IC/95/193

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

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ABSTRACT

Hawking radiation is observed in a general spacetime which includes all of the black hole spacetimes as well as various types of other spacetimes which are not interesting from the physical point of view like black hole spacetimes. Even Hawking radiation is observed in NUT spacetime which is sometimes considered as unphysical. So naturally arises the question whether Hawking radiation is physically reasonable.

MIRAMARE – TRIESTE
July 1995

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1 Introduction

In 1969 Penrose proposed a classical gedanken experiment for showing that energy could be extracted from a rotating black hole. The wave analogue of this process is known as "Superradiance": incident waves in certain modes would be amplified (rather than absorbed) by a rotating black hole, and would carry away some of the black hole's rotational energy.

In 1971 Zel'dovich considered this amplification, on quantum level, as stimulated emission and therefore there ought to be a corresponding rate of spontaneous emission in these modes, produced by vacuum fluctuations of the field under consideration. In 1973 this was further confirmed by Unruh. About the same time, Hawking's discovery that there should be an additional spontaneous emission in all modes, even if the black hole was not rotating, attracted widespread attention.

Hawking's (1974) investigations of quantum effects interpretated as the emission of a thermal spectrum of particles near the black hole event horizon has been extended by Gibbons and Hawking (1977) to the spacetimes of cosmological horizons. Hawking's investigation (1974) has been carried on by Liu Liao and Xu Dianyan (1980) into Kerr black hole spacetime and by Zhao Zheng *et at.* (1981) into Kerr–Newman black holes spacetime. Ahmed (1987) extended Hawking's investigation (1974) to the combined NUT–Kerr–Newman spacetimes which includes not only the physically interesting black hole solutions but also the NUT solution which is sometimes considered as unphysical. According to Misner (1963), the NUT spacetime is one which does not admit an interpretation without a periodic time coordinate, a spacetime without reasonable spacelike surfaces, and an asymptotically zero curvature spacetime which apparently does not admit asymptotically rectangular coordinates. McGuire and Ruffini (1975) suggested that the solutions endowed with NUT parameter should never be directly physically interpretated. So the Hawking radiation from the unphysical NUT–Kerr–Newman spacetime (Ahmed 1987) is interesting. Ahmed and Mondal (1992) further investigated Hawking radiation in Kasner type spacetime. Kasner spacetime arises due to the contraction of Schwarzschild spacetime. So the particle creation in Kasner spacetime goes beyond the idea that in the contraction phase, it is necessary that matter should disappear. So the particle creation in all of these diverse situations points out the fact that Hawking radiation is not only true for the black hole spacetimes but also in other spacetimes which may be sometimes considered as unphysical.

In this paper, we would like to extend Hawking's result in a spacetime which is rotating, charged and uniformly accelerated. In Section 2, we give some preliminaries regarding a rotating, charged and uniformly accelerated spacetime. Then in Section 3, we obtain Hawking radiation of Dirac particles in the said spacetime. And in the final Section 4, we add some discussion.

2 Preliminaries

In terms of the Boyer coordinates (p, q, σ, τ) where $q \sim r$ and $p \sim \cos\theta$, the solution of Einstein–Maxwell equations given by the line element

$$ds^2 = (1-pq)^{-2}\left(\frac{p^2+q^2}{X}\,dp^2 + \frac{p^2+q^2}{Y}\,dq^2 + \frac{X}{p^2+q^2}\,(d\tau + q^2\,d\sigma)^2 - \frac{Y}{p^2+q^2}\,(d\tau - p^2\,d\sigma)^2\right) \tag{1}$$

where

$$\begin{aligned} X &= X(p) = (-g^2+\gamma-\Lambda/6) + 2np - \in p^2 + 2mp^3 - (e^2+\gamma+\Lambda/6)p^4 \\ Y &= Y(q) = (e^2+\gamma-\Lambda/6) - 2mq + \in q^2 - 2nq^3 + (g^2-\gamma-\Lambda/6)q^4 \end{aligned}$$

was given by Plebanski and Demianski (PD) (1976). In certain ranges of the coordinates, the vector ∂_τ is timelike and may be interpreted as an azimuthal coordinate: the solutions are stationary and axisymmetric. In addition to the cosmological constant Λ, the PD solution (1) includes six real parameters: m and n are the mass and the NUT parameters, γ and $\in$, are related to the angular momentum per unit mass a and the acceleration parameters b, while e and g are the electric and magnetic monopoles. The only non vanishing tetrad component of the Weyl tensor is

$$\psi_2 = -(m+in)\left(\frac{1-pq}{q+ip}\right)^3 + (e^2+g^2)\left(\frac{1-pq}{q+ip}\right)\left(\frac{1+pq}{q-ip}\right) \tag{2}$$

and consequently the PD solution is of Petrov type D. The PD metric describes a rotating, charged and uniformly accelerated frame.

