MESON PRODUCTION FROM A RAPIDLY HADRONIZING QUARK GLUON PLASMA: A KINEMATICAL APPROACH

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ABSTRACT

Production of mesons from an equilibrated but rapidly hadronizing quark gluon plasma (QGP) is studied. The proposed model assumes the formation of hadrons such that the constituent quarks from the hadronizing QGP combine to form an on-shell particle. The QGP is assumed to hadronize without undergoing through an equilibrated hadron gas (HG) phase. The results obtained are compared with the case of a pure HG without QGP formation.

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$T > 250$ MeV in the initial QGP phase the relaxation time for the $s(8)$ quark density to saturate is quite small (i.e. $\leq 3$ fm/c) than the expected life-time of the QGP [4, 7, 16, 20] and the strange quark chemical potential relaxes to zero, while for the light $(u, d)$ quarks the relaxation time is even smaller. Matsui et al. have used $m_{s, s}$ as the mass of the $s(8)$ quark and $m_{s} = 150$ MeV in their calculation while Koch et al. have treated the light quarks as massless. In this model we expect that the possible signals of QGP formation will have a good chance to survive. In the above picture of the rapidly hadronizing QGP the formation of hadrons is assumed to take place from the entire volume of the quark matter glob at the point of disintegration of the plasma. This will essentially happen when the Debye screening of the colour charges will become less and less effective and start permitting the formation of the bound states of the quarks [22, 23]. For the sake of simplicity we will neglect the possible surface effects. Bertel et al. [24] have used a complementary picture where the hadronization transition is a pure surface effect and proceeds via the radiation of hadrons from the surface of the quark matter glob.

A crucial feature of the quark-hadron confinement phase transition concerns the conservation of entropy. If the number of the quark constituents does not change during an equilibrium transition from the QGP to the HG phase at a constant temperature then it will result in a sharp decrease of entropy. One possible suggested mechanism to conserve entropy for such a phase transition is to allow the gluons to fragment thereby creating extra quark-antiquark pairs which produce additional hadrons at the time of QGP breakup. In the present model we however do not allow for an equilibrium QGP $\rightarrow$ HG phase transition. However, in order to conserve the entropy generated in the QGP phase during the processes leading to its equilibration [4, 7, 20] the gluons can be allowed to fragment to create extra quark-antiquark pairs when the QGP starts disintegrating and the hadrons form. This fragmentation at the hadronization point, where the QGP temperature has reached the critical value ($T_c$) due to expansion and cooling, will result in the production of extra mesons at the same temperature ($T_c$). However we do not expect this mechanism to affect the "relative" abundances of the mesons to any significant level.

In the present paper we will concentrate exclusively on the production of $K$, $K^*$ and $\phi$ mesons from the rapidly hadronizing QGP. In many previous studies [5-7, 18] the relative abundance of mesons and baryons was obtained in a static picture by assuming it to be proportional to the number densities of the corresponding constituent quarks. For example the ratio

$$K^*/K = n_{q}(n_{s}) / n_{s}(n_{s}),$$

where $n_{q}(n_{s})$ and $n_{s}(n_{s})$ are the equilibrium number densities of the light quark (antiquark) and strange quark (antiquark), respectively. These are obtained by integrating the corresponding quantum distribution functions of the quarks (antiquarks) in the entire phase space. However, in this picture the effect of the mass of the final state hadron forming via the coalescence of the quark and antiquark is not taken into account. This has a serious consequence on the evaluation of the relative abundances of the mesons with different masses, e.g. it gives the ratio $K^*/K = 1$ always at all temperatures and chemical potentials, which is absurd. Since this ratio must always be less than unity (ignoring the degeneracy factors) and only slowly approach unity when the temperature of the QGP is raised to a high value, as we shall see later. Similarly all other ratios of the particles with different masses are wrongly obtained in this picture. This happens because the criteria of a fixed given invariant mass to the hadronizing quark-antiquark pairs is not imposed. We will now impose this essential criteria and as a result the number $N_m$ of the meson with rest mass $m$ emitted from a hadronizing QGP in the present model will be given as

$$N_m \sim \int d^4 p_1 f(p_1) \int d^4 p_2 f(p_2) \delta(m_1^2 + m_2^2 + 2 E_1 E_2 - 2|p_1| |p_2| \cos \theta - m^2)$$

We are guided to the above expression by the fact that the two quarks with masses $m_1$ and $m_2$ and having the three-momenta $p_1$ and $p_2$ in the rest frame of the QGP glob will have an invariant mass $M_{inv}$ given by

$$M_{inv}^2 = m_1^2 + m_2^2 + 2 E_1 E_2 - 2|p_1| |p_2| \cos \theta$$

where $\theta$ is the relative angle between $p_1$ and $p_2$ while $E_1$ and $E_2$ are given by

$$E_1^2 = p_1^2 + m_1^2; \quad E_2^2 = p_2^2 + m_2^2$$

In the above $p_1$ and $p_2$ now indicate the three-momenta of the respective quarks, unless specified otherwise. The $f(p_1)$ and $f(p_2)$ are the Fermi-Dirac distribution functions for the quark and antiquark.

