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LINEARIZED DYNAMICAL APPROACH TO CURRENT ALGEBRA

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ABSTRACT

We study the original motivations searching for a nonlinear chiral lagrangian to replace the linear sigma model while manifesting all the successful properties of current algebra and partial conservation of axial currents (PCAC).

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In principle, a *linearized* (and renormalized) chiral lagrangian should be a prime concern for field theorists seeking to maximize the predictive power of chiral symmetry as characterized by the current algebra method of the 1960s. The original motivation for focusing instead on a more complicated *nonlinear* and nonrenormalizable chiral lagrangian (ultimately used to generate modern chiral perturbation theory [1]) was outlined in the Weinberg letter of 1967 [2]. In this letter, Weinberg sought to replace the canonical (linear) field theory at that time-the linear σ model [3,LσM]: "The time-honored example of a lagrangian satisfying partially conserved axial currents (PCAC) and the chiral commutation relations (the current algebra method) is that of the (linear) σ model." But hereafter in [2] he pointed out the apparent deficiencies of the LσM as understood in 1967:

a) The LσM is unrenormalized.

b) 'At the nucleon level [3,4], the LσM involves a (renormalizable) *pseudoscalar* coupling $\bar{N}\gamma_5\vec{\tau}\cdot\vec{\pi}N$ along with (rapidly varying) pole graphs not directly related to the current algebra, rather than the more streamlined (nonlinear) current algebra employing instead a (nonrenormalizable) *pseudovector* coupling $\bar{N}\ \not{\partial}\gamma_5\vec{\tau}\cdot\vec{\pi}N$ involving powers of derivatives [2].'

c) 'The LσM is not useful as a phenomenology of strong interactions because it predicts in "narrow width" approximation a large width $\Gamma(\sigma\to\pi\pi) > m_\sigma$ for scalar masses $m_\sigma \geq 500$ MeV, and such a broad s-wave resonance seems unlikely (in 1967) [2].'

d) 'For processes involving more than one soft-pion emission, the LσM (apparently) generates graphs with pions emitted from internal lines [2]. In effect the current algebra method suffers because it is apparently not symmetric in multiple soft- pion emission [2].'

In this note we attempt to answer Weinberg's above four objections to the LσM (only with hindsight 28 years later).

a') Because the original nucleon-level LσM [3,4] was only taken at tree-level, renormalization issues did not arise (instead the Goldberger-Treiman relation [GTR] $f_\pi g_{\pi NN} = m_N$ (with $g_A = 1$) directly followed). The more modern quark-level LσM, when dimensionally regularized [5], is instead chirally renormalized at one- loop level based on the original $Z = 0$ compositeness conditions [6] not distinguishing a scalar σ meson as elementary (as in the LσM [3-5]) or as a $\bar{q}q$ bound state (as in the NJL four-fermion theory [7]). In fact, the one-loop order LσM at the quark level also manifests the Goldstone theorem [5,8]

$$m_\pi^2 = 0_{qk\ loops} + 0_{\pi\ loops} + 0_{\sigma\ loops} = 0, \tag{1}$$

without further renormalization due to (quadratically divergent but negative) chiral tadpole diagrams. Since the crucial techniques of dimensional regularization [9] and the Lee null tadpole condition [10] were not known until the early 1970s, the Weinberg letter of 1967 [2] could not have accounted for these LσM advances.

b') Pseudoscalar (LσM) coupling $\bar{N}\gamma_5\vec{\tau}\cdot\vec{\pi}N$ in tree-level pole diagrams naturally suggests using dispersion relations in the presence of rapidly varying poles when computing e.g. πN scattering lengths [11]. The analogue meson-quark chiral coupling $g\bar{\psi}(\sigma + i\gamma_5\vec{\tau}\cdot\vec{\pi})\psi$ as driven by the GTR $f_\pi g = m_q$ is the primary term in the quark-level LσM lagrangian requiring $\partial\vec{A} = 0 = m_\pi$ in the chiral limit.

c') The dynamically generated quark-level LσM lagrangian [5] is quite useful as a phenomenological tool. After shifting to the true vacuum with expection values $\langle\sigma\rangle = \langle\vec{\pi}\rangle = 0$, the interacting part of the SU(2) LσM lagrangian density is

$$\mathcal{L}_{int} = g\bar{\psi}(\sigma + i\gamma_5\vec{\tau}\cdot\vec{\pi})\psi + g'\sigma(\sigma^2 + \vec{\pi}^2) - \lambda(\sigma^2 + \vec{\pi}^2)^2/4 - f_\pi g\bar{\psi}\psi \qquad (2a)$$

with chiral couplings [3,4]

$$g = m_q/f_\pi, \qquad g' = m_\sigma^2/2f_\pi = \lambda f_\pi, \qquad (2b)$$

for $f_\pi \approx 90$ MeV and (constituent) quark mass m_q in the chiral limit (CL). Dimensional regularization, however, requires two additional dynamical constraints in the CL [5]

