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Isovector meson-exchange currents in the light-front dynamics

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Abstract

In the light-front dynamics, there is no pair term that plays the role of the dominant isovector pion exchange current. This current gives rise to the large and experimentally observed contribution to the deuteron electrodisintegration cross-section near threshold for pseudo-scalar πNN coupling. We show analytically that in leading 1/m order the amplitude in the light-front dynamics coincides, however, with the one given by the pair term. At high Q^2, it consists of two equal parts. One comes from extra components of the deuteron and final state relativistic wave functions. The other results from the contact $NN\pi\gamma$ interaction which appears in the light-front dynamics. This provides a transparent link between relativistic and non-relativistic approaches.

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1 Introduction

The contribution of Meson Exchange Currents (MEC) to the deuteron electrodisintegration cross-section near threshold [1] is firmly established. The cross-section in the impulse approximation has a minimum at about $Q^2 \approx 0.5$ $(GeV/c)^2$, contrary to experimental data (see refs.[2] for a review). Isovector MEC associated with the exchange of a pion fill this minimum and are mandatory to understand the data. In first order in a $1/m$ expansion, the dominant contribution comes from the pair term illustrated in fig.1. This term corresponds to the creation of a $N\overline{N}$-pair by the virtual photon.

However, these first order calculations, especially transparent for physical interpretation, may be insufficient to analyse the data at high momentum transfer in the region to be covered at CEBAF, for example. The light-front dynamics provides a general framework in which a self-consistent analysis beyond first relativistic corrections can be achieved. It has been developed in an explicitly covariant form adapted to practical use (see for a review ref.[3]).

In the impulse approximation, the light-front dynamics was already applied to the deuteron electrodisintegration amplitude in ref.[4] and, in its explicitly covariant form, in ref.[5]. *In these studies the non-relativistic wave functions for the deuteron and for the* final $^1S_0$-state were used. Relativistic effects were found to be small in both calculations. This may not be a surprise since these studies did not account for the dominant MEC contributions which are also of relativistic origin.

In order to achieve a consistent calculation, several relativistic effects should be considered. The first one is the modification of the non-relativistic wave function and the appearance of extra components. The second one is the contribution of a contact $NN\pi\gamma$ interaction (see fig.4 below) which is a new feature of the light-front dynamics.

Extra components of the light-front wave function appear because of its dynamical dependence on the position of the light front. The usual light-front dynamics deals with the state vector defined on the surface $t+z=0$. In the covariant version the state vector is defined on the light front characterized by its general position $\omega\cdot x=0$, where $\omega=(\omega_0,\vec{\omega})$ with $\omega^2=0$. The dynamical dependence of the wave function on the light-front position, manifesting itself in the case of $t+z=0$ as a lack of covariance, is now explicitly parametrized in terms of a four-vector ω without loss of covariance. The dependence of the wave function (but not of the physical amplitude) on the four-vector ω leads to extra spin structures and increases the number of components of the deuteron wave function from two (S- and D- states) up to six. Similarly, the relativistic continuous spectrum wave function of the np-system (corresponding to the $^1S_0$-state in non-relativistic limit) is determined by two components instead of one in the non-relativistic limit.

In the one-boson-exchange approximation, these new components were estimated from a perturbative calculation for the deuteron [6] and for the continuous $^1S_0$-state [7]. It was found in particular that for the deuteron wave function one of the component, called f_5, is rather important. The decomposition of the wave function incorporating the dominant components, usual S- and D-waves and the extra component f_5, has the form:

$$\vec{\psi}(\vec{k},\vec{n}) = \frac{1}{\sqrt{2}}\vec{\sigma}u_S(k) - \frac{1}{2}\left(\frac{3(\vec{\sigma}\vec{k})\vec{k}}{\vec{k}^2} - \vec{\sigma}\right)u_D(k) + \sqrt{\frac{3}{2}}\frac{i}{k}[\vec{k}\times\vec{n}]f_5(k) \,, \tag{1}$$

with $\vec{n} = \vec{\omega}/\omega_0$, where we choose for convenience $\omega_0 > 0$. In this equation, $\vec{k}$ is the relative momentum between the two nucleons in their center of mass frame and $k = |\vec{k}|$. At $k > 0.5$ GeV already, f_5 exceeds all other components including the S- and D-waves. The same was found for the extra component g_2 of the continuous spectrum wave function with zero total angular momentum:

$$\psi(\vec{k},\vec{n}) = g_1(k) + \frac{i}{k}\vec{\sigma}[\vec{k}\times\vec{n}]g_2(k) \,. \tag{2}$$

To avoid any misunderstanding, we emphasize that the parametrization of the light-front wave functions in terms of the four-vector ω in eqs.(1,2) is not the reason of the origin of these extra components, but rather a convenient method for their representation.

The contact interaction originates from the diagram in the old fashioned perturbation theory corresponding to the creation of a nucleon-antinucleon pair from the vacuum. In the infinite momentum frame, this contribution for scalar particles disappears due to the increase of the denominator containing the difference of energies between the intermediate state and the vacuum. For fermions (nucleons), this increase of the denominator is compensated by an increase of the numerator. The finite result is a contact-like interaction, which can be introduced in the light-front Lagrangian from the very beginning [8, 9].

Relativistic effects in the deuteron wave function exert considerable influence on the deuteron form factors [10]. The analysis of the data beyond the non-relativistic expansion is the subject of numerous calculations of relativistic wave functions and observables (deuteron form factors, deuteron electrodisintegration, etc). In almost all relativistic approaches, the link with non-relativistic calculations is not at all clear. This is rather unfortunate if one wants to incorporate our knowledge of the non-relativistic phenomenology developed over the last 20 years into the relativistic formulation of few-body systems and their electromagnetic interactions. This is also important to understand the qualitative features of the relativistic approaches (the light-front dynamics in our case), and especially their link to MEC contributions.

