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J.A. Holt, M.A. Martens, L. Michelotti and G. Goderre

Fermi National Accelerator Laboratory
P.O. Box 500, Batavia, Illinois 60510

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J. A. Holt, M. A. Martens, L. Michelotti, G. Goderre
Fermi National Accelerator Laboratory†
P. O. Box 500, Batavia, IL 60510, USA

Abstract

The traditional formula for calculating luminosity assumes an uncoupled lattice and makes use of one-degree-of-freedom lattice functions, β_H and β_V, for relating transverse beam widths to emittances. Strong coupling requires changing this approach. It is simplest to employ directly the linear normal form coordinates of the one turn map. An equilibrium distribution in phase space is expressed as a function of the Jacobian's eigenvectors and beam size parameters or emittances. Using the equilibrium distributions an expression for the luminosity was derived and applied to the Tevatron lattice, which was coupled due to a quadrupole roll.

I. Introduction

The Tevatron lattice for collider operations at Fermilab is designed to give the same lattice functions and luminosity at the two interaction regions CDF and D0. During the first part of Collider Run IB however, the ratio of measured luminosities at CDF and D0 was about $\mathcal{L}_{\rm CDF}/\mathcal{L}_{\rm D0} = 0.75$. In addition to a lower luminosity, the longitudinal distribution of luminosity at CDF was not symmetric as expected. These discrepancies were reduced when a low beta quadrupole near the CDF interaction region was found to be rolled by 8 mrad and subsequently re-aligned. After the re-alignment of the low beta quad, the ratio of measured luminosity changed to about $\mathcal{L}_{\rm CDF}/\mathcal{L}_{\rm D0} = 1.1$ and luminosity distribution at CDF also became symmetric as expected.

Since the effect of a rolled quadrupole on luminosity cannot be explained by using the standard β function treatment of uncoupled machines we develop the formulation for calculating luminosity in a coupled machine. Others have developed a set of general lattice functions which can be used to describe the lattice of coupled machines [2] but we choose instead to use linear normal form analysis. We present the development of an expression for the luminosity in a coupled machine based on linear normal forms and give results of luminosity calculations based on models of the Tevatron with and without the rolled quad.

II. Theory

In this section we will lay out the expressions used to calculate luminosity in the presence of strong coupling.

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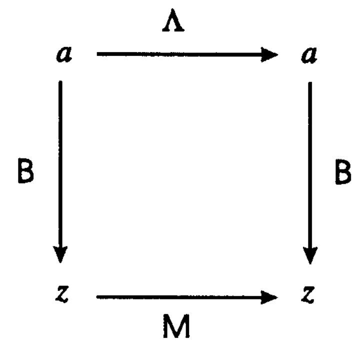

Figure. 1. Commutative diagram showing the action of a one-turn matrix in the normal form representation.

A. Linear normal form coordinates

As usual, we write the state of a particle as an array, $\underline{u}^T = (x, y, c\Delta t; x', y', \delta)$, by referring a coordinate chart to a local "design" fiducial curve in phase space, normally a segment of the closed orbit. $\underline{u}$ contains the coordinates of a particle as it crosses a plane transverse to this local fiducial curve. The coordinates, x, y, x', and y' are the transverse position and momentum* of a particle, relative to the curve, at the instant the particle crosses the plane while $\delta = \Delta p/\bar{p}$ is its momentum offset. The coordinate Δt is the time, relative to the reference time, at which the plane is crossed, so that particles with $\Delta t > 0$ arrive late.