$$m_\sigma = 2m_q, \qquad g = 2\pi/\sqrt{3} \approx 3.6276, \qquad (3)$$

for color number $N_c = 3$. Then the quark and scalar masses have the expected values [5]

$$m_q = f_\pi\, 2\pi/\sqrt{3} \approx 325 \text{ MeV}, \qquad \mathrm{m}_\sigma = 2\mathrm{m}_\mathrm{q} \approx 650\mathrm{MeV}, \qquad (4)$$

and the πNN coupling is then predicted to be

$$g_{\pi NN} = 3gg_A \approx 13.6, \qquad (5)$$

for $g_A = 1.25$, near $g_{\pi NN} \approx 13.4$ found from πN scattering and dispersion relations [12]. Given $\sigma(650)$ in (4), the LσM $\sigma \to \pi\pi$ width for $p \approx 266$ MeV is

$$\Gamma(\sigma \to \pi\pi) = \frac{3p}{16\pi m_\sigma^2}|m_\sigma^2/f_\pi|^2 \sim 700 \text{ MeV}, \qquad (6a)$$

which Weinberg [13] now finds instead using mended chiral symmetry (MCS)

$$\Gamma_\sigma = \left(\tfrac{9}{2}\right)\Gamma_\rho \sim 700 \text{ MeV}. \qquad (6b)$$

In fact recent experiments [14] may also suggest such a broad s-wave scalar, so the LσM-MCS predictions in eqs.(6) may be reflecting nature.

There are further phenomenological consequences of the quark-level LσM due to the $Z = 0$ compositeness condition which "bootstraps" quark loops to trees, such as [5]

$$g_{\sigma\pi\pi}^{loop} = g' = m_\sigma^2/2f_\pi\ , \qquad g_{\rho\pi\pi}^{loop} = g_\rho\ , \qquad (7)$$

the latter relation being Sakurai's, vector meson dominance (VMD) universality condition [15]. Indeed the latter VMD scale can in fact be dynamically generated in the LσM to be [5,16]

$$g_\rho = \sqrt{3}g = 2\pi, \qquad (8)$$

close to the $\rho \to \pi\pi$ coupling determined from the ρ rate $g_\rho^2/4\pi \approx 3.0$ or $|g_{\rho\pi\pi}| \approx 6.1$. Lastly the chiral KSRF relation [17]

$$m_\rho = \sqrt{2}g_\rho f_\pi \sim 800\text{MeV} \qquad (9)$$

for the CL value $f_\pi \approx 90$ MeV is now also dynamically generated via (8).

d') Weinberg's LσM deficiencies (d) above have been circumvented for low energy $\pi\pi$ scattering in ref.[4] at tree order by adding to the $\lambda\phi^4/4$ LσM contact term the 3 (narrow width) σ poles in the s,t,u channels to give the low energy $\pi\pi$ amplitude

$$T_{\pi\pi} = -2\lambda(\delta_{ab}\delta_{cd} + \delta_{ac}\delta_{bd} + \delta_{ad}\delta_{bc}) \tag{10}$$
$$+4\lambda^2 f_\pi^2 \left(\frac{\delta_{ab}\delta_{cd}}{m_\sigma^2 - s} + \frac{\delta_{ac}\delta_{bd}}{m_\sigma^2 - t} + \frac{\delta_{ad}\delta_{bc}}{m_\sigma^2 - u}\right).$$

Expanding around $s = t = u = 0 = m_\pi^2$ in the CL and using the LσM relation $m_\sigma^2/2f_\pi = \lambda f_\pi$ in (2b), ref.[4] notes the "miraculous" cancellation between the two terms in (10), leading back again to Weinberg's current algebra $\pi\pi$ amplitude [18] for $m_\pi = 0$: $T_{\pi\pi} = (s - m_\pi^2) f_\pi^{-2}\delta_{ab}\delta_{cd} + \ldots$

In the context of the quark-level LσM theory, $\pi\pi$ quark box graphs "shrink" back to tree diagrams due to the $Z = 0$ condition [5]. Thus one need not go beyond the original nucleon (tree)-level LσM [3,4] as recently emphasized by Ko and Rudaz [19]. This leads to a slight LσM correction to Weinberg's current algebra $\pi\pi$ amplitude [18] above. Specifically ref.[19] follows Weinberg and expresses the $\pi\pi$ amplitude as

$$T_{ab,cd} = A(s,t,u)\delta_{ab}\delta_{cd} + A(t,u,s)\delta_{ac}\delta_{bd} + A(u,t,s)\delta_{ad}\delta_{bc}, \tag{11}$$

but uses the (tree-level) LσM result

$$A(s,t,u) = -2\lambda\left[1 - \frac{2\lambda f_\pi^2}{m_\sigma^2 - s}\right]. \tag{12a}$$