One of the striking features of the comparison between light-front dynamics and non-relativistic approaches is in the fact that the pair term indicated in fig.1 does not contribute in the light-front dynamics. Indeed, ω can be chosen transverse relative to the photon momentum: $\omega \cdot q = 0$. In the non-covariant approach $\omega = (1, 0, 0, -1)$, this corresponds to the usual condition $q_+ = q_0 + q_z = 0$. However, since all the particles are on their mass shells (but off energy shell), the $N\overline{N}$-pair has a positive invariant mass $(p_N + p_{\overline{N}})^2 \geq 4m^2$, and, hence, $\omega(p_N + p_{\overline{N}}) > 0$. This pair cannot be created by the photon with $\omega \cdot q = 0$.

So, where has the dominant contribution from the pair term gone? In the present paper we show analytically how in leading $1/m$ order the light-front dynamics provides the contribution to the deuteron electrodisintegration amplitude coinciding with the well known corrections from the pair term. At high Q^2, this amplitude consists of two equal parts: the first one comes from the contribution of the dominant extra components f_5 and g_2, the second one results from the contact $NN\pi\gamma$ interaction. Preliminary results were published in ref.[11]. For simplicity, we concentrate on the leading contribution from π-exchange with pseudo-scalar coupling. We shall comment in the last section about the extension of our results to the pseudo-vector coupling and to other contributions to MEC.

The plan of the paper is the following. In section 2 we calculate the deuteron electrodisintegration amplitude taking into account, in addition to S- and D-waves, the extra components f_5 and g_2 in the deuteron and continuum state wave functions respectively. In section 3 we find analytical expression for this extra component. In section 4 the contribution of the contact $NN\pi\gamma$ interaction is calculated. Section 5 contains concluding remarks.

2 Deuteron electrodisintegration amplitude

The amplitude of the transition $\gamma^* d \to np$ $(^1S_0)$ has the following general form [5]:

$$F_{\mu\rho} = \frac{1}{2m^2} e_{\rho\mu\nu\gamma} q_\nu p_\gamma A \ , \tag{3}$$

where q is the four-momentum transfer, p is the deuteron momentum, μ and ρ are four-dimensional indices for the deuteron and the virtual photon respectively. The amplitude

(3) is automatically gauge invariant and is determined by the only invariant function A. The cross-section has the standard form which we give to fix the normalization

$$\frac{d\sigma}{d\Omega_e dE'} = \left(\frac{d\sigma}{d\Omega}\right)_0 \left[W_2 + 2\tan^2(\frac{1}{2}\theta)W_1\right] \ , \tag{4}$$

with:

$$W_2 = \frac{\pi p^* Q^2}{24m^2}|A|^2, \quad W_1 = (1+\nu^2/Q^2)W_2 \ , \tag{5}$$

where $Q^2 = -q^2 = -(k_e - k'_e)^2$, $\nu = pq/M_d$ and p^* is the momentum of one of the final nucleons in their c.m.-system.

We calculate below the invariant amplitude A. For simplicity, we shall not take into account first the final state interaction and incorporate it later. The vertex $\gamma^* d \rightarrow np\ (^1S_0)$ is represented graphically in fig.2. This graph corresponds to a special graph technique developed by Kadyshevsky [12] and applied to the light-front dynamics [3]. While all particles are on their respective mass shell, the vertices containing the dashed lines are off energy shell. For convenience, we represent the deuteron wave function in the four-dimensional form [6]:

$$\psi_\mu = \overline{u}(k_1)O_\mu U_c \overline{u}(k_2) \ , \tag{6}$$

where U_c is the charge conjugation matrix and O_μ will be detailed at the end of this section. Using these rules we obtain for the diagram of fig.2 the following expression:

$$G_{\mu\rho} = im^{-1/2}2^{-2}\pi^{-3/2}Tr\left[\gamma_5(\hat{k}_f + m)\Gamma^V_\rho(\hat{k}_2 + m)O_\mu(\hat{k}_1 - m)\right] \ , \tag{7}$$

where Γ^V_ρ is the isovector nucleon electromagnetic vertex:

$$\Gamma^V_\rho = \gamma_\rho F^V_1 + \frac{i}{2m}\sigma_{\rho\nu}q_\nu F^V_2 , \tag{8}$$

with $\sigma_{\rho\nu} = \frac{i}{2}[\gamma_\rho\gamma_\nu - \gamma_\nu\gamma_\rho]$ and $\hat{k} \equiv \gamma_\mu k^\mu$. The superscript V refers to the isovector part of the form factors. We neglect in the amplitude the relative momentum of final nucleons, i.e. put $p^* = 0$, so that $k_1 = k_f = (p+q)/2$. The relative momentum will be taken into account later in the final state interaction wave function. Note that the matrix γ_5 appears in (7) due to the relativistic spin wave function of the $^1S_0$ state [5].

The expression of the electromagnetic current used here is different from the one with Sachs form factors employed in a previous paper [5]. These two expressions are not equivalent to each other off-energy shell. The present one for its F_1 part is consistent with the minimal substitution in the Dirac equation. Sachs form factors may be used, but extra terms have then to be introduced in the current.