Let $\underline{\mathcal{M}}(\underline{u}; s)$ be the one-turn map at the point marked with arclength coordinate s. That is, in one revolution starting from s, $\underline{u} \mapsto \underline{\mathcal{M}}(\underline{u}; s)$. We are interested in the linear part of $\mathcal{M}$ obtained by taking the Jacobian: $\underline{\underline{M}}(s) = (\partial \underline{\mathcal{M}}/\partial \underline{u})_{\underline{u}=0}$, and $\underline{u} \mapsto \underline{\underline{M}}(s) \cdot \underline{u}$, in linear approximation. Let $\underline{\underline{B}}(s)$ be the matrix whose columns are eigenvectors of $\underline{\underline{M}}(s)$, so that,

$$\underline{\underline{M}}(s) \cdot \underline{\underline{B}}(s) = \underline{\underline{B}}(s) \cdot \underline{\underline{\Lambda}} \quad , \tag{1}$$

where $\underline{\underline{\Lambda}} = \mathrm{diag}(e^{\pm i 2\pi\nu_k})$. The conversion to linear normal form (Weyl) coordinates, $\underline{a}$, is given by

$$\underline{u} = \underline{\underline{B}}(s) \cdot \underline{a} \quad .$$

These are complex coordinates, and from the commutative diagram (Fig. 1) based on Eq.(1), we see that in one turn, $a_k \mapsto \Lambda_{kk} a_k = \exp(\pm i 2\pi\nu_k)\, a_k$. Thus, each $|a_k|$ is an invariant, and *with proper normalization,* such as either auxiliary

*Actually, $x' = p_x/\bar{p}$ and $y' = p_y/\bar{p}$, where $\bar{p}$ is the reference momentum.

condition,

$$-i\underline{\underline{B}}^T(s)\,\underline{\underline{J}}\,\underline{\underline{B}}(s)\,\underline{\underline{J}}=\underline{\underline{1}}\ \ ,\ \text{ or }\ \det\underline{\underline{B}}=1\ \ ,$$

these magnitudes are easily related both to Hamiltonian action coordinates, I_k, and to physical "emittances" of the particle, $\epsilon_k = 2\pi I_k = 2\pi|a_k|^2$.

We want to calculate the state covariance matrix,

$$\underline{\underline{C}}(s) \equiv \langle \underline{u}\,\underline{u}^T\rangle = \underline{\underline{B}}(s)\cdot\langle \underline{a}\,\underline{a}^\dagger\rangle\cdot\underline{\underline{B}}^\dagger(s)\ \ ,$$

of a stationary, equilibrium distribution. With this condition, the angle variables must be uniformly distributed, so that $\langle a_k a_m^*\rangle = \delta_{km} I_k$, and

$$C_{ij}(s) = \langle u_i u_j\rangle = \frac{1}{2\pi}\sum_k B_{ik}(s)B_{jk}^*(s)\langle\epsilon_k\rangle\ . \qquad (2)$$

Coefficients of the $\langle\epsilon_k\rangle$ in this expression are what *should be* meant by "lattice functions" in a coupled machine. They are the numbers which relate invariant emittances to observable properties: the transverse widths of bunch distributions.

B. Luminosity integrals

We now use $\underline{\underline{C}}(s)$ to evaluate luminosity in the presence of coupling. The general expression for the luminosity of two bunches colliding head on with velocity $v=\beta c$ is given by the overlap integral [1],

$$\mathcal{L} = 2\beta c f_{rev}\iint f_1(x,y,z;t)f_2(x,y,z;t)\,dVdt \qquad (3)$$

where f_{rev} is the revolution frequency, and $f_1(x,y,z;t)$ and $f_2(x,y,z;t)$ are the volume density distributions of the two colliding bunches.

To simplify Eq.(3) we assume a Gaussian for the equilibrium distribution in $\underline{u}$ phase space,

$$\rho_s(\underline{u};s) = \frac{N}{\sqrt{2\pi}^3(\det\underline{\underline{C}}(s))^{\frac{1}{2}}}e^{-\frac{1}{2}\underline{u}^T\cdot\underline{\underline{C}}^{-1}(s)\cdot\underline{u}} \qquad (4)$$

where N is the number of particles in a bunch, and $\underline{\underline{C}}(s)$ is the covariance matrix defined in Eq.(2).