Some well-known physically interesting solutions, namely the black hole solutions which are asymptotically flat as well as asymptotically de Sitter could be obtained from

the PD solution by an appropriate limiting procedure. Moreover NUT and Kasner solutions which are considered as anisotropic model of the early universe could also be obtained as special cases of the PD solutions. The C-metric can also be deduced from the PD metric. In a special case, the C-metric describes the gravitational field generated by two uniformly accelerating mass points (Kinnersley and Walker 1970). Besides, some well known solutions associated with the names of Kinnersley (1969), Plebanski (1975), Demianski and Newman (1966), Carter (1973), Brill (1964), Newman *et al.* (1965), Bertotti (1959), Robinson (1959), Reissner (1916) and Nordstrom (1918) can be obtained from the PD solutions.

There are apparent singularities in the PD metric at the value of $q \sim r$ for which $Y(r) = 0$ and can be measured by using appropriate coordinate patches. The surfaces $Y(r) = 0$ along which the PD metric includes the apparent singularity is interpreted as horizons.

The equation $Y(r) = 0$ has four distinct roots: r_{--}, r_-, r_+ and r_{++}. The surfaces $r = r_+$ and $r = r_{++}$ are event horizon and cosmological horizon for observers moving on world lines of constant r between r_+ and r_{++} respectively. The surface $r = r_-$ is an inner Cauchy horizon. Passing through this, one comes to the ring singularity at $r = 0$, on the other side of which there is Cauchy cosmological horizon at $r = r_{--}$.

3 Particle Creation in PD Spacetime

With these preliminaries completed, we now turn to our main problem. Kamran and MacLenaghan (1984) obtained the separation of Dirac equation in a general background of Petrov type D. From Kamran–MacLenaghan's pertinent equation, we obtain the radial decoupled Dirac equation for the electron in the PD spacetime as follows:

$$\begin{aligned} &\left[Y\,\frac{d^2}{dr^2} + \left(Y^{1/2}\,\frac{d}{dr}\,(Y^{1/2}) - \frac{im_e\,Y}{\lambda + im_e r}\right)\frac{d}{dr}\right. \\ &\left.+\left(K^2Y^{-1} - \lambda^2 - m_e^2 r^2 - \frac{m_e\,K}{\lambda + im_e r}\right. + iY^{1/2}\,\frac{d}{dr}\,(K\,Y^{1/2})\right] R(r) = 0 \end{aligned} \tag{3}$$

with

$$K = (r^2 + n^2 + \gamma^2)\alpha - (\gamma m_0 - ee_0 r)$$

where α is a positive constant, m_0 is another constant, e_0 is the electric charge of Dirac particles, and λ and m_e are related to separation constant and mass of the electron respectively.

We now introduce the coordinate transformation

$$\frac{d}{dr_*} = \frac{Y}{r^2+n^2+\gamma^2}\frac{d}{dr} \tag{4}$$

and obtain from (3)

$$\begin{aligned} L^2\frac{d^2R}{dr_*^2} &+ \left(2rY - \frac{1}{2}Y'L - \frac{Lm_e Y}{m_e r - i\lambda}\right)\frac{dR}{dr_*} \\ &+Y\left[\frac{K^2}{Y} - \lambda^2 - m_e^2 r^2 - \frac{m_e K}{\lambda + i m_e R}\right. \\ &\left.-i\left(-ee_0 + 2r\alpha - \frac{1}{2}K\frac{Y'}{Y}\right)\right]R = 0 \end{aligned} \tag{5}$$

where a prime denotes differentiation with respect to the argument.

Near the horizon $Y = 0$, Eq.(5) reduces to the form

$$\frac{d^2R}{dr_*^2} + \frac{K^2}{L^2}R = 0 \tag{6}$$

The solution of the wave equation (6) is given as

$$R \sim \exp\left[\pm i(\alpha - \alpha_0)r_*\right] \tag{7}$$

with

$$\alpha_0 = \frac{(2r^2+n^2)\alpha}{r_+^2+n^2+\gamma^2} + \frac{ee_0 r_+}{r_+^2+n^2+\gamma^2}$$

where r_+, the smaller of the two positive values of r at which $Y = 0$ is the event horizon of the spacetime. The larger positive value of $Y = 0$ denoted by r_{++} represents the cosmological horizon.

Now we write the radial wave function as

$$\psi_r = \exp\left[-i\alpha(t + \hat{r}_*)\right] \tag{8}$$

where

$$\hat{r}_* = \frac{\alpha - \alpha_0}{\alpha r_*}$$

We resolve ψ_r into ingoing and outgoing waves as

$$\psi_r^{in} \sim \exp[-i\alpha(t + \hat{r}_*)] \tag{9}$$

$$\psi_r^{out} \sim \exp[-i\alpha(t - \hat{r}_*)] \tag{10}$$

Introducing the Eddington Finkelstein coordinates

$$V = t + \hat{r}_* \tag{11}$$

we have

$$\psi_r^{in} \sim \exp(-\alpha V) \tag{12}$$

$$\psi_r^{out} \sim \exp[-i\alpha V + 2i(\alpha - \alpha_0)r_*] \tag{13}$$

Near $r = r_+$, from (4), we obtain

$$dr_* = -\frac{3}{\Lambda}\frac{(r_+^2+n^2+\gamma^2)dr}{(r_+ - r_{++})(r_+ - r_-)(r_+ - r_{--})(r - r_+)} \tag{14}$$

where r_+, r_-, r_{++}, r_{--} are the four roots of $Y = 0$.