The amplitude $G_{\mu\rho}$ in eq.(7) does not coincide with $F_{\mu\rho}$ in eq.(3). The tensor $G_{\mu\rho}$ depends on ω and, due to its decomposition on the general invariant amplitudes, has the form:

$$G_{\mu\rho} = \frac{1}{2m^2} e_{\rho\mu\nu\gamma} q_\nu p_\gamma A + e_{\rho\mu\nu\gamma} q_\nu \omega_\gamma B_1 + e_{\rho\mu\nu\gamma} p_\nu \omega_\gamma B_2 \tag{9}$$
$$+(V_\mu q_\rho + V_\rho q_\mu) B_3 + (V_\mu \omega_\rho + V_\rho \omega_\mu) B_4 + (V_\mu p_\rho + V_\rho p_\mu) B_5 \ ,$$

where $V_\mu = e_{\mu\alpha\beta\gamma}\ \omega_\alpha\ q_\beta\ p_\gamma$. The decomposition (9) contains the symmetric structures in front of the functions $B_{3,4,5}$. Corresponding antisymmetric terms (like $V_\mu q_\rho - V_\rho q_\mu$) are not independent and can be expressed through the first three items. We emphasize that $G_{\mu\nu}$ depends on ω even for the components of the wave function which do not depend on ω. In the latter case, ω enters through the rules of the graph technique for the amplitude of fig.2.

The dependence of the electromagnetic vertex on ω was discussed in refs.[13]. The decomposition (9) enables us to separate immediately the ω-independent parts from unphysical ω-dependent ones so that one can extract directly the physical form factors from the initial tensor $G_{\mu\rho}$. From eq.(9) we immediately find[1]:

$$A = \frac{m^2}{Q^2\ (\omega\cdot p)} e_{\mu\rho\nu\gamma} q_\nu \omega_\gamma G_{\mu\rho}\ . \tag{10}$$

The deuteron vertex function O_μ, which enters in eq.(6), has the general form:

$$O_\mu = \varphi_1 \frac{(k_1-k_2)_\mu}{2m^2} + \varphi_2 \frac{\gamma_\mu}{m} - \varphi_5 \frac{i}{m^2(\omega\cdot p)} \gamma_5 e_{\mu\alpha\beta\gamma}\ k_{1\alpha} k_{2\beta} \omega_\gamma\ . \tag{11}$$

We keep in eq.(11) the dominant functions only. They are expressed through u_S, u_D and f_5 defined[2] in eq.(1). In leading $1/m$ order one has:

$$\varphi_1 = \frac{\sqrt{2}}{8} u_S(k) - \left(\frac{1}{8} + \frac{3m^2}{4k^2}\right) u_D(k), \quad \varphi_2 = \frac{1}{4}\left(\sqrt{2} u_S(k) + u_D(k)\right), \quad \varphi_5 = \frac{1}{2}\sqrt{\frac{3}{2}} \frac{m}{k} f_5(k)\ . \tag{12}$$

From eqs.(7,10), we thus get in a $1/m$ expansion:

$$A = \frac{m^{3/2}}{\pi^{3/2}\sqrt{2}} \left\{ G_M^V \left[u_S(\Delta/2) + \frac{1}{\sqrt{2}} u_D(\Delta/2)\right] - G_C^V \frac{\sqrt{3}\ m}{\Delta} f_5(\Delta/2) \right\}, \tag{13}$$

[1]In ref.[5] the symmetrization of the last three items in eq.(9) was not taken into account and consequently the expression for A differs from (10). This fact, however, has no influence on the conclusion of ref.[5]. Using eq.(10) we have not found neither any difference in leading order nor any noticeable numerical difference in the cross-section in the interval $0 < Q^2 < 10\ (GeV/c)^2$.

[2]The definition of the wave function used in [5] differs from the present paper by the sign in front of u_D.

where $\Delta = \sqrt{Q^2}$ and $k = \Delta/2$. We introduce here the magnetic and charge nucleon form factors $G_M^V = F_1^V + F_2^V$ and $G_C^V = F_1^V - Q^2/(4m^2)F_2^V \approx F_1^V$. In the plane wave approximation with $p^* = 0$, the function φ_1 does not contribute to (13). Let us emphasize that the contribution of the extra component f_5 is proportional to the charge form factor G_C^V (or F_1^V in leading $1/m$ order). Neglecting f_5 in eq.(13), we recover the usual expression in the plane wave approximation given in refs.[1, 5].

3 The deuteron relativistic wave function

In order to make the link with the usual non-relativistic approach, we calculate below analytically the expression for the extra component f_5 in leading $1/m$ order. We start with the equation for the wave function [6]:

$$(\vec{k}^2 + \kappa^2)\vec{\psi}(\vec{k},\vec{n}) = -m \int \vec{\psi}(\vec{k}',\vec{n})\sigma_y V(\vec{k}',\vec{k},\vec{n})\sigma_y \frac{d^3k'}{(2\pi)^3} , \tag{14}$$

with $\kappa^2 = m|\epsilon_d|$. In eq.(14) the energy $\varepsilon' = \sqrt{m^2 + \vec{k}^2}$ has been already replaced by the mass m of the nucleon. Higher order corrections will be neglected below. Like in ref.[6], we substitute in the right hand side of this equation the relativistic kernel V (for π-exchange only) and the non-relativistic wave functions containing u_S and u_D only instead of the complete $\vec{\psi}(\vec{k}',\vec{n})$. The extra components of the wave function are generated by the relativistic kernel. We shall concentrate in the following on the term which has the structure $[\vec{k} \times \vec{n}]$, see eq.(1), and calculate its coefficient.