Next we have imposed the criteria that this invariant mass must always be equal to the rest mass of the meson, $m$, formed by the coalescence of the quarks. The occurrence of the $\delta$-function in Eq. (1) is to guarantee this fact. However, the phase-space integrals of the two quarks distribution functions allow the formed meson itself to be emitted at all possible momenta. In other words the hadronizing quark pair although will always have the same invariant mass ($M_{inv}^2 = m^2$) but the pair as a whole can fly out of the QGP region at all possible momenta. In this way we have not restricted the meson emitted due to the hadronization process to any particular limited phase-space sector, i.e. we cover the entire phase-space of the emitted meson. We notice that the two phase-space integrals over the variables $p_1$ and $p_2$ are not completely independent now and are coupled through the occurrence of the $\delta$-function. The roots of the argument of the $\delta$-function can be found for $\cos \theta$ i.e.

$$\cos \theta = \frac{1}{2} \frac{2|p_1^2 + m_1^2|^{1/2}(p_1^2 + m_1^2)^{1/2} + m_2^2 + m_2^2 - m^2/2|p_1| p_2}$$

We therefore notice that the sine-moments $p_1$ and $p_2$ do not run freely since they must be such that $|\cos \theta| \leq 1$ always. Thus mass of the meson plays an important role here in governing the number of the emitted mesons as clearly seen from Eq.(4). Using the above method the relative $K$, $K^*$ and $\phi$ multiplicities were obtained numerically. We choose the light quark mass $m_{q, d} = 8$ MeV and the strange quark mass $m_s = 150$ MeV. On the other hand for the sake of comparison we also consider the case of a pure HG phase with no QGP formation. Using the model which has been extensively used to obtain multiplicities of various hadronic species emitted from a hot HG phase we can write the number density of a given hadronic species, $h$, (in the Boltzmann approximation) as follows [5-7]

$$n_h = \frac{g_h}{2\pi^2} \lambda_h W(m_h/T) T^3$$

where $W(m_h/T) = (m_h/T)^2 K_3(m_h/T)$ and $\lambda_h = \exp(\mu_h/T)$. In the above expression $g_h$, $\lambda_h$ and $m_h$ are the (spin $\times$ isospin) degeneracy factor, fugacity, chemical potential and mass of the given hadronic species $h$ and $T$ is the temperature of the system in thermal and chemical equilibrium. Using Eq.(5), one can obtain the various particle ratios. In the following we discuss the results.
In Fig. 1 the variation of the $K^* (892)/K (493)$ ratio is plotted with the temperature $T$ of the system. Keeping in mind that the future LHC experiments will be able to create almost baryon free hot matter we choose the value of the baryon chemical potential $\mu = 0$. The dashed curve shows the variation of the $K'/K$ ratio with temperature $T$ of the system at freeze-out when it is in a pure HG phase i.e. without QGP formation. The solid line shows the QGP contribution at the time of its disintegration. A very interesting thing which emerges from the calculation is that the $K^*/K$ ratio from QGP can never exceed unity, while in the HG phase it can become quite larger than unity also. This happens because in the HG phase the $K^*$ abundance at lower temperatures is suppressed than the $K$ due to the larger mass (892 MeV). But with increasing temperature the effect of the suppression due to the larger mass is overtaken and at a certain temperature $T \approx 200$ MeV the $K^*$ and $K$ abundances become almost equal. Here it must be noted that in case of an equilibrated HG the particle abundances also depend on the degeneracy factors $g$, e.g. the $K^*$ being a vector meson has a larger $(\text{spin} \times \text{isospin})$ degeneracy factor $g_{K^*} = 6$ than that of $K$ which has $g_K = 2$. Thus at very high temperatures where the mass suppression effect is almost completely overtaken, the $K'/K$ ratio would become larger than unity and continue to rise asymptotically and ultimately saturate at the value $g_{K'}/g_{K} = 3$. This is also evident from Eq.(5) since $\lambda_{K^*} = \lambda_{K}$ [2,7]. However, for the case of the rapid QGP hadronization the situation is completely different. Since the final state hadrons do not form an equilibrated HG the emitted hadrons would directly reflect the quark composition of the QGP at the time of its breakup at the critical temperature and the corresponding degeneracy factors of $K^*$ and $K$ mesons will have no effect on their final state abundances. In fact given the spin of the hadron its abundance due to QGP hadronization process will be governed only by the number of possible spin combinations of the constituent quarks which coalesce to form the given hadronic species. We find that this number happens to be same for the $K^*$, $K$ and $\phi$ mesons, e.g. the quarks which coalesce to form the $K^*$ and $\phi$ must have their spins parallel while those coalescing to form $K$ must have their spins opposite. Thus for the $K$ meson (here $K$ stands for $K^*$ and $K^0$ which is a $q \bar{q}$ state, the $q$ and $\bar{q}$ must have their spins opposite. Thus two combinations 1) where $q$ has spin up and $\bar{q}$ has down and 2) where $q$ has spin down and $\bar{q}$ has up are possible. Similarly one will again find only two possible spin combinations for the case of the vector mesons $K^*$ (i.e. $K^{*+}$ and $K^{*-}$) and $\phi$. As a result although the $K'/K$ ratio from the hadronizing QGP increases asymptotically with the temperature as in the case of a HG also but it ultimately saturates at a value = 1 at large temperatures. In Fig. 1 we also notice that the $K'/K$ ratio gets saturated very rapidly in the QGP phase i.e. at temperatures $\geq 200$ MeV while in the HG phase it would require a larger temperature for the $K'/K$ ratio to saturate. Hence we find that in the framework of the given model for the QGP hadronization, the formation of QGP at critical temperatures $T_C > 250$ MeV will result in a sharp drop of the $K'/K$ ratio and this ratio can never become larger than unity even when the temperature is increased to very high values. However for temperatures $\leq 250$ MeV the two phases are almost indistinguishable.

Next we discuss the results obtained for the $\phi/(K + \bar{K})$ ratio. Here it should be noted that in the calculation the final state $K$ and $\bar{K}$ abundances also contain the decay contributions of the $K^*$ and $K^0$ mesons after freeze-out [2,5]. The $\phi/(K + \bar{K})$ ratio in both the phases is seen to saturate at large temperatures. This happens again due to the fact that at large temperatures the abundance of all the mesons (with varying masses) per degree of freedom will become almost same in the HG, see e.g. Eq.(5). Thus in the HG the ratio $\phi/(K + \bar{K})$ ultimately saturates at $g_{\phi}/(g_{K} + g_{\bar{K}} + g_{\phi}) \approx 0.19$, while in the case of QGP formation this ratio saturates at the value of 0.125 (due to the proper counting of the spins and isospins i.e. flavours of the constituent quarks). However, the $\phi/(K + \bar{K})$ ratio in the QGP is found to saturate more rapidly than the HG phase, as in the previous case of the $K'/K$ ratio. One finds that in the range of $150 < T_C < 250$ MeV the QGP formula will result in a slight enhancement of the $\phi/(K + \bar{K})$ ratio while for temperatures $> 350$ MeV it will result in a sudden drop of this ratio. In the range of $250 < T_C < 350$ MeV it may however be difficult to distinguish the QGP and the HG phases by measuring this ratio.

In summary a method is presented to calculate the relative abundances of mesons from an equilibrated but rapidly disintegrating QGP without undergoing through an equilibrated HG phase. The constituent quarks from the QGP combine to form the on-shell particles. The calculated $K'/K$ ratio from the QGP and a pure HG phase are compared. It is found that at the critical temperatures $T_C > 250$ MeV for the QGP hadronization the formation of QGP will result in a sharp decrease of this ratio and it will saturate at unity even when the temperature of the system is raised further. On the other hand one finds that in a pure HG phase also the ratio $K'/K$ saturates but the values are consistently larger than the QGP phase. The saturation occurs at a much larger value ($\approx 3$) and requires a much larger temperature than the QGP phase. The calculated $\phi/(K + \bar{K})$ ratio is found to be slightly larger in the QGP phase than the HG for temperatures $< 300$ MeV. We however also notice that if the formation of QGP takes place at temperatures $> 350$ MeV then it will cause a significant and sudden decrease in the $\phi/(K + \bar{K})$ ratio. In calculating the particle ratios for the QGP formation case the care of proper spin and isospin (i.e. quark flavour) counting has been taken. As a future exercise it may be worthwhile to extend the given QGP hadronization model in order to obtain the strange antihyperon to hyperon ratios and test the validity of the suggestions that the enhanced $A/A$ etc. ratios can be a definite signature of the QGP formation [4-7]. This will require the study of the three quarks (antiquarks) bound states production from a rapidly disintegrating QGP.

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References


Figure Captions

Fig. 1 Variation of the $K^*/K$ ratio with temperature $T$ for a baryonless matter with $\mu = 0$. The solid curve shows the contribution when the matter is in the QGP phase. The dashed curve is for the case when the hot matter does not suffer a phase transition to QGP and remains in a pure HG phase.

Fig. 2 Variation of $\phi/(K + \bar{K})$ ratio with temperature $T$ for $\mu = 0$. The two cases are the same as before. The $K$ and $\bar{K}$ abundances here also contain the strong decay contributions of $K^*$ and $\bar{K}^*$ for both the cases.