To find the volume density distribution $f_1(x,y,z;t)$ we first need to convert Eq.(4) from the s-representation to the time or t-representation. What we are interested in is the position of a particle at a given instant of time. However the s-representation describes the state of a particle as it crosses the transverse plane at position s. Therefore we need to "propagate" the particle away from the local transverse plane. In a drift space this "propagation" is simple since particles travel in straight lines. Without giving the details, the resulting volume density distribution is

$$f_1(x,y,z;t) = \frac{N_1}{\sqrt{2\pi}^3(\det\underline{\underline{C}}_1)^{\frac{1}{2}}}e^{-\frac{1}{2}\underline{\zeta}_1^T\cdot\underline{\underline{C}}_1^{-1}\cdot\underline{\zeta}_1}. \qquad (5)$$

In this expression $\underline{\zeta}_1^T = \rho(x,y,\beta ct - z)$ and $\underline{\underline{C}}_1$ is the 3×3 matrix composed of the elements of the position sector of $\underline{\underline{C}}$. The matrix $\underline{\underline{C}}_1$ (obtained by projecting out the momentum and energy components) describes the beam footprint in the local fiducial chart, that is, in local transverse coordinates.

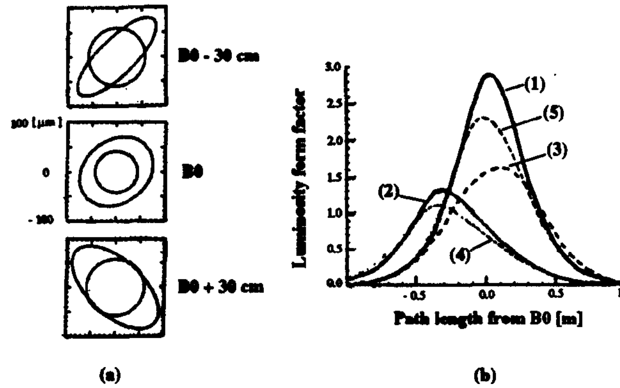

Figure. 2. Astigmatism in the vicinity of B0 resulting from a 9 mrad roll of a low beta quad.

The integrals over x, y, and t in Eq.(3) can be done analytically and the luminosity reduced to an integral over z. First define

$$\underline{d} \ = \ \text{third column of } (\underline{\underline{C}}_1^{-1} - \underline{\underline{C}}_2^{-1})/\sqrt{[\underline{\underline{C}}_1^{-1}+\underline{\underline{C}}_2^{-1}]_{33}}$$

$$\underline{\underline{D}} \ = \ (\underline{\underline{C}}_1^{-1}+\underline{\underline{C}}_2^{-1}-\underline{d}\cdot\underline{d}^T)^{-1}\ \ ,\ \ \Sigma_z^2 \ = \ [\underline{\underline{D}}]_{33}.$$

Then the expression for the luminosity becomes

$$\mathcal{L} = \frac{2f_{rev}N_1N_2}{\sqrt{2\pi}^3}\int\frac{(\det\underline{\underline{C}}_1^{-1}\cdot\underline{\underline{C}}_2^{-1}\cdot\underline{\underline{D}})^{\frac{1}{2}}}{\Sigma_z\sqrt{[\underline{\underline{C}}_1^{-1}+\underline{\underline{C}}_2^{-1}]_{33}}}e^{-\frac{(z-z_0)^2}{2\Sigma_z^2}}\,dz. \qquad (6)$$

where $\underline{\underline{D}}, \underline{\underline{C}}_1$, and $\underline{\underline{C}}_2$ are all functions of z. The emittances, ϵ_k, are determined by measuring the beam profile at three different locations in the accelerator s_k, $k=1,2,3$. From our lattice model we have $\underline{\underline{B}}(s_k)$ and from the measurements we have a component of $\underline{\underline{C}}(s_k)$. Thus we have three equations of the form given in Eq.(2) and can calculate the ϵ_k.