The expression for $\vec{\psi}\sigma_y V \sigma_y$ in the case of the pseudo-scalar coupling reads:

$$\vec{\psi}\sigma_y V \sigma_y = -\frac{\pi}{m^2}\frac{(-3g_\pi^2)}{\mu^2 + (\vec{k} - \vec{k}')^2}\vec{\sigma}\cdot(\vec{k}_2 - \vec{k}_2{}')\,\vec{\psi}\,\vec{\sigma}\cdot(\vec{k}_1 - \vec{k}_1{}') , \tag{15}$$

where μ is the pion mass. Our notations for the kernel are indicated in fig.3. The factor -3 in (15) incorporates the action of the isospin operator $\vec{\tau}_1\vec{\tau}_2$ (not included explicitly) on the deuteron state. Note that we use the definition of coupling constants with $g_\pi^2 \approx 14$, and not with $g_\pi^2/4\pi \approx 14$.

It is convenient to work in the system of reference where $\vec{k}_1 + \vec{k}_2 = 0$, and thus $\vec{k}_1 = \vec{k} = -\vec{k}_2$. The vectors $\vec{k}_1{}'$ and $\vec{k}_2{}'$ are equal to $\vec{k}'$ and $-\vec{k}'$ in the system where $\vec{k}_1{}' + \vec{k}_2{}' = 0$ but not in the system where $\vec{k}_1 + \vec{k}_2 = 0$. Let us thus express them through $\vec{k}, \vec{k}', \vec{n}$.

In the light-front dynamics all the four-momenta are on their corresponding mass shells but generally off energy shell. The latter means that the four-dimensional momenta satisfy the following conservation law:

$$k_1 + k_2 - \omega\tau = k_1' + k_2' - \omega\tau' \ . \tag{16}$$

The scalar parameters τ, τ' are responsible for the off-energy shell effects. This is the reason why the system at rest for the final particles ($\vec{k}_1 + \vec{k}_2 = 0$) does not coincide with that for intermediate particles ($\vec{k}_1{}' + \vec{k}_2{}' \neq 0$). From eq.(16) it follows that $\vec{k}_1{}' + \vec{k}_2{}' = \vec{\omega}(\tau' - \tau)$. The zeroth component of (16) gives $\omega_0(\tau' - \tau) = (\vec{k}'^2 - \vec{k}^2)/m$. In first approximation, the total momentum $\vec{\omega}(\tau' - \tau) = \vec{n}\omega_0(\tau' - \tau)$ is evidently shared equally between $\vec{k}_1{}'$ and $\vec{k}_2{}'$. Hence, we obtain in first $1/m$ order:

$$\vec{k}_1{}' = \vec{k}' + \vec{n}(\vec{k}'^2 - \vec{k}^2)/(2m), \quad \vec{k}_2{}' = -\vec{k}' + \vec{n}(\vec{k}'^2 - \vec{k}^2)/(2m) \ . \tag{17}$$

Substituting these expressions for the momenta $\vec{k}_{1,2}$, $\vec{k}_{1,2}{}'$ and for ψ in eq.(15), neglecting f_5 in the integrand and after integrating over d^3k' which generates the structure $[\vec{k} \times \vec{n}]_i$, one gets:

$$f_5[\vec{k} \times \vec{n}]_i/k = -\frac{3g_\pi^2\sqrt{2}}{8\sqrt{3}\pi^2m^2}\int \frac{(\vec{k}^2 - \vec{k}'^2)}{(\vec{k}^2 + \kappa^2)}\frac{d^3k'}{\mu^2 + (\vec{k} - \vec{k}')^2}$$
$$\times \left[\frac{1}{\sqrt{2}}u_S(k')\delta_{ij} - \frac{1}{2}(\frac{3k_i'k_j'}{\vec{k}'^2} - \delta_{ij})u_D(k')\right][(\vec{k} - \vec{k}') \times \vec{n}]_j \ . \tag{18}$$

Up to a coefficient, this component gives rise to an amplitude similar to the pair contribution, if one neglects in eq.(18) the factor $(\vec{k}^2 - \vec{k}'^2)/(\vec{k}^2 + \kappa^2)$. Since k' is restricted by the integration domain to low momenta, this factor at $k \gg k' \sim \kappa$ is close to 1. From eq.(18) it follows:

$$f_5(k) = -\frac{3g_\pi^2\sqrt{2}}{8\sqrt{3}\pi^2m^2k}\int \frac{d^3k'}{\mu^2 + (\vec{k} - \vec{k}')^2}\left\{\frac{1}{\sqrt{2}}(\vec{k} - \vec{k}')\cdot\vec{k}u_S(k')\right.$$
$$\left. - \frac{1}{4}[(\vec{k} - \vec{k}')\cdot\vec{k} - 3(\vec{k} - \vec{k}')\cdot\vec{k}' \ (\vec{k}'\vec{k})/\vec{k}'^2]u_D(k')\right\} . \tag{19}$$

Here the factor $(\vec{k}^2 - \vec{k}'^2)/(\vec{k}^2 + \kappa^2)$ is omitted. We shall come back to this point in the last section.

Transforming eq.(19) to coordinate space, we find for f_5 the following expression:

$$f_5(k) = -\frac{\sqrt{\pi}3g_\pi^2}{2\sqrt{3}m^2}\int_0^\infty \frac{\exp(-\mu r)}{r}(\mu r + 1)j_1(kr)\left[u(r) + \frac{1}{\sqrt{2}}w(r)\right]dr \ . \tag{20}$$

where $u(r)$ and $w(r)$ are the usual S- and D-state wave functions in r-space. Generally speaking, the function f_5, as well as the relativistic extention of S- and D- state wave functions depend on the scalar product $\vec{n}\vec{k}$. In first approximation this dependence is absent and all the $\vec{n}$-dependence of the deuteron wave function is reduced to the vector product $[\vec{k}\times\vec{n}]$ in eq.(1).

