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ON MINIMAL ENERGY HARTREE–FOCK STATES FOR THE 2DEG AT FRACTIONAL FILLINGS[1]

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ABSTRACT

Approximate minimal energy solutions of the previously discussed general class of Hartree-Fock (HF) states of the 2DEG at 1/3 and 2/3 filling factors are determined. Their selfenergy spectrum is evaluated. Wannier states associated to the filled Bloch states are introduced in a lattice having three flux quanta per cell. They allow to rewrite approximately the $\nu = 1/3$ HF hamiltonian as a sum of three independent tigth-binding model hamiltonians, one describing the dynamics in the band of occupied states and the other ones in the two bands of excited states. The magnitude of the hopping integral indicates the enhanced role which should have the correlation energy in the present situation with respect to the case of the Yoshioka and Lee second order energy calcualtion for the lowest energy HF state. Finally, the discussion also suggests the Wannier function, which spreads an electron into a three flux quanta area, as a physical model for the composite fermion mean field one particle state.

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1 Introduction

In previous works a general class of extremals of the Hartree- Fock problem in connection with rational filling factors of the 2DEG in a magnetic field was determined [1],[2]. The basis set of Bloch functions used in the construction has the property of being eigenfunctions of all the magnetic translations in a cell having one flux quantum. A close related basis was also discussed in [3]-[5]. A nice property of those one particle states is that it is possible to fill them in an arbitrary way and you continue to have an extremal of the Hartree-Fock equations non-necessarely of minimal energy. In this sense they realize in this context, the analogous property of the Fourier functions of the normal translationally invariant states in the absence of magnetic field. Then, a required question to answer in connection with the clarification of the physical meaning of these HF states is the determination of their minimal energy configurations.

In this letter, the discussion of this problem is started for the special filling factor values 1/3 and 2/3. The particular case of an hexagonal lattice is assumed in order to make use of the symmetry in determining approximate minimal energy HF states. The haxagonal symmetry of the lattice allows to conclude that the fermi surface should be close to an hexagon (or a circle) having 1/3 or 2/3 of the Brillouin cell area formed by the set of all momentum quantum numbers of the one particle wave functions. The arising analytical expression for selfenergy spectrum in this approximation reproduces the exact one qualitatively.

In second place, it is argued that the approximate one particle hamiltonian for these near minimal energy Hartree-Fock states is equivalent to a localized states tight binding problem in a lattice having three flux quanta per unit cell. However the hoping integral is not vanishingly small as it would be the case in a similar picture for the global minimal energy HF states [6],[7]. The appreciable value of the hopping integral also suggests this representation as able in allowing for commensuration processes which could appear through second order correlation corrections to the energy. Already in [1], possible links among this HF problem and the cooperative rings of exchange, Hall crystal and Tao-Thouless approaches were mentioned. Such connection, if existing, looks as being related with this representation [8]-[11]. The calculation of the above mentioned second order contributions to the energy is in progress.

Finally, on purely physical grounds suggested by the spreading of the electron over a three flux quanta area in a Wannier state, the possible connection of them with a composite fermion mean-field one particle wave function is argumented.

In Section 2, previous results are reviewed and conditions for a minimal energy stated. In Section 3 the approximate HF states are introduced and their selfenergy spectrum and fermi surface determined. Section 4 is devoted to define the Wannier states related to the the approximate minimal energy states and use them to argument the link of the HF hamiltonian with nearest neighbor tight-binding models . The possible connections with the composite fermion approach is discussed in Section 5.

2 Review and minimal energy criteria

The notation to be used will be the same introduced in Ref. [2]. Concretly, the vectors of the hexagonal spatial lattice $\mathcal{L}$ to be considered will be defined by

$$\vec{l} = l_1\vec{b_1} + l_2\vec{b_2}, \quad l_1, l_2 = -\infty \cdots \infty, \tag{1}$$

in which

$$\vec{b_1} = r_0\sqrt{\frac{4\pi}{\sqrt{(3)}}}(1,0) \tag{2}$$

$$\vec{b_2} = r_0\sqrt{\frac{4\pi}{\sqrt{(3)}}}(cos(\frac{\pi}{3}), sin(\frac{\pi}{3})) \tag{3}$$

and r_0 is the magnetic lenght.
The wavefunctions of the basis can be written in the compact way

$$\phi_{\vec{p}}(\vec{x}) = \frac{1}{2\sqrt{q\mathcal{N}_{\vec{p}}}}\sum_{\vec{l}}(-1)^{l_1 l_2}\exp(i\vec{p}.\vec{l})T_{\vec{l}}\phi(\vec{x}), \tag{4}$$

$$\mathcal{N}_{\vec{p}} = \frac{N^2}{4}\sum_{\vec{l}}(-1)^{l_1 l_2}\exp[-\frac{1}{4r_0^2}\vec{l}^2 + i\vec{p}.\vec{l}]. \tag{5}$$

where $T_{\vec{l}}$ is the magnetic translation operator in a vector $\vec{l}$, q is an arbitrary odd number and N^2 is the number of auxiliary unit cells having each of them q flux quanta passing through which was introduced in [1],[2].
Such representation makes clear that the states are invariant under a shift in $\vec{p}$ given by the lattice vectors

$$\delta\vec{p} = 4\pi(n_1\vec{\sigma_1} + n_2\vec{\sigma_2}), \tag{6}$$

where the reciprocal lattice unit cell vectors satisfy

$$\vec{\sigma_i}.\vec{b_j} = \frac{1}{2}\delta_{ij}, \quad i = 1, 2. \tag{7}$$

The functions (4), under the action of magnetic translation in the spatial lattice $\mathcal{L}$, have eigenvalues of the form [2]

$$\lambda^{\vec{p}}(\vec{l}) = (-1)^{l_1 l_2}\exp(-i\vec{p}.\vec{l}). \tag{8}$$

It also follows that the density, for any way of filling of the basis states, has lattice periodicity which reflects in the Fourier expansion of the kind

$$\rho(\vec{x}) = \sum_{\vec{Q}}\rho(\vec{Q})exp(i\vec{Q}.\vec{x}), \tag{9}$$

where the wavevectors take values given by

$$\vec{Q} = 4\pi(n_1\vec{\sigma_1} + n_2\vec{\sigma_2}), \tag{10}$$

and the unit cell vectors of the reciprocal lattice in (10) also obey the relations

$$\vec{\sigma_1} = -\frac{\vec{n}\times\vec{b_2}}{4\pi r_0^2}, \quad \vec{\sigma_2} = \frac{\vec{n}\times\vec{b_1}}{4\pi r_0^2}, \tag{11}$$

and

$$\vec{b_1} = -4\pi r_0^2\vec{n}\times\vec{\sigma_2}, \quad \vec{b_2} = 4\pi r_0^2\vec{n}\times\vec{\sigma_1}. \tag{12}$$

with $\vec{n}$ being the normal to the 2DEG plane unit vector.
The Fourier component of the density was calculated in [2] for a general way of filling of the basis through the formula

$$\rho(\vec{Q}) = \frac{1}{V_N}\int_{V_N} d\vec{x}\rho(\vec{x})exp(-i\vec{Q}.\vec{x})$$
$$= \frac{1}{V_N}\sum_{\{c\}_f}\int_{V_N} d\vec{x}[\phi^c(\vec{x})]^*\phi^c(\vec{x})exp(-i\vec{Q}.\vec{x}), \quad (13)$$

where $V_N = 2\pi r_0^2 qN^2$, is the area of the region of N^2 cells on which periodicity boundary conditions were imposed in ref.[1], and the quantum numbers of the basis functions (4) are designed by c. The summation over $\{c\}_f$ in (13) runs over all the values corresponding to the filled one electron states in the considered Hartree-Fock solution. In this work c will coincide with the momentum quantum number $\vec{p}$.
The order parameter Δ is introduced in the usual way as

$$\Delta(\vec{Q}) = 2\pi r_0^2 exp(\frac{r_0^2\vec{Q}^2}{4})\rho(\vec{Q}). \quad (14)$$

and in [2] the following remarkable simple formula for this quantity was derived

$$\Delta(\vec{Q}) = \frac{2\pi r_0^2}{V_N}\sum_{\{\vec{p}\}_f}\lambda^{\vec{p}}(\vec{x}^*)$$
$$= \frac{2\pi r_0^2}{V_N}(-1)^{n_1 n_2}\sum_{\{\vec{p}\}_f}\exp(-i\vec{p}.\vec{x}^*). \quad (15)$$

where the dependence in $\vec{Q}$ has been expressed in terms of a dual spatial lattice vector $\vec{x}^*$ defined as follows

$$\vec{x}^* = r_0^2\,\vec{n}\times\vec{Q} = -n_2\vec{b_1} + n_1\vec{b_2},$$
$$\vec{Q} = \frac{1}{r_0^2}\,\vec{x}^*\times\vec{n} = 4\pi(n_1\vec{\sigma_1} + n_2\vec{\sigma_2}). \quad (16)$$

The condition for $\Delta(0)$ to be the filling factor ν and the sum rule derived in [7] for arbitrary HF states are directly satisfied by (15) .
After reviewing the main previous results, the question of appropiate requirements for defining minimal energy HF states will be discussed.

a. Minimal energy criteria

Before continuing, is useful to determine some rules for deciding wether a definite Slater determinant formed with the considered functions is near a minimal energy. Here, only the wide but not complete case of continuous selfenergy surfaces as functions of the selected wave function indices will be considered. A discontinuos selfenergy function can be imagined to occur upon a non appropiate selection of the labeling of the states which are filled.

It seems clear that a necessary condition for a minimal energy state in the present problem, is the selfconsistent property that the fermi surface should coincide exactly with a constant energy curve. If not so is the case, by moving a particle to another point of the curve where its selfenergy is lower, would reduce the total energy. Note that the effect of the modification of the selfenergy function produced by changing the occupation of the considered particle should be disregarded in the thermodinamical limit.
Another suplementary condition should be satisfied. It is that the filling of the momentum space in regions inside the fermi surface should be compact if the selfenergy in all the points is lesser than the fermi energy. In other case the process of translating a particle from the boundary to an empty state inside the region will again reduce the energy.

3 Approximate minimal energy HF states

Let us consider now special HF state at $\nu = 1/3$ obtained by filling all the one particle orbits being inside an hexagon H_0. This region will be selected also as the Brillouin cell corresponding to a sublattice $\mathcal{R}$ of the set of vectors $\mathcal{L}$. The vectors $\vec{R}$ in $\mathcal{R}$ will have the particular form

$$\vec{R} = (2n_1 - n_2)\vec{b_1} + (2n_2 - n_1)\vec{b_2}, \quad n_1, n_2 = -\infty\cdots\infty. \quad (17)$$

Notice that $\nu = 1/3$ corresponds to q=3 in the general formulae of Section 1.
For later use it will be helpful to also introduce two more hexagonal regions in momentum space H_1, H_2 which are basically equal to H_0 but displaced in such a way that the three hexagons H_0, H_1 and H_2 have a common vertex and any two of them also have a common side. The region formed in such a way is also a unit cell in

the momentum quantum number plane and then the empty states can be labeled by points in the hexagons H_1 and H_2.
The vectors which implement the mentioned translations have the following form

$$\vec{T}_1 = \frac{4\pi}{3}(2\vec{\sigma}_1 + \vec{\sigma}_2), \tag{18}$$

$$\vec{T}_2 = \frac{4\pi}{3}(2\vec{\sigma}_2 + \vec{\sigma}_2). \tag{19}$$

For the order parameter of this state the following expression can be written

$$\Delta(\vec{Q}) = 2\pi r_0^2(-1)^{n_1 n_2}\int_{H_0}\frac{d\vec{p}}{(2\pi)^2}exp(-i\vec{q}.\vec{x}^*). \tag{20}$$

The hexagon H_0 in the integration in (20) can be expected to be reasonably well approximated by a circle. This approximation, as simplifying explicit evaluations, will be considered below for qualitative reasonings. Note that making it corresponds to consider a state in which all the electron orbits in a circle are filled.
The momentum inside this circle satisfies

$$|\vec{p}| \leq \sqrt{2/3}\frac{1}{r_0}. \tag{21}$$

In this case the spherical symmetry allows to explicitely calculate the following formula for the order parameter [2]

$$\begin{aligned}\Delta(\vec{Q}) &= 2\pi r_0^2(-1)^{n_1 n_2}\int_{\vec{p}\leq\sqrt{2/3}/r_0}\frac{d\vec{p}}{(2\pi)^2}exp(-i\vec{q}.\vec{x}^*) \\ &= \frac{2}{3}(-1)^{n_1 n_2}\frac{J_1(\sqrt{\frac{8\pi}{3\sqrt{3}}}\sqrt{n_1^2+n_2^2-n_1 n_2})}{\sqrt{\frac{8\pi}{3\sqrt{3}}}\sqrt{n_1^2+n_2^2-n_1 n_2}},\end{aligned} \tag{22}$$

where n_1, n_2 are defined by (10).
Relation (22) in turn simplifies the calculation of the selfenergy spectrum for which the following formula can be used [2]

$$\epsilon(\vec{p}) = \frac{e^2}{r_0}\frac{V_N}{(2\pi r_0^2)}\sum_{\vec{Q}}\Delta^{\vec{p}}(\vec{Q})\Delta^*(\vec{Q})exp(-\frac{r_0^2\vec{Q}^2}{4})$$
$$\left[\frac{(1-\delta_{\vec{Q},0})}{r_0|\vec{Q}|}exp(-\frac{r_0^2\vec{Q}^2}{4}) - (\pi/2)^{1/2}I_o(\frac{r_0^2\vec{Q}^2}{4})\right], \tag{23}$$

where

$$\Delta^{\vec{p}}(\vec{Q}) = \frac{2\pi r_0^2}{V_N}\lambda^{\vec{p}}(\vec{x}^*), \tag{24}$$

and the $\vec{x}^*$ is the function of $\vec{Q}$ given in (16).
The graphical representation of the modulus of the order parameter (22) is given in Fig 1. From this picture it can be observed that the leading terms in the dependence on $\vec{Q}$ are the the zero value of the wavevector and its nearest neighbors. The reduced contribution of the higher harmonics is even more pronounced for such physical quantities as the particle density and selfenergy. This fact can be observed from the analogous plot in Fig. 2 for the Fourier transform of the density. In those pictures the curve values at the integer coordinates are the Fourier components.

The selfenergy dependence on the momentun $\vec{p}$ can be calculated from expression (23). It shows a general appeareance as ilustrated in Fig. 3 which was obtained by an approximate expression to be discussed below. The sixfold simmetry can be observed as well as the smooth angular dependence around the symmetry axis in the momentum regions inside the fermi surface. These two properties indicate the considered states as being approximate to the exact minimal wavefunction. This can be concluded because, the filling of orbits was by construction compact inside the circle defined by (21). In addition the sixfold symmetry of the constant energy curve and its smooth angular dependence near the fermi surface makes it to be close to the selected circular or hexagonal regions of filled states.

In what follows let us assume a simplified version of the expression for the order parameter (22) which only consider the zero momentum wavevector and its nearest neighbors. It will allow a clearer discussion of some properties of the considered HF state. Then, the parameter Δ will be taken in the form

$$\Delta(\vec{Q}) = \frac{r_0^2}{2\pi}(-1)^{n_1+n_2}\int d\vec{p} f(\vec{p}) exp(-i\vec{q}.\vec{x^*}) = \frac{1}{3}\delta_{\vec{Q},0} + 0.168560(-1)^{n_1+n_2}\sum_{i=0}^{5}\delta_{\vec{Q}_i,\vec{Q}}. \quad (25)$$

After substituting (25) in (23) the following approximate but simple expression for the selfenergy is received

$$\varepsilon(\vec{p}) = -\frac{e^2}{r_0}[0.41771 + 0.06748\sum_{i=0}^{5}\cos(\vec{p}.\vec{x_i^*})] = \frac{e^2}{r_0}[\varepsilon(0) + \varepsilon(1)\sum_{i=0}^{5}\cos(\vec{p}.\vec{x_i^*})], \quad (26)$$

in which the $\vec{x_i^*}, i = 0, ..., 5$ vectors are the six neighbors of the origin in the hexagonal lattice $\mathcal{L}$.
As it was noticed above, the plot of (26) in Fig. 3 shows a close resemblance with the numerical evaluation of the exact relation (23). An auxiliary plane is traced at an energy value corresponding to a momentum within the boundary of the circle (21). As remarked before, this plane intercepts the surface in a curve of hexagonal symmetry which is nearly a circular or hexagonal form. In the next section these approximations for the order parameter and one-particle energies will be employed for expressing the HF hamiltonian problem as a collection of three independent nearest neighbors tight binding models.

4 Wannier states and Tight-Binding model.

The aim of in this section is to construct an approximate lattice model representation of the Hartree-Fock hamiltonian corresponding with the mean field state constructed by filling completely all the orbits which have momentum inside an hexagon Ho having 1/3 of all the quantum numbers. The hexagon Ho, by construction is the Brillouin cell for the lattice $\mathcal{R}$ defined in (17) and which is included in $\mathcal{L}$. In addition the unit cell of this lattice have three flux quanta passing from it. Then, it is possible to define a complete set of orthogonal Wannier states through

$$\Phi^i_{\vec{R}}(\vec{x}) = \frac{3r_o^2}{2\pi}\int_{H_o} dp\ \exp(-i\vec{p}.\vec{R})\,\phi_{\vec{p}}(\vec{x}), \qquad i = 0, 1, 2. \quad (27)$$

That is, three localized states are defined for each lattice position $\vec{R}$.
The special case of a 1/3 filling is being considering here, but similar considerations for $\nu = 2/3$ are also feasible.
By the definition (27), it follows that under magnetic translations in a vector of the type $\vec{R}$ these functions transform as

$$T_{\vec{R}'}\Phi^i_{\vec{R}}(\vec{x}) = (-1)^{l_1^{R'}l_2^{R'}}\Phi^i_{\vec{R}-\vec{R}'}(\vec{x}), \quad (28)$$

where l_1^R, l_2^R are the entire components of $\vec{R}$ expressed as a vector of $\mathcal{L}$ through (1).
For the matrix elements of the HF one particle hamiltonian between the Wannier states the following relation arises after considering that the Bloch functions are exact eigenvectorsof the hamiltonian

$$< i', \vec{R}'|H_{HF}|i, \vec{R} >= \delta_{i'i}\frac{3r_0{}^2}{2\pi}\int_{H_o} dp\,\varepsilon(\vec{p})\exp(-i\vec{p}.(\vec{R} - \vec{R}')). \quad (29)$$

The Dirac's bracket notation is used in (29) and below.
After substituting $\varepsilon_i(\vec{p})$ by its approximate expression (26) the following relation follows

$$< i', \vec{R}'|H_{HF}|i, \vec{R} >= \delta_{i'i}\left[\varepsilon(0)\delta_{\vec{R},\vec{R}'} + \frac{3r_o{}^2}{2\pi}\sum_{j=0}^{5}\int_{H_o} dp\,\varepsilon(1)\exp(-i\vec{p}.(\vec{R} - \vec{R}' - \vec{x_j^*}))\right]. \quad (30)$$

Moreover, relation (30) is again expressed in terms of the order parameter which, in the considered approximation, can be substituted by its formula (25). After employing (25) in (30) the matrix elements of the HF hamiltonian between the Wannier states can be written as follows

$$< i', \vec{R}'|H_{HF}|i, \vec{R} > = \delta_{i'i}\left[\varepsilon^i(s)\delta_{\vec{R},\vec{R}'} + \sum_{j=0}^{5}\varepsilon^i(h)\delta_{\vec{R},\vec{R}'+\vec{R_j^*}}\right],$$

$$\begin{aligned} \varepsilon^0(s) &= -0.6224\,\frac{e^2}{r_0}, \quad \varepsilon^0(h) = -0.0682\,\frac{e^2}{r_0}, \\ \varepsilon^{1,2}(s) &= -0.3154\,\frac{e^2}{r_0}, \quad \varepsilon^{1,2}(h) = 0.03411\,\frac{e^2}{r_0}, \end{aligned} \tag{31}$$

where $\vec{R}_j^*, j = 0, ..., 5$ are the nearest neighbors for the origin in the lattice of vectors $\vec{R}$. In (31) the onsite $\varepsilon^i(s)$ and hopping $\varepsilon^i(h)$ for each band of Wannier states have been introduced. Notice that the two sets of site and hopping parameters corresponding to the Wannier states defined in terms the empty Bloch like orbits coincide. Also, as it should be expected, the site energy of Wannier states coming from the filled band H_o is lower in energy. The sign of the hopping parameter also changes between the two kind of bands. Also, it follows that only nearest neighbors within the same band are coupled through hopping.

Now, it is of interest to rewrite (31) in the coordinate representation by considering again the action of the magnetic translations on the Wannier states

$$\begin{aligned} T_{\vec{R}'}\,\Phi^i_{\vec{R}}(\vec{x}) &= \exp(i\vec{G}.\vec{R}')\Phi^i_{\vec{R}}(\vec{x}) \\ &= (-1)^{i_1^{R'} i_2^{R'}}\,\Phi^i_{\vec{R}-\vec{R}'}(\vec{x}) \end{aligned} \tag{32}$$

where $\vec{G}$ are the generators of the magnetic translations.
Expression (32) and the fact that the HF hamiltonian have matrix elements only between Wannier state both pertaining to the same sublattice, allow to obtain for the coordinate representation kernel

$$\begin{aligned} H_{HF}(\vec{x}', \vec{x}) &= < \vec{x}'|i', \vec{R}' >< i', \vec{R}'|H_{HF}|i, \vec{R} >< i, \vec{R}|\vec{x} > \\ &= \sum_{i=0}^{2} \left[\varepsilon^i(s) P^i(\vec{x}, \vec{x}') + \sum_{j=0}^{5} \varepsilon^i(h)\,(-1)^{i_1^{R_j^*} i_2^{R_j^*}} < \vec{x}'|P^i \exp(-i\vec{G}.\vec{R}_j^*) P^i|\vec{x} > \right], \end{aligned} \tag{33}$$

where P^i are the projection operators in each of the Wannier subspaces giving the full projection operator in the first Landau level upon their addition.

5 Wannier vs. Composite fermion state

The three flux quanta per unit cell of the lattice formed by the considered Wannier states and the spread of the electron over all that unit cell, takes to the mind the possibility for a link of these states with the composite fermion wave function in the mean field approximation. An overcomplete basis of one particle orbits for composite fermions are simply the zero angular momentum first Landau level eigenfunction for one third of the external magnetic fiel value and all its magnetic translations. Since any of such functions have appreciable values only inside a region having three flux quanta passing through it, their projection in the first Landau level should have the leading components along only three Wannier states whenever the center of charge c the composite fermion state coincides with a lattice point in $\mathcal{R}$. Therefore, as both the HF and the mean field composite fermion manybody states are Slater determinants, it follows that the later could be imagined as an idealization of a HF minimal energy wavefunction. A signal for such possibility, would be the similarity of the zero angular momentum composite fermion one particle state with its corresponding Wannier components in the first Landau level.
Here, in order to consider the above question, a crude estimate of the Wannier function spatial dependence in the above defined nearest neighbor approximation is done.
After substituting the Bloch eigenfunction expression (4) in the Wannier state definiton (27) the following intermediate relation follows

$$\Phi_{\vec{R}}(\vec{x}) = \frac{1}{2\sqrt{3}} \sum_{\vec{l}} (-1)^{i_1 i_2} \exp(i\vec{p}.\vec{l}) T_{\vec{l}} \phi(\vec{x}) \int_{H_i} dp \frac{3r_o^2}{2\pi \mathcal{N}_{\vec{p}}} \exp(i\vec{p}.(\vec{l} - \vec{R})). \tag{34}$$

The denominator in the momentum integral in (34) is proportional to the Bloch like wave function normalization factor. The most drastic approximation to be done here is to disregard the momentum dependence of this factor in order to estimate qualitatively and analytically the spatial dependence of the Wannier function. Through the graphical representation of the norm as a funtion of the momentum in the periodicity region, shown in Fig. 4, it can be seen that this quantity while being a smooth function evidently varies with the momentum. After roughly assuming a mean constant value for the norm, the momentum integrals over the regions H_i are evaluated by also assuming as before that they have their leading values when $\vec{l}$ coincides with $\vec{R}$ or becomes a nearest neighbor. Then, the following expression for the estimated Wannier orbitals arises

$$\Phi^1_{\vec{R}}(\vec{x}) \approx (-1)^{l_1^R l_2^R} T_{\vec{R}} \left[\phi(\vec{x}) + 0.505682 \sum_{j=0}^{5} (-1)^{l_1^j l_2^j} \exp(i\vec{T}_1.\vec{x}_j^*) T_{\vec{x}_j^*} \phi(\vec{x}) \right], \qquad (35)$$

where as defined before l_1^j, l_2^j and l_1^R, l_2^R are the entire components of x_j^* and $\vec{R}$ as vectors of the lattice $\mathcal{L}$.
The graphical representation of the charge density associated to the Wannier state (35) at $\vec{R} = 0$ is shown in Fig. 5a) as function of the coordinates. The corresponding density plot for the composite fermion mean-field one particle state is depicted in Fig. 5b). It can be observed that the form of the density is not very similar. However, the overlap of the regions of maximal probability for the electron position is appreciable. The non coincidence of the shape is partially due to the lack of sixfold rotational symmetry of the considered Wannier function. Such a difficulty, would be solved by using the fact that the relation (27) defining the Wannier orbital is not unique. This arbitrariness can be used to construct more symmetric Wannier states. Such a possibility is also supported by the fact a linear combination of the the Wannier orbitals at $\vec{R} = 0$ has a sixfold symmetry. However, it is needed to find such symmetric Wannier orbitals which are expressed as superpositions of the filled one electron states. This question is being considered.

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Figure Captions

Fig. 1. The plot of (22) for order parameter as a function of the discrete coordinate n_1 for $n_2 = 0$. Only values at integer n_1 are meaningful.

Fig. 2. The plot of the Fourier coefficients of the density, in units of the density of a filled Landau level, as a function of the discrete coordinate n_1 for $n_2 = 0$. Only values at integer n_1 are meaningful. Note that in this case the first Fourier harmonics are even more reduced with respect to the component at zero momentum than in Fig. 1. The charge density modulation in the considered state is then highly diminished with respect to the one present in the global minimum HF state [7].

Fig. 3. The approximate selfenergy momentum dependence ($r_0 = 1$) defined by (26). It reproduces very well in symmetry and form and quantitatively up two around %10 the results from the exact expression (23).

Fig. 4. The norm of the Bloch like wave functions (4) as a function of the oblique components of the momentum along the unit cell vectors $4\pi\vec{\sigma}_1$ and $4\pi\vec{\sigma}_2$.

Fig. 5a) The spatial dependence ($r_0 = 1$) of the electron density for the approximate Wannier orbital at the origin and $i = 0$. Note that, unfortunately, the selected way of construction of the Wannier states through (27) has not produced a sixfold axes of symmetrym around the origin.

Fig. 5b) The similar plot for the zero angular momentum mean-field composite fermion state. In spite of the lack of symmetry coincidence with the density shown in Fig.5a), the extension of the region for the most probable finding of the electron is similar.

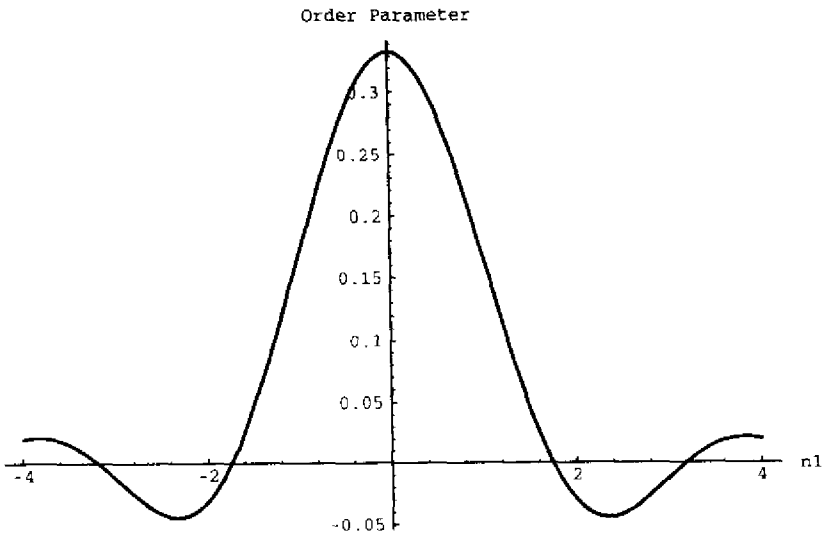

Fig.1

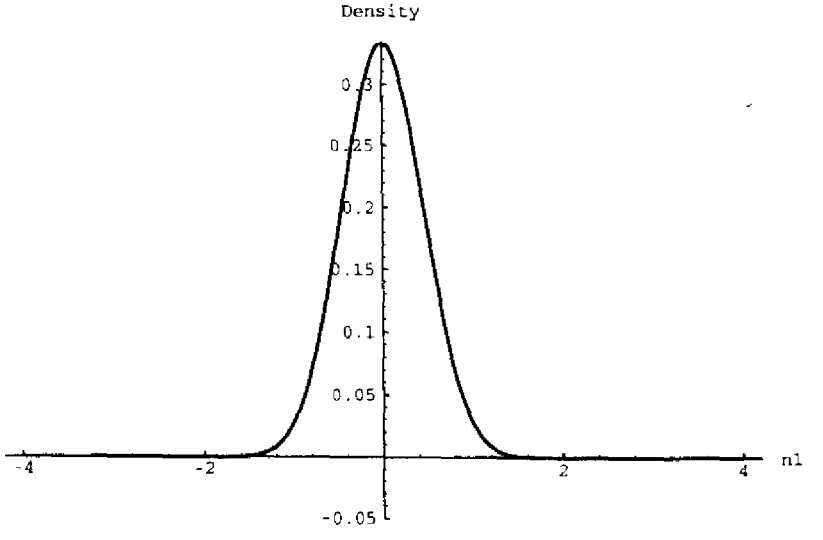

Fig.2

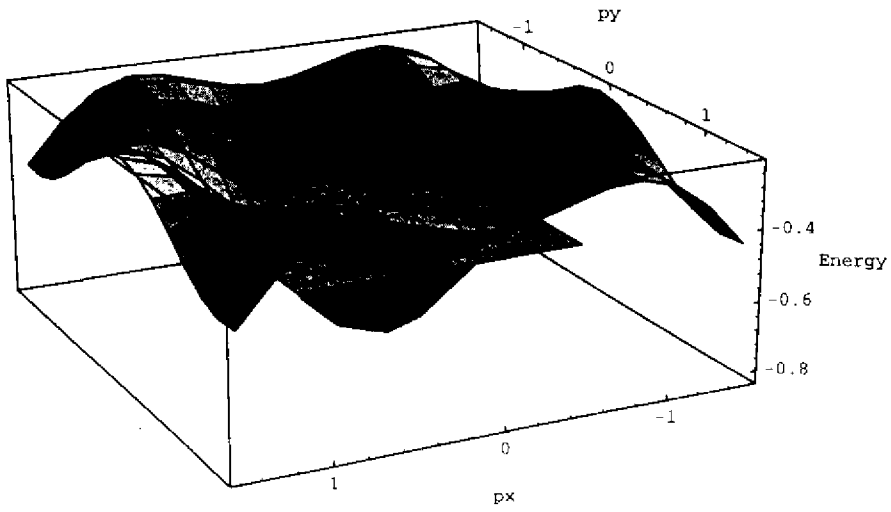

Fig.3

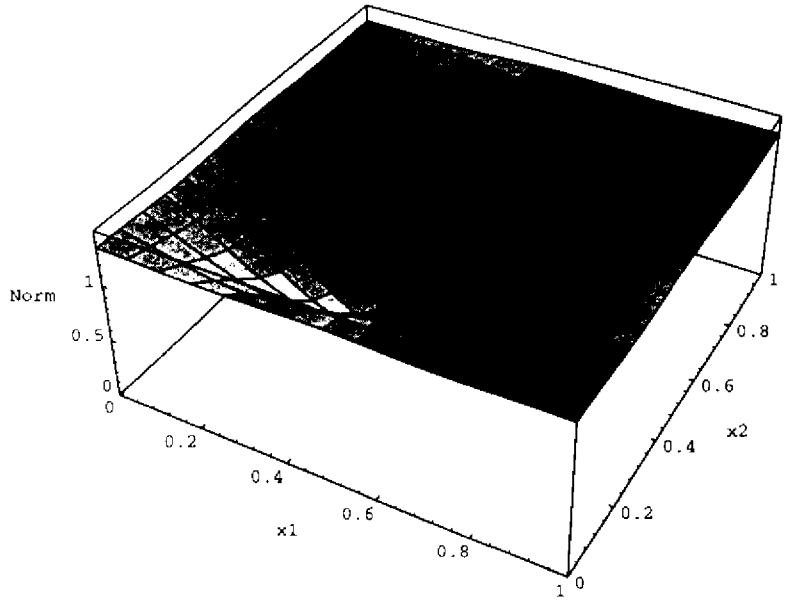

Fig.4

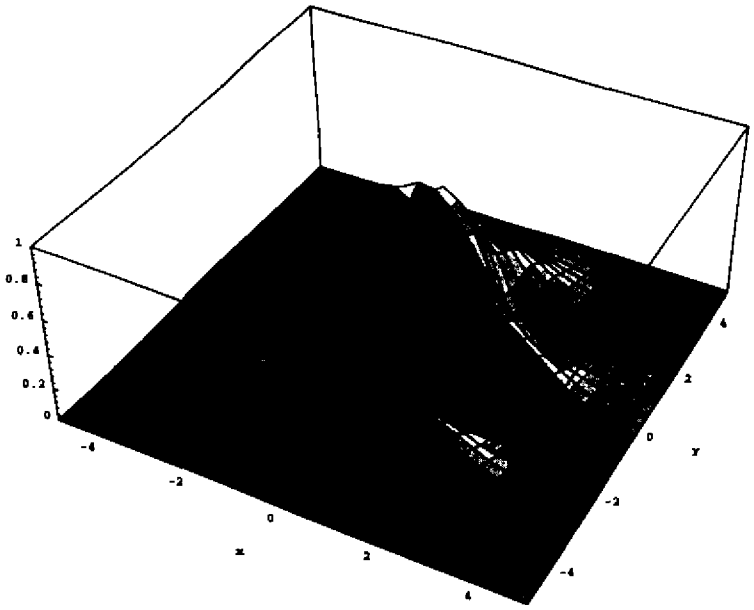

Fig.5a

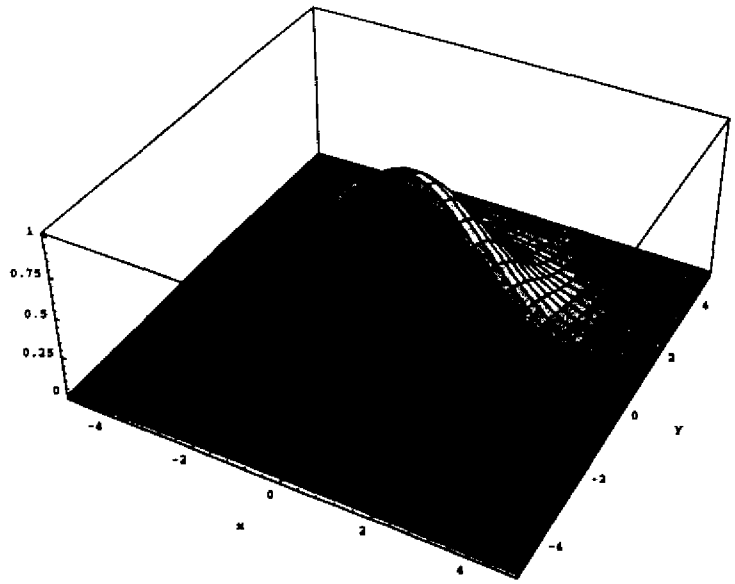

Fig.5b