III. Calculations and Measurements

A. Astigmatic focus

A first application of Eq.(2) to the vicinity of B0 is shown in Figure 2(a). For simplicity, only the transverse dimensions have been taken into account, and we assume the values $\langle\epsilon_1\rangle = \langle\epsilon_2\rangle = 30\pi$ mm-mr $/\beta\gamma$ for the transverse expected emittances appearing in Eq.(2). (These numbers are actually the nominal 95% emittance values.) Drawn at three locations in Part (a) are ellipses corresponding to the projection of the covariance matrix, $\underline{\underline{C}}$, into the transverse position sector. They represent the footprint of the bunch as it passes through a plane at each location. The circular loci were obtained by assuming that the accelerator hardware was perfectly aligned, and the more eccentric ones, by introducing a 9 mrad roll in the upstream B0Q2 quadrupole, (the low beta quad near CDF) that was discovered to have this roll. Two effects occur: (a) the focus itself has expanded, and (b) the orientation of the footprint rotates as one travels downstream – the bunch "twists." Optically, these characterize a condition of astigmatism; the quad roll produced an astigmatic focus.

Of course, the emittances that appear in Eq.(2) do not really refer to the "horizontal" and "vertical" planes. They refer to invariant planes in six dimensions, with "longitudinal" entering into the mix. For the above example we simply assumed that the emittances were $\langle \epsilon_1 \rangle = \langle \epsilon_2 \rangle = 30\pi$ mm-mr $/\beta\gamma$. A more correct way of doing this calculation would be to infer the true emittances from three flying wire measurements, as described in the preceding section, but the results would not change significantly.

B. Luminosity Profiles

A more direct way of estimating the effect on luminosity is shown in Figure 2(b). The expression $\exp(-s^2/2\sigma_z^2)/(\det \underline{\underline{C}}(s))^{\frac{1}{2}}$ is plotted within a two meter interval of B0. This profiles the integrand of the luminosity integral at the instant when the centroids of the proton and antiproton bunches meet. The two solid curves correspond to (1) the ideal case of no quad roll and (2) the "actual" case, corresponding to a 9 mrad quad roll with skew quad settings as they actually existed during the run up to July 20, 1995. The dashed curves illustrate three hypothetical scenarios: (3) a "best-case" scenario, in which the effect of the quad roll is compensated by the SQB0 skew-quad correction circuit only, (4) a "worst-case" scenario, in which the SQA0 circuit was not used to compensate, and (5) an attempt to mimic the quad roll with skew quad circuits, without rolling the quad. Notice that (5) comes nowhere near the other dashed curves, or curve (2).

C. Online 6-D Calculation

Eq.(6) has been implemented in conjunction with an online, interactive, six-dimensional model of the Tevatron using the C++ class libraries MXYZPTLK and BEAMLINE [3]. Using the design lattice, a comparison was made using the beam conditions of the present collider run. Two Tevatron collider stores were chosen; one before the quadrupole roll was discovered and one after. In the model the quadrupole was rolled 8 mrad. The invariants (I_k) have been calculated using the sigmas from three Tevatron flying wire measurements. The ratio of measured luminosities at CDF and D0 before the rolled quadrupole discovered was 0.76; the model predicts 0.70. For a store after the roll was corrected the measured ratio was 1.13; the model predicts 1.03. In the rolled quadrupole case, the luminosity distribution at CDF is skewed towards the upstream end of the interaction region. This is in agreement with both the model and with Figure 2.

Several effects have been neglected in the calculation. The electrostatic separators were assumed to be zero and no account was taken of RF bucket cogging. Some of the Tevatron low-β quadrupoles run at values slightly different from the design in order to change the longitudinal position of the minimum β. Inclusion of these effects should bring the model calculation in closer agreement with experimental data.

IV. Conclusions

Linear normal form analysis has been used to develop an expression for the luminosity in a coupled machine. Comparison of the calculation result with experimental data shows that the model can reproduce the qualitative features of the data. Work is in progress to implement a more accurate lattice model of the Tevatron and to include electrostatic separators.

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