We should also take into account the extra component g_2 of the final state wave function indicated in eq.(2). However, there is no need to repeat the calculations. The contributions including f_5 and g_2 differ from each other by the opposite order in time of γ- and π-exchanges. In the leading order static limit, the result will be the same except for the isospin factor and sign. The isospin operator $\vec{\tau}_1\vec{\tau}_2$ acting on the ${}^1S_0$-state with T=1 gives 1 instead of -3. Another factor -1 appears from opposite order of γ-matrices. As a result, the g_2-contribution can be incorporated by the replacement $-3g_\pi^2 \rightarrow -(3+1)g_\pi^2 = -4g_\pi^2$ in the amplitude.

Making this replacement in (20), from eq.(13) we find:

$$A = \frac{m^{3/2}\sqrt{2}}{\pi}\left\{G_M^V\int_0^\infty\left[u(r)j_0(\frac{r\Delta}{2}) - \frac{1}{\sqrt{2}}w(r)j_2(\frac{r\Delta}{2})\right]rdr\right.$$
$$\left. + \frac{G_C^V g_\pi^2}{m\Delta}\int_0^\infty \frac{\exp(-\mu r)}{r}(\mu r+1)\left[u(r)+\frac{1}{\sqrt{2}}w(r)\right]j_1(\frac{r\Delta}{2})dr\right\}. \qquad (21)$$

The form of the second term coincides with the one given by the pair term contribution [1, 14] (for a plane wave in the final state). However, the coefficient of this term is smaller by a factor of 2. We show below that an equal contribution is provided by the contact term.

4 Contribution of the contact $NN\pi\gamma$ interaction

The diagram corresponding to the left contact term is shown in fig.4. We associate with the crossed line the factor [9]: $-\hat{\omega}/[4\pi(\omega\cdot l)]$, where l is the four-momentum transfered via the crossed line. For $\omega = (1,0,0,-1)$ this factor is proportional to $(\gamma_0+\gamma_z)$ given for the fermion contact term in ref.[8]. An analogous instantaneous interaction appears in QED in the infinite momentum frame [15]. The relative time order of the $NN\pi$-vertices in fig.3 is irrelevant in the static limit. Besides this diagram, we will take into account later the diagram with the right contact term, obtained from fig.3 by changing the relative time order of the π- and γ-exchanges.

The contribution of the diagram of fig.3 to the amplitude A has the form:

$$A^{cont} = \frac{-3g_\pi^2}{8\pi^{5/2}m^{3/2}}\frac{m^2}{Q^2}\frac{1}{(\omega\cdot p)}e_{\mu\rho\nu\gamma}\, q_\nu\omega_\gamma\int\frac{d^3k'}{\mu^2+(\vec{k}-\vec{k}')^2} \qquad (22)$$

$$\times Tr\left[-i\gamma_5(\hat{k}_f+m)\Gamma^V_\rho\left(-\frac{\hat{\omega}}{4\pi(\omega\cdot k_f)}\right)i\gamma_5(\hat{k}'_2+m)O_\mu(\hat{k}'_1-m)i\gamma_5(\hat{k}_1-m)\right].$$

Since we still keep a plane wave in the final state, the integration in eq.(22) corresponds to the left loop of the diagram in fig.3. The scalar products of the four-vectors appearing after the trace calculation are expressed approximately through $\vec{k}=\vec{q}/2, \vec{k}'$ and $\vec{n}$. We put, in particular, $x'=(\omega\cdot k'_1)/(\omega\cdot p)\approx 1/2-\vec{n}\cdot\vec{k}'/(2m)$. For free final particles with zero relative energy, $x=(\omega\cdot k_1)/(\omega\cdot p)=1-(\omega\cdot k_f)/(\omega\cdot p)=1/2$. By this way, we obtain in leading $1/m$ order:

$$A^{cont}=\frac{3g_\pi^2G_C^Vm^{1/2}}{32\pi^{7/2}\vec{k}^2}\int\frac{d^3k'}{\mu^2+(\vec{k}-\vec{k}')^2}\left\{\sqrt{2}(\vec{k}-\vec{k}')\vec{k}u_S(k')\right.$$
$$\left.-\left[(3(\vec{n}\vec{k}')^2/\vec{k}'^2-1)\vec{k}^2+(\vec{k}\vec{k}')\right]u_D(k')\right\}. \qquad (23)$$

The integrand (23) contains $(\vec{n}\vec{k}')^2$ and after integrating over d^3k' the amplitude A^{cont} could depend on $\vec{n}$ through $\vec{n}\vec{k}$ only. However, since $\omega\cdot q=0$, we get in leading order $\vec{n}\vec{k}=0$ (this is equivalent to $x=1/2$). So, in spite of the presence of $\vec{n}$ in eq.(23), the amplitude A^{cont} does not depend on $\vec{n}$. To eliminate the fictitious $\vec{n}$-dependence it is convenient to average (23) over $\vec{n}$- directions in the plane orthogonal to $\vec{k}$. This is equivalent to the replacement: $(\vec{n}\vec{k}')^2\to(\vec{k}'^2-(\vec{k}'\vec{k})^2/\vec{k}^2)/2$, after which the integrands of eqs.(23) and (19) coincide with each other. Note that the factor $(\vec{k}^2-\vec{k}'^2)/(\vec{k}^2+\kappa^2)$, which has been neglected in the transformation from eq.(18) to eq.(20), is absent in eq.(23) from the very beginning. We thus get:

$$A^{cont}=-\frac{\sqrt{3}m^{5/2}}{\pi^{3/2}2\sqrt{2}k}G_C^Vf_5(k) \qquad (24)$$

with f_5 given by eq.(19) and $k=\Delta/2$. Comparing (24) with (13) we find that the contribution from the left contact term coincides exactly with the contribution of the extra component f_5.