On integration we have

$$\begin{aligned} In\,(r - r_+) &= -\frac{1}{3}\Lambda\frac{(r_- r_{++})(r_+ - r_-)(r_+ - r_{--})r_*}{r_+^2+n^2+r^2} \\ &= 2\kappa_+ r_* \end{aligned}$$

i.e.

$$r - r_+ = \exp(2\kappa_+ r_*) \tag{15}$$

where

$$\kappa_+ = -\frac{1}{6}\frac{\Lambda}{r_+^2+n^2-\gamma^2}(r_+ - r_{++})(r_+ - r_-)(r_+ - r_{--}) \tag{16}$$

is the surface gravity of the event horizon of the PD spacetime. Just outside the event horizon

$$\psi_r^{out} \sim \exp\left(-i\alpha V(r - r_+)\frac{i(\alpha - \alpha_0)}{\kappa_+}\right) \tag{17}$$

We now extend the outgoing wave outside the horizon to the region inside. Since on the horizon the outgoing wave function is not analytic, we can make it analytic by going round the horizon in the complex plane.

Hence inside the horizon

$$\psi_r^{out} \sim \exp\left(-i\alpha V(r_+ - r)\frac{i(\alpha - \alpha_0)}{\kappa_+}\right)\exp\left(\frac{\pi}{\kappa_+}(\alpha - \alpha_0)\right) \tag{18}$$

We now introduce the step function

$$y(x)\begin{cases} = 1 & \text{for } x > 0 \\ = 0 & \text{for } x < 0 \end{cases} \tag{19}$$

and write the outgoing wave function as

$$\phi_r^{out} = N_r\Big[y(r-r_+)\ \psi_r^{out}\ (r-r_+) + y(r_+-r)\ \psi_r^{out}(r_+-r)\exp\left(\frac{\pi}{\kappa_+}\ (\alpha-\alpha_0)\right)\Big] \tag{20}$$

where ψ_r^{out} is the normalized Dirac function. Eq.(2) represents the outward flow from the horizon of positive energy particles of intensity N_r^2. Inside the horizon, due to the interchange of time and space, r represents the time axis, and Eq.(20) represents a flow of positive energy particles propagating in a gravitational field in the reversed time i.e. a flow in time of negative energy antiparticles towards the singularity region. This shows that Dirac particle–antiparticle pairs are created on the horizon as being interpretated by Derulle and Ruffini (1975). Obviously, from the normalization condition, we have

$$(\phi_r^{out},\phi_r^{out}) = N_r^2\left[\exp\left(\frac{2\pi(\alpha-\alpha_0)}{\kappa_+}\right)+1\right] = 1 \tag{21}$$

or

$$N_r^2 = \left[\exp\left(\frac{2\pi(\alpha-\alpha_0)}{\kappa_+}\right)+1\right]^{-1} = \left[\exp\left(\frac{\omega-\omega_0}{k_B T_+}\right)+1\right]^{-1} \tag{22}$$

where

$$T_+ = \frac{\kappa_+}{2\pi\ k_B} \tag{23}$$

T_+ being the temperature of the region inside the event horizon. k_B is Boltzmann's constant. Eq.(22) is the formula for Hawking radiation of Dirac particles in the PD spacetime.

In a similar way, one could obtain

$$T_{++} = \frac{K_{++}}{2\pi k_B} \tag{24}$$

where

$$\kappa_{++} = \Lambda\frac{(r_{++}-r_+)(r_{++}-r_-)(r_{++}-r_{--})}{6(r^2+n^2+\gamma^2)} \tag{25}$$

is the surface gravity of the cosmological horizon.

4 Discussion

In this work we observe Hawking radiation in a spacetime which includes various types of spacetimes as special cases such as Kinnersley (1969), Plebanski (1975), Demianski and Newman (1966), Carter (1973), Newman *et al.* (1965). The spacetime given by Newman *et al.* (1965) is known as the Kerr–Newman spacetime which includes all the black hole spacetimes. The black hole spacetimes were generalized by Carter (1973) to include a cosmological constant. The black hole spacetimes generalized with the cosmological constant are considered as the black holes in the early universe as the cosmological constant is found to be present in the inflationary scenario of the early universe. In the inflationary scenario the universe undergoes a stage where it is geometrically similar to de Sitter spacetime. The spacetime given by Plebanski (1975) includes NUT and Kasner spacetimes as special cases. The Demianski and Newman spacetimes (1966) is actually the combined NUT–Kerr–Newman spacetimes. The C–metric is a special case of PD metric. The C–metric describes the gravitational field generated by two uniformly accelerated mass points (Kinnersley and Walker 1970). So the Hawking radiation which is true for the PD spacetimes will be true for all these special cases of PD spacetimes. It is interesting to note that Hawking radiation is true for the NUT spacetime which is sometimes considered as unphysical. So the Hawking radiation in the unphysical spacetime like NUT arises the question whether Hawking radiation is physically reasonable.

Acknowledgments

The author would like to thank the International Centre for Theoretical Physics, Trieste, for hospitality.

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