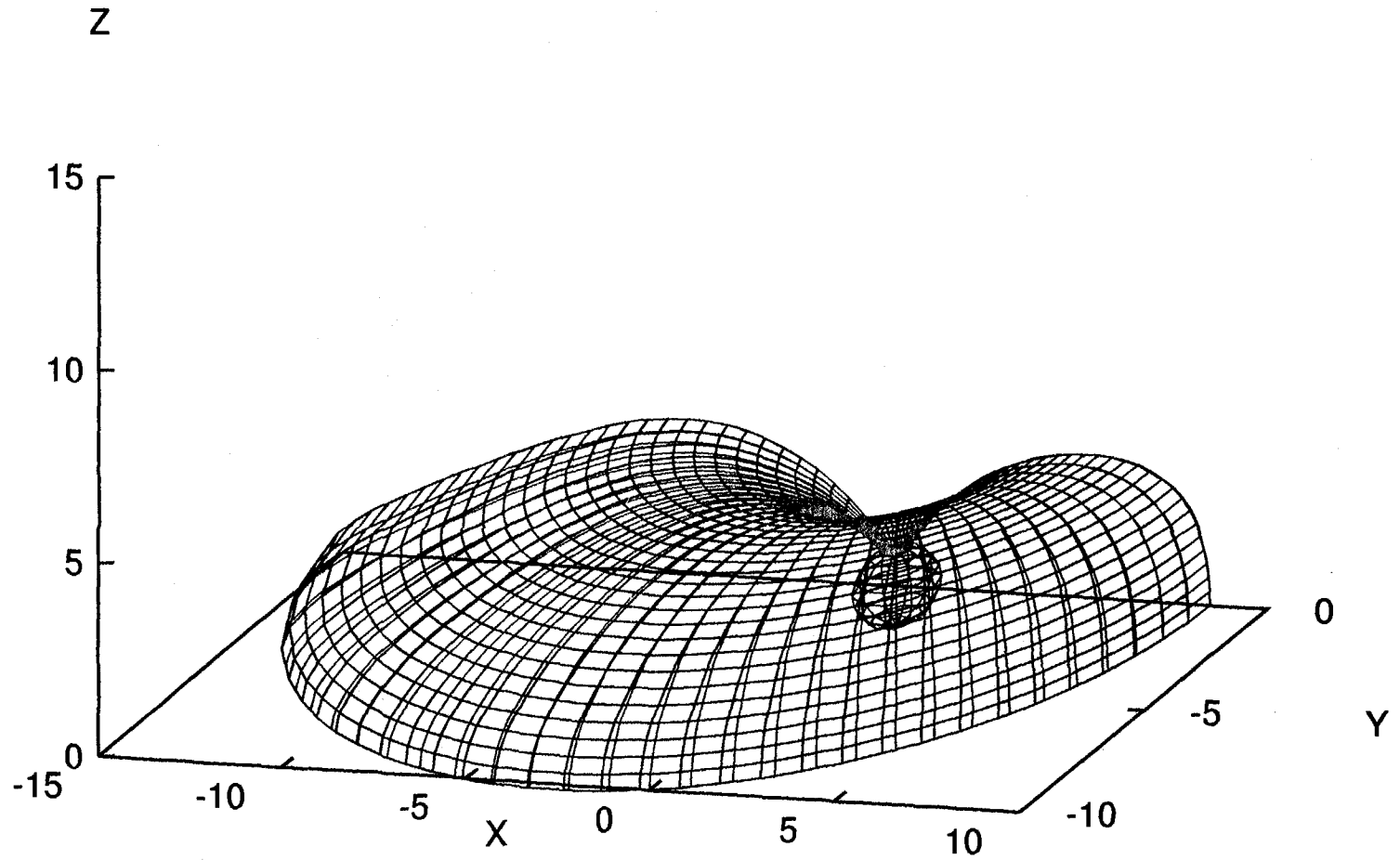


Fig. 2

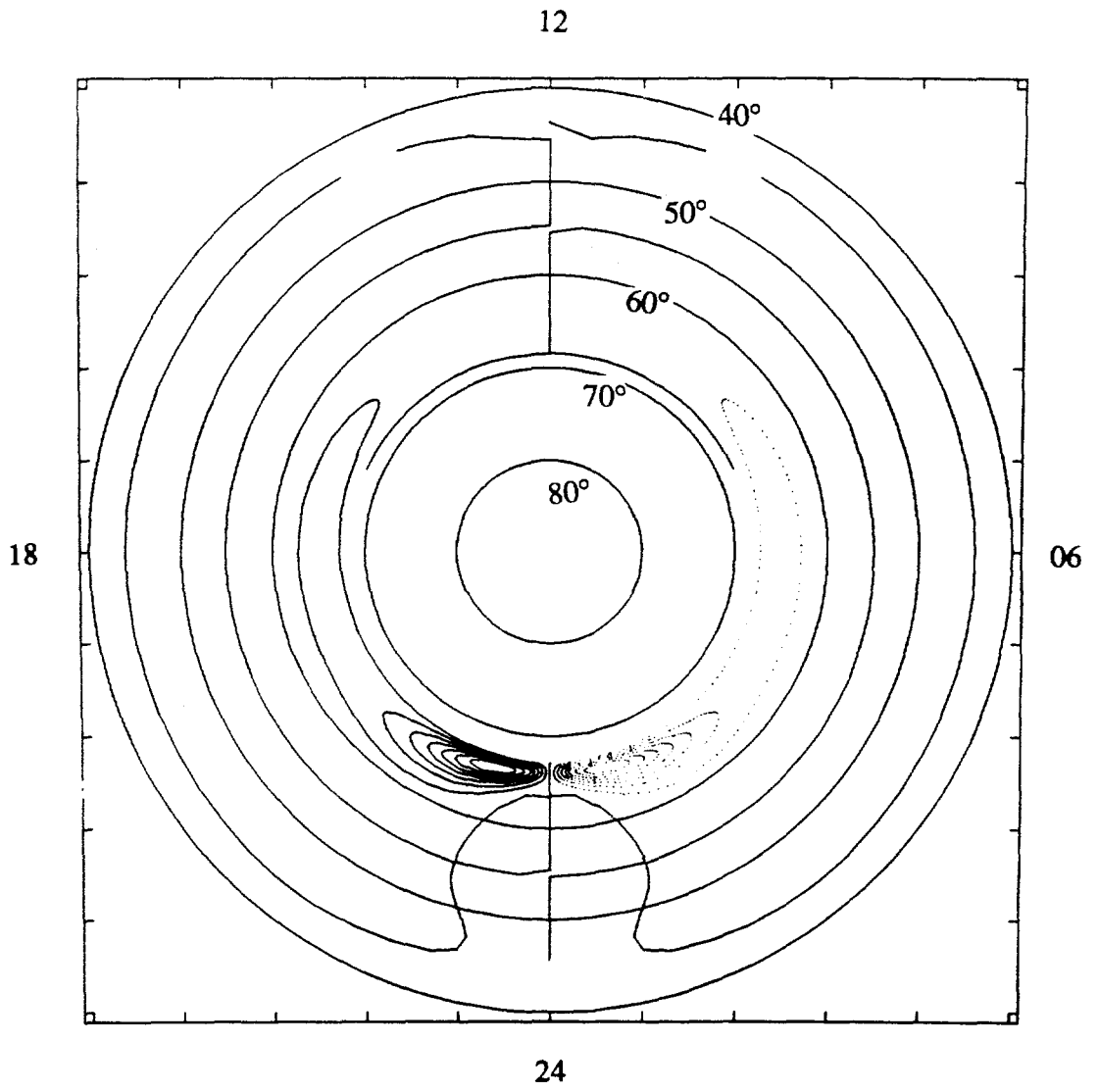


Fig. 3

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