The right contact term is given by eq.(22) with the replacement:

$$\Gamma^V_\rho\left(-\frac{\hat{\omega}}{4\pi(\omega\cdot k_f)}\right)i\gamma_5\to i\gamma_5\left(-\frac{\hat{\omega}}{4\pi(\omega\cdot k'_2)}\right)\Gamma^V_\rho.$$

Without any isospin factors the right and left contact terms give opposite contributions. Therefore, like in the case of the extra component g_2, the right contact term can be incorporated by the replacement $3g_\pi^2\to 4g_\pi^2$. We thus find that the total contribution of the contact interaction equals the contribution from the extra deuteron and final state components and, hence, increases the coefficient $G_C^Vg_\pi^2/(m\Delta)$ in (21) by a factor of 2.

The incorporation of the final state interaction (neglected throughout above for simplicity) results in multiplying the integrands of eq.(21) by the final state wave function $\varphi_{p^*}(r)$, with the asymptotic normalization $\sin(p^*r+\delta)/(p^*r)$, where δ is the $^1S_0$ phase shift.

With the coefficient $2G_C^V g_\pi^2/(m\Delta)$ and with the incorporation of $\varphi_{p^*}(r)$, eq.(21) exactly coincides with the pair term contribution [1, 14].

We would like to conclude our derivation by two comments.

First, it is known that MEC of interest here arise from the charged pion exchange. Since the f_5 component in the deuteron state, as well as the g_2 component in the scattering state, are calculated from the full π exchange contribution, they involve the charged as well as the neutral π exchange. A detailed examination of their contributions shows that for f_5 these contributions are respectively proportional to -2 and -1 making -3 altogether while for g_2 they are proportional to -2 and $+1$. When adding these two contributions, the part corresponding to the neutral pion exchange cancels out, in complete agreement with the MEC approach. Similarly, it can be shown that the contribution of the η meson, which contributes to both f_5 and g_2, cancels out in the total result in leading $1/m$ order.

The second comment originates from the identity of the light-front dynamics approach with the non-relativistic approach completed by MEC. This identity suggests the existence of a unitary transformation to go from one to another[3]. The identification of the transformation is similar to what was done by one of the authors, in a different context however [16]. The idea is to remove part of the wave function (the f_5 and g_2 components in the present case), which contributes to the one-body current, and to transform the corresponding contribution in the form of a two-body current. The following developments, valid to lowest $1/m$ order, are made using notations commonly applied to the non-relativistic approach.

Starting from the expression of the π-exchange interaction and using (17), one can easily calculate the new term which appears in the NN interaction. This one gives rise to the f_5 (or g_2) component. It can be written as:

$$\delta V^\pi(\vec{k},\vec{k}\,') = \frac{(\vec{k}^{\,2}-\vec{k}\,'^2)}{m}\frac{4\pi g_\pi^2}{8m^2}(\vec{\tau}_1\vec{\tau}_2)\frac{[\vec{\sigma}_1\times\vec{\sigma}_2]\cdot[(\vec{k}-\vec{k}\,')\times\vec{n}]}{\mu^2+(\vec{k}-\vec{k}\,')^2}\ . \tag{25}$$

In r-space it is given by:

$$\delta V^\pi(\vec{r}) = [H_0, -iU(\vec{r})]\ , \tag{26}$$

[3]This possibility was mentioned by H.J.Weber to one of the authors (V.A.K.) while it was independently explored by another one (B.D.).

where H_0 is the kinetic energy part of the Hamiltonian and $U(\vec{r})$ is defined by:

$$U(\vec{r}) = \frac{g_\pi^2}{8m^2}(\vec{\tau}_1 \cdot \vec{\tau}_2)[\vec{\sigma}_1 \times \vec{\sigma}_2] \cdot \left[\vec{\partial}_{\vec{r}}(\frac{\exp(-\mu r)}{r}) \times \vec{n}\right] , \tag{27}$$

with $\vec{r} = \vec{r}_1 - \vec{r}_2$.

One can now performs on the total Hamiltonian, $H = H_0 + V + \delta V^\pi + H_{el}$, the transformation:

$$H \rightarrow \exp(-iU) H \exp(iU) = H_0 + V + H_{el} - [iU, H_{el}] + O(g^4) . \tag{28}$$

In writing the last formula we have taken into account the relation (26) which allows one to eliminate δV^π from the total Hamiltonian. The counterpart of this transformation is the appearence of terms of order g^4 in the NN interaction which can be considered as higher order terms, and, more importantly, terms of order g_π^2 in the electromagnetic interaction. The part of interest comes from the charge density and, more specifically, from its isovector part which does not commute with $U(\vec{r})$. The result is:

$$\delta H_{el} = -[iU, H_{el}] = \frac{g_\pi^2}{8m^2}[\vec{\sigma}_1 \times \vec{\sigma}_2] \cdot \left[\vec{\partial}_{\vec{r}}\left(\frac{\exp(-\mu r)}{r}\right) \times \vec{n}\epsilon_0\right] [\vec{\tau}_1 \times \vec{\tau}_2]_z G_C^V(Q^2) \times (\exp(i\vec{q}\vec{r}_1) - \exp(i\vec{q}\vec{r}_2)) , \tag{29}$$

where ϵ is the photon polarization vector. In the same notations and approximations the contact term is given by:

$$H_{el}^{cont} = \frac{g_\pi^2}{8m^2}[\vec{\sigma}_1 \times \vec{\sigma}_2] \cdot \left[\vec{\partial}_{\vec{r}}\left(\frac{\exp(-\mu r)}{r}\right) \times (\vec{\epsilon} - \vec{n}\varepsilon_0)\right] [\vec{\tau}_1 \times \vec{\tau}_2]_z G_C^V(Q^2)(\exp(i\vec{q}\vec{r}_1) - \exp(i\vec{q}\vec{r}_2)). \tag{30}$$