Then away from the CL $m_\pi \neq 0$, the quartic coupling $\lambda = (m_\sigma^2 - m_\pi^2)/2f_\pi^2$ converts (12a) to a slightly modified Weinberg structure

$$A(s,t,u) = \left(\frac{m_\sigma^2 - m_\pi^2}{m_\sigma^2 - s}\right)\left(\frac{s - m_\pi^2}{f_\pi^2}\right). \tag{12b}$$

This in turn increases Weinberg's $I = 0$ s-wave scattering length prediction [17] $a_{\pi\pi}^{(0)} = \frac{7m_\pi}{32\pi f_\pi^2} \approx 0.16 m_\pi^{-1}$ to

$$a_{\pi\pi}^{(0)}|_{L\sigma M} \approx \left(\frac{7+\varepsilon}{1-4\varepsilon}\right)\frac{m_\pi}{32\pi f_\pi^2} \approx (1.23)\frac{7m_\pi}{32\pi f_\pi^2} \approx 0.20 m_\pi^{-1}, \tag{13}$$

where $\varepsilon = m_\pi^2/m_\sigma^2 \approx 0.046$ from (4).

Such a 23% enhancement of Weinberg's $a_{\pi\pi}^{(0)}$ in (13) is also found from a (tedious) higher-loop chiral perturbation theory (ChPT) calculation [20]. While it is satisfying that the *simple* tree-level result (13) numerically agrees with the far more complicated ChPT analysis, the point of Weinberg's 1967 program [2] was to search for the easiest (field-theoretic) scheme to manifest and now improve the original chiral current algebra. Certainly a linear tree-level theory (the dynamically generated quark- level LσM) which is chirally renormalized is easier to handle than a nonlinear ChPT approach which has to be constantly renormalized in each loop order with an ever-increasing number of arbitrary parameters.

To return to other successful phenomenological calculations, we concentrate on the pion charge radius, respectively measured to be [21,22]

$$r_\pi = (0.66 \pm 0.02)\text{fm}, \quad \text{or} \quad r_\pi = (0.63 \pm 0.01)\text{fm}. \tag{14}$$

Recall the amazingly accurate VMD prediction [15]

$$r_\pi^{VMD} = \sqrt{6}/m_\rho \approx 0.63\text{fm}, \tag{15}$$

and also the quark-level LσM prediction [23]

$$r_\pi^{L\sigma M} = \sqrt{3}/2\pi f_\pi \approx 0.60\text{fm} \tag{16}$$

for the CL value $f_\pi \approx 90$ MeV. Since the $Z = 0$ condition for the LσM recovers the VMD universality relation (7) $g_{\rho\pi\pi} = g_\rho$, it should not be surprising that r_π^{VMD} in (15) also reduces to $r_\pi^{L\sigma M}$ in (16). This in fact follows from the LσM dynamically generated relation (8), $g_\rho = 2\pi$, combined with the KSRF relation (9):

$$r_\pi^{VMD} = \sqrt{6}/m_\rho = \sqrt{3}/g_\rho f_\pi = \sqrt{3}/2\pi f_\pi = r_\pi^{L\sigma M}. \tag{17}$$

Contrast this simple LσM-VMD result in (17) with the ChPT version of the pion charge radius [24]:

$$r_\pi = 12L_9/f_\pi^2 + \text{chiral logs}, \tag{18}$$

where L_9 is one of ten parameters $L_1 - L_{10}$ needed to regularize the *nonlinear* ChPT loops. Ignoring at first the chiral log term in (18), ChPT enthusiasts *fit* L_9 to the measured r_π in (14) or effectively to the VMD or LσM values of r_π in (15-17). While such a procedure already weakens the predictive power of ChPT relative to VMD or the LσM, a more important difference is due to the chiral log term in (18), which is known to diverge in the CL [25], presumably reflecting the effect of charged pion loops.

The question then is why the VMD and LσM values for r_π in (15-17) do not involve such (singular) pion loop effects. Of course VMD is a tree-level phenomenology so pion loops are then circumvented. As for the one-loop quark-level LσM field theory, even without a virtual off-shell vector current (photon), the *sum* of (quadratically divergent) loops contributing to the pion mass in eq.(1) *vanish*, whether they be quark or boson loops, a LσM version of the Goldstone theorem [8]. For the case of the pion charge radius r_π, one must consider u and d quark loops only even in a LσM (or quark model) context, with quartic pion loops directly vanishing. Thus the VMD and LσM finite versions of r_π in (15-17) are not contaminated by (formally divergent CL) chiral logs.

In conclusion then, we have attempted to reply to each of the four objections (a-d) that Weinberg gave in 1967 [2] against the LσM. In our reply (a'-d') we find that the quark-level dynamically generated LσM answers all of Weinberg's reservations. In effect, the latter LσM remains the 'time-honored example of a lagrangian satisfying PCAC and current algebra.' Even for the tests of $a_{\pi\pi}^{(0)}$ or r_π, the simple (tree-or one loop level) *linear* σ model matches or surpasses the far more complicated *nonlinear* and higher loop order chiral perturbation theory approach [26].

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