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RADIATION PROTECTION AND FUZZY SET THEORY

Prof. Dr.Dr. Yasushi Nishiwaki
University of Vienna, Austria
Address: Jagdschlossgasse 91
A-1130 Vienna, Austria

1. Introduction

The survivors and those who visited Hiroshima immediately after the atomic bombing could have been subjected to a number of other possible noxious effects in addition to atomic radiation. Hospitals, laboratories, drugstores, chemists, pharmaceutical works, storehouses of chemicals, factories etc. that were situated close to the hypocentre were all completely destroyed and various mutagenic, carcinogenic or teratogenic substances must have been released. There was no medical care and no food in the region of high dose exposure and the drinking water was contaminated. There would have been various possibilities of infection. Mental stress would also have been much higher in the survivors closer to the hypocentre. It is confusing which factor played a dominant role. In addition, there would be problems in accurately recording the position of the exposed persons at the time of the atomic bombing and also in estimating the shielding factors. There may be considerable uncertainty in human memory under such conditions.

In the case of the Bikini accident, a Japanese fishing boat was showered by strongly radioactive ash from hydrogen bomb testing whilst engaged in fishing in the Bikini area on March 1, 1954 at about 150 km from the test site. When the ship came back two weeks later, stronger radioactivity was found in such easily movable things as dust, gloves, ropes and clothing rather than on the smooth surface of the ship. Therefore, the exact dose of a particular person was difficult to estimate, although the range of probable gamma external dose was estimated to be about 170 to 600 rad. The degree of uncertainty was considered far greater for internal dose estimation. The long-lived radionuclides detected many weeks later in organs such as the liver, could not be considered the only sources of internal exposure. Depending on the assumed degree of initial incorporation of short-lived radionuclides, a wide range of estimates was possible. (10 to 10^4 rad).

When the Three Mile Island accident occurred it was 04:00 hours, a time when the error potential of operators would have been very high. When the Chernobyl accident occurred it was 01:23 hours. This is a time when the level of consciousness is about to drop and the error potential begins to increase. To describe this type of uncertainty, which differs from random uncertainty, requires special mathematic tools. The fuzzy set theory is an attempt to provide such a tool for the consideration of subjective uncertainties in human judgement in the field of radiation protection and nuclear safety.

To describe these situations, what is needed is not the 'yes' or 'no' judgement by ordinary binary logic "0" or "1", but the fuzzy logic which allows the degree of expert judgement between "0" and "1". From the cognition or perception point of view, fuzziness may be divided into two: subjective or intrinsic fuzziness on the part of the man receiving information and objective or extrinsic fuzziness on the part of the object or associated with the incoming information.

2. Fuzzy Set and Fuzzy Measure

The uncertainties in estimating dose-effect relationships in Hiroshima and Nagasaki have been repeatedly discussed by Professor Kishiwaki since the first meeting on the medical and pathological effects of atomic bombings held at the Department of Pathology of the late Professor Ryojun Kinoshita, the then Professor of Pathology, Faculty of Medicine of Osaka University in 1945.

According to Kruse [1], the mathematical concept of fuzziness, which is a modality of uncertainty related to the subjective fuzziness of a human being, may be classified into two categories, one is the fuzzy set proposed by Zadeh [2] and the other the fuzzy measure proposed by Sugeno [3] and Terano and Sugeno [4].

Assuming the universal set $U = \{x_1, x_2, ...\}$ let us consider the relation between the subset and the elements. The fuzzy set deals with the relation between the non-fuzzy element x and the fuzzy subset $\tilde{A}$. If $U = \{1, 2, ..., 10\}$ and fuzzy subset $\tilde{A}$ = a set of large numbers, the fuzzy concept "large number" may be defined mathematically by the membership function $\tilde{A}: U \rightarrow [0, 1]$ such as $\tilde{A}_{(1)} = \tilde{A}_{(2)} = \tilde{A}_{(3)} = 0$, $\tilde{A}_{(4)} = 0.2$, $\tilde{A}_{(5)} = 0.4$, $\tilde{A}_{(6)} = 0.5$, $\tilde{A}_{(7)} = 0.7$, $\tilde{A}_{(8)} = 0.9$, $\tilde{A}_{(9)} = \tilde{A}_{(10)} = 1$. On the other hand, the fuzzy measure deals with the relation between the fuzzy element $\tilde{x}$ and the non-fuzzy subset B of U. This concerns the subjective guessing of whether an a priori non-located element $\tilde{x}$ belongs to the subset B. Suppose balls, each bearing one number between 1 and 10, are placed in a box, and predict whether the number of a ball taken from the box coincides with the number in $B = \{3,4,5\}$. The degree of coincidence of the prediction may be expressed by $Gr(\tilde{x} \in B)$, (Gr = Grade). Gr takes the value '1' if it is sure, and the value '0' if it is impossible. If we consider the degree to be represented by the ratio, it may be expressed as $Gr(\tilde{x} \in B) = 3/10$. In general the following relations may hold:

$$\text{(i)} \quad Gr(\tilde{x} \in U) = 1, \quad Gr(\tilde{x} \in \emptyset) = 0$$
$$\text{(ii)} \quad \text{if } A \subset B, \ Gr(\tilde{x} \in A) \leqslant Gr(\tilde{x} \in B) \qquad (1)$$

where $\emptyset$ is an empty set. The fuzzy measure is the one with which the number in the interval [0, 1] is made to correspond to a subset of the universal set X. It is represented by g(·). It is a set-function which satisfies the following relations:

$$\text{(i)} \quad g(X) = 1, \ g(\emptyset) = 0 \qquad \text{(Boundedness)}$$
$$\text{(ii)} \quad A \subset B \Rightarrow g(A) \leqslant g(B) \qquad \text{(Monotonicity)} \qquad (2)$$
$$\text{(iii)} \quad \text{when } A_1 \subset A_2 \subset \ldots \subset A_n \subset \ldots, \text{ or } A_1 \supset A_2 \supset \ldots \supset A_n \supset \ldots,$$
$$\lim_{n \to \infty} g(A_n) = g(\lim_{n \to \infty} A_n) \qquad \text{(Continuity)}$$

A probability measure is the one which satisfies the relation

$$\text{(i)} \quad P(X) = 1, \ P(\emptyset) = 0$$
$$\text{(ii)} \quad A_i \cap A_j = \emptyset \text{ for } i \neq j \Rightarrow P(\bigcup_{n=1}^{\infty} A_n) = \sum_{n=1}^{\infty} P(A_n) \qquad (3)$$

3. Fuzzy Integral

The fuzzy integral [3–6] is a concept of integrals using fuzzy measure and may be considered an extension of the Lebesgue integral. It is an evaluation of objects by human subjectivity measure. The fuzzy integral for the function $h: X \rightarrow [0,1]$ may be defined by

$$\int_X h(x) \circ g(\cdot) = \sup_{\alpha \in [0,1]} [\alpha \wedge g(F_\alpha)],\ F_\alpha \triangleq \{x \mid h(x) \geqslant \alpha\} \quad (4)$$

where 'sup' means least upper bound and '$\circ$' sup-min. composition of the fuzzy relations, '$\wedge$' min (conjunction), '$\vee$' max (disjunction). For simplicity, assuming

$$X = \{x_1, x_2, \ldots, x_n\},\ h(x_1) \geqslant h(x_2) \geqslant \ldots \geqslant h(x_n) \quad (5)$$

it may be expressed as

$$\int_X h(x) \circ g(\cdot) = \bigvee_{i=1}^{n} [h(x_i) \wedge g(F_i)],\ F_i \triangleq \{x_1, x_2, \ldots, x_i\}$$

where $a \vee b = \max(a,b)$, $a \wedge b = \min(a,b)$, $\bigvee_{i=1}^{n} a_i = \max_{1 \leqslant i \leqslant n} \{a_i\}$. The value of the integral is like the expectation in probability and may be called 'fuzzy expectation'.

Sugeno, Tsukamoto and Terano [11] applied the fuzzy integral model to the subjective evaluation of the human face and residence and Onisawa, Sugeno and Nishiwaki [12] to the public attitude analysis on nuclear energy. Nishiwaki [13] recently proposed the possible use of the fuzzy integral model for the evaluation of sites for nuclear facilities.

In these examples, the attributes or elements of a certain object is expressed by $H = \{a_1, a_2, \ldots\}$ and the grade of importance about a set of attributes at evaluation by a person by g. The characteristic function h_A about a concrete object A may be expressed by a number in $[0,1]$. In this case, if the grade of importance is given, the overall evaluation of the person towards the object A may be expressed by $\int h_A(a) \circ g(\cdot)$.

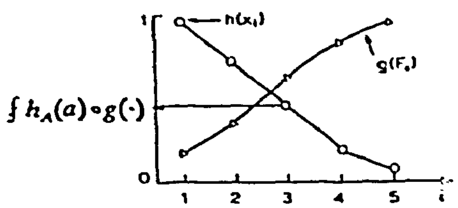

Fig. 1. An example of a fuzzy integral.

In Fig. 1, an example of fuzzy integral for subjective evaluation is shown. In these cases, the additivity may not hold

$$g(\{a_1, a_2\}) \neq g(\{a_1\}) + g(\{a_2\}) \quad (6)$$

However, the monotonicity may hold

$$g(\{a_1\}) \leqslant g(\{a_1, a_2\}) \leqslant \cdots \leqslant g(\{a_1 \cdots a_n\}) \quad (7)$$

4. Conclusion

It has been pointed out by Nishiwaki that the estimation of causal relationships between the initial physical damage caused by ionization or excitation of molecules and the biological end effects in radiation biology may also be considered more or less a problem of fuzzy inference, or fuzzy reasoning in fuzzy logic in some cases. If the fuzzy set theory is applied to the target theory of radiation biophysics as a model of working hypothesis, it may be called fuzzy target theory.

In radiation protection we encounter a variety of sources of uncertainties which are due to fuzziness in our cognition or perception of objects. For systematic treatment of this type of uncertainty, the concepts of fuzzy sets or fuzzy measures could be applied to construct system models, which may take into consideration both subjective or intrinsic fuzziness and objective or extrinsic fuzziness. The theory of fuzzy sets and fuzzy measures is still in a developing stage, but its concept may be applied to various problems of subjective perception of risk, nuclear safety, radiation protection and also to the problems of man-machine interface and human factor engineering or ergonomics. [7–10]

5. References

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