It is easily seen that the term containing the factor $\vec{n}\epsilon_0$ in eq.(30) is cancelled by the term (29) arising from the unitary transformation, leaving the term proportional to $\vec{\epsilon}$. This last term is nothing but the usual pion pair term current. The above example also illustrates in a particular case how the physical amplitude turns out to be independent of $\vec{n}$. This means also that the functions B_{1-5} in eq.(9), after incorporating the impulse approximation and the contact terms, have to be of higher order than $1/m$.

On the other hand, while the disappearence of the charge density term proportional to ϵ_0 is consistent with the non-relativistic expectation that the deuteron electrodisintegration is a transverse (magnetic) process, the appearence of the charge density in δH_{el} and H_{el}^{cont} in intermediate steps of the calculation should not be a surprise. In a covariant approach, like the one we started from, and provided the amplitude in a given approximation does not depend on ω, the calculations of the only relevant form factor can be performed equivalently from the spatial or time component of the current.

5 Conclusion

We have shown that the light-front dynamics, despite the absence of explicit contributions from the excitation of pair terms by the virtual photon, reproduces the well-known non-relativistic phenomenology, including the main features of MEC. In the standard approach, this contribution is dominated at moderate Q^2 by π-exchange and is associated with $N\overline{N}$-intermediate state. In the light-front dynamics, half of this contribution is given by a new component in the relativistic NN-wave function containing no other intermediate states except for NN. The second half is not included in the wave function, and comes from a contact $NN\pi\gamma$ interaction which arises in the light-front dynamics.

The absence of a minimum in the experimental deuteron electrodisintegration cross-section at small momentum tranfer, half of which is filled by the contribution from the f_5 component in the light-front dynamics, is already a strong indication of the presence of this extra component in the relativistic deuteron wave function.

We have not considered in this paper the contribution to MEC coming from the direct coupling of the photon to the pion in flight (the so-called mesonic current). This contribution is of course also present in the light-front formulation. It is however identical, in leading $1/m$ order, to the non-relativistic contribution. This is why it has not been discussed in this paper. Moreover, other mesons are expected to contribute at large momentum transfer. They can easily be included in the calculation of the relativistic two-body wave functions, as it is done in refs.[6, 7].

It is also well known that, in leading $1/m$ order, the pair contribution associated with pseudo-vector πNN coupling is zero. The deuteron electrodisintegration amplitude is however equivalent to the one given in the pseudo-scalar representation since in that case the photon can couple by minimal substitution on the πNN vertex, generating a genuine $NN\pi\gamma$ current. In the light-front dynamics, the equivalence between the two representations is realized in the following way. In the pseudo-vector representation with π-exchange only, the f_5 component in the deuteron wave function is strictly zero in leading order. This is due to the off-shell condition at the πNN vertex. The contribution from the light-front contact term is also of higher $1/m$ order for this representation. On the other hand, the current originating from direct coupling of the photon to the pseudo-vector πNN vertex has its analogue in the light-front dynamics, providing the equivalence between relativistic and non-relativistic formulations in leading $1/m$ order.

Our result is a very good illustration of the "duality" in relativistic nuclear physics where one and the same contribution in different approaches (i.e. in different representations) is obtained from different physical starting points. We have found an approximate unitary transformation which connects the wave functions relative to these representations as well as the electromagnetic current operators. In leading $1/m$ order and in lowest order in g^2 this transformation eliminates extra components of the wave functions and turns their contribution to the one-body current into a two-body one.

We would like to come back finally to the approximation we made by replacing the factor $(\vec{k}^2 - \vec{k}'^2)/(\vec{k}^2 + \kappa^2)$ in (18) by 1. This approximation is valid as soon as the momentum transfered by the photon is larger than typical momenta involved in the non-relativistic deuteron (or final state) wave functions. If this is not the case, our procedure to calculate the f_5 component is not adequate, since higher order meson exchange contributions to the irreducible NN interaction kernel should be considered. Such corrections should be included in an exact calculation of the wave functions and electromagnetic observables in the light-front dynamics, but is of no importance in our formal comparison.

For the sake of comparison with non-relativistic approaches, we took into account in our analytical derivation the leading $1/m$ order only. However, the formulation of the few-body system wave functions and electromagnetic observables can be done exactly in the light-front dynamics. This formalism provides a completely coherent relativistic framework in which the forthcoming data at high momentum transfer can be safely analyzed.

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Figure captions

Figure 1 : Contribution of the pair term to the deuteron electrodisintegration amplitude in the non-relativistic framework.

Figure 2 : Impulse approximation contribution in the light-front graph technique.

Figure 3 : π-exchange kernel in the calculation of the relativistic deuteron wave function.

Figure 4 : Contribution containing the contact $NN\pi\gamma$ interaction (crossed line).

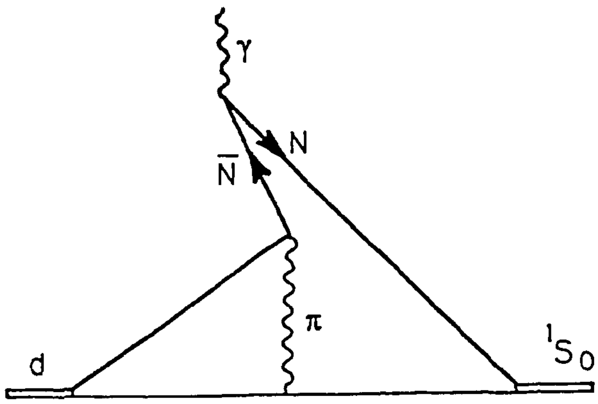

Fig. 1

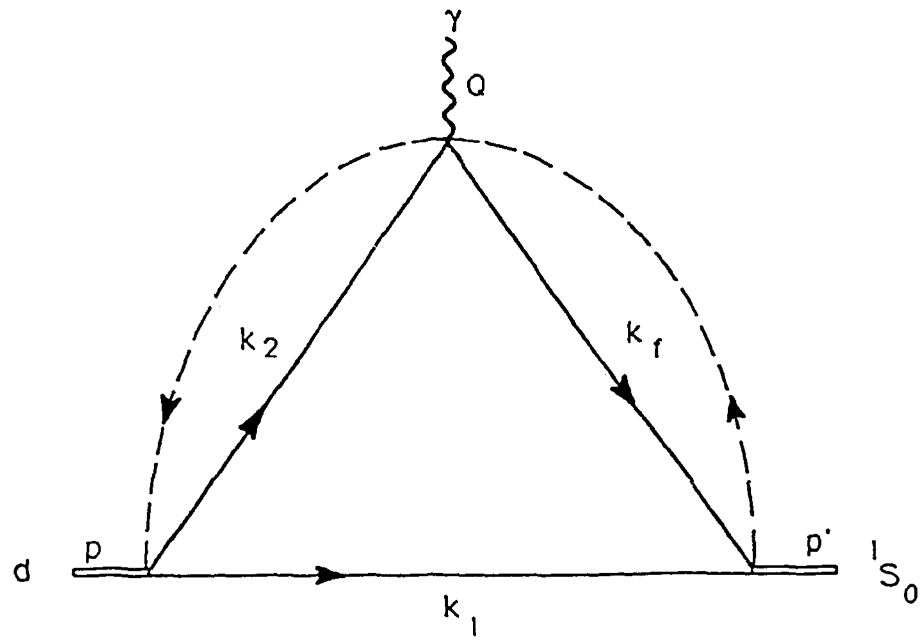

Fig. 2

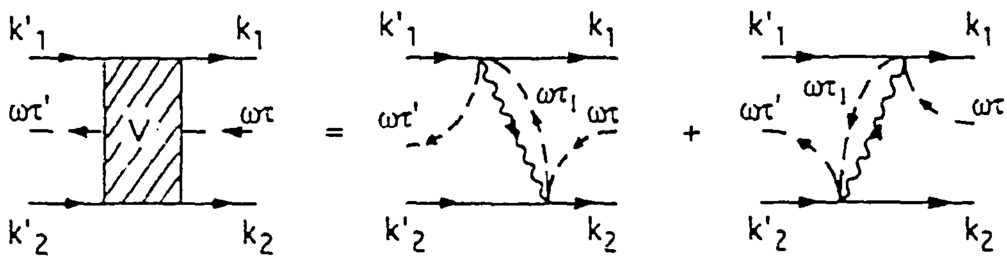

Fig. 3

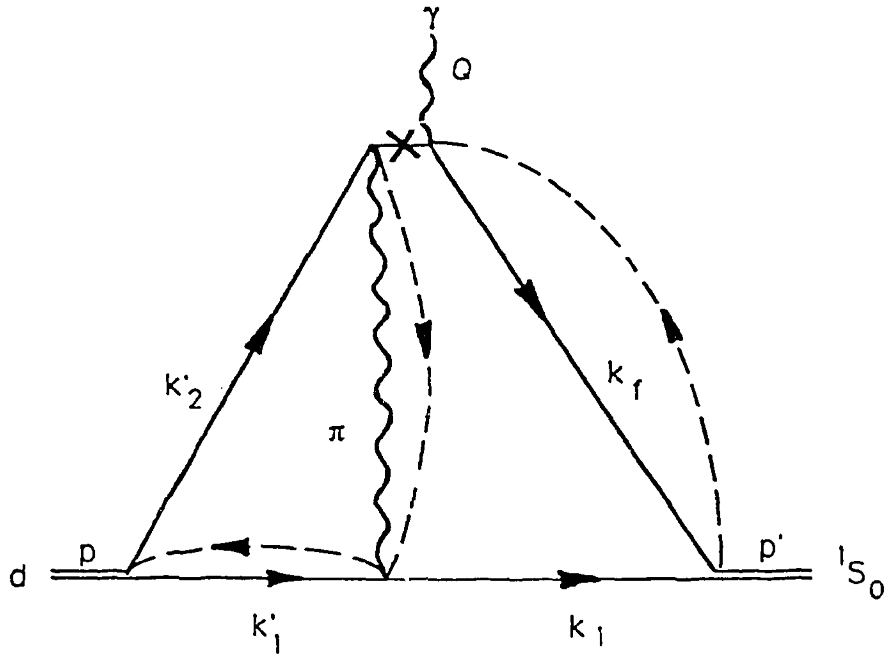

Fig. 4