IC/95/230
INTERNAL REPORT
(Limited Distribution)

International Atomic Energy Agency
and
United Nations Educational Scientific and Cultural Organization
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

THE SUSCEPTIBILITIES IN THE SPIN-S ISING MODEL

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ABSTRACT

The susceptibilities of the spin-S Ising model are evaluated using the effective field theory introduced by Tucker et al. for studying general spin-S Ising model. The susceptibilities are studied for all spin values from $S = 1/2$ to $S = 5/2$.

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August 1995

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1 Introduction

The study of the transverse spin-S Ising model has received much attention because of its simplicity and effectiveness. Many theoretical techniques have been applied to these problems [1-10]. Recently the effective field theory has been applied to the transverse Ising model with arbitrary spin [10]. The dependence of the transition temperature was studied as a function of the transverse field strength for all spin values from $S = \frac{1}{2}$ to $S = \frac{5}{2}$. The purpose of this work is to study the longitudinal and transverse susceptibilities for a transverse spin-S Ising model on an hypercubic lattice with coordination number N using the effective field theory [11]. In Section 2, we briefly present the basic framework of the effective field theory of the transverse spin-S Ising model in an applied magnetic field. In Section 3, the longitudinal and transverse susceptibilities are studied.

2 The effective field equations

The transverse spin-S Ising model in an applied magnetic field may be described by the hamiltonian

$$H = -\sum_{<i,j>} J_{ij} S_{iz} S_{jz} - h \sum_i S_{iz} - \Omega \sum_i S_{ix} \tag{1}$$

in equation (1), S_{ix} and S_{iz} are respectively the x and z components of a quantum spin $\vec{S}_i$ of magnitude S at site i, h and Ω are respectively the magnitude of the longitudinal and transverse fields and J_{ij} is the exchange interaction with $J_{ij} = J$ for i and j nearest neighbours and $J_{ij} = 0$ otherwise. In essence, the theory to be used is the simple-site cluster theory in which attention is focused on a cluster comprising just a single selected spin, labelled 0, and the neighbouring spins with which it directly interacts. To this end the hamiltonian is split into two parts $H = H_0 + H'$, where H_0 is that part of the hamiltonian containing spin 0, namely

$$H_0 = -AS_{0z} - BS_{0x} \tag{2}$$

with

$$A = \sum_j JS_{jz} + h \tag{3}$$

and

$$B = \Omega \tag{4}$$

Following Sà Barreto et al. [1], one then employs the approximate relation

$$< m_{p\alpha} > = < S_{0\alpha}^p > = < \frac{trace_0[S_{0\alpha}^p \exp(-\beta H_0)]}{trace_0[\exp(-\beta H_0)]} > \tag{5}$$

where $p = 1, 2, ..., 2S$; $\beta = 1/k_B T$ and $\alpha = z, x$.
The equation (4) neglects the fact that H_0 and H' do not commute. In the limit $\Omega = 0$,

the hamiltonian contains only S_z components and the relationship (4) is then exact. Evaluation of the traces on the right-hand side of this equation yields [10]

Spin 1/2

$$\begin{aligned} m_{1z} &= < \frac{As_1}{2\Delta c_1} > \\ &= < F_{1\alpha}^{(\frac{1}{2})}(A,B) > \end{aligned} \tag{6}$$

Spin 1

$$\begin{aligned} m_{1z} &= < \frac{2As_2}{\Delta(2c_2+1)} > \\ &= < F_{1\alpha}^{(1)}(A,B) > \end{aligned} \tag{7}$$

$$\begin{aligned} m_{2z} &= < \frac{(2A^2+B^2)c_2}{\Delta^2(2c_2+1)} \\ &\quad + \frac{B^2}{\Delta^2(2c_2+1)} > \\ &= < F_{2\alpha}^{(1)}(A,B) > \end{aligned} \tag{8}$$

Spin 3/2

$$\begin{aligned} m_{1z} &= < \frac{A(3s_3+s_1)}{2\Delta(c_3+c_1)} > \\ &= < F_{1\alpha}^{(\frac{3}{2})}(A,B) > \end{aligned} \tag{9}$$

$$\begin{aligned} m_{2z} &= < \frac{3(3A^2+B^2)c_3}{4\Delta^2(c_3+c_1)} \\ &\quad + \frac{(A^2+7B^2)c_1}{4\Delta^2(c_3+c_1)} > \\ &= < F_{2\alpha}^{(\frac{3}{2})}(A,B) > \end{aligned} \tag{10}$$

$$\begin{aligned} m_{3z} &= < \frac{3(9A^2+7B^2)s_3}{8\Delta^3(c_3+c_1)} \\ &\quad + \frac{(A^2+19B^2)s_1}{8\Delta^3(c_3+c_1)} > \\ &= < F_{3\alpha}^{(\frac{3}{2})}(A,B) > \end{aligned} \tag{11}$$

Spin 2

$$\begin{aligned} m_{1z} &= < \frac{2A(2s_4+s_2)}{\Delta(2c_4+2c_2+1)} > \\ &= < F_{1\alpha}^{(2)}(A,B) > \end{aligned} \tag{12}$$

$$\begin{aligned} m_{2z} &= < \frac{2(4A^2+B^2)c_4}{\Delta^2(2c_4+2c_2+1)} \\ &\quad + \frac{(2A^2+5B^2)c_2+3B^2}{\Delta^2(2c_4+2c_2+1)} > \\ &= < F_{2\alpha}^{(2)}(A,B) > \end{aligned} \tag{13}$$

$$\begin{aligned} m_{3z} &= < \frac{2A(8A^2+5B^2)s_4}{\Delta^3(2c_4+2c_2+1)} \\ &\quad + \frac{(A^2+7B^2)s_2}{\Delta^3(2c_4+2c_2+1)} > \\ &= < F_{3\alpha}^{(2)}(A,B) > \end{aligned} \tag{14}$$

$$\begin{aligned} m_{4z} &= < \frac{(32A^4+34A^2B^2+5B^4)c_4}{\Delta^4(2c_4+2c_2+1)} \\ &\quad + \frac{(2A^4+31A^2B^2+17B^4)c_2}{\Delta^4(2c_4+2c_2+1)} \\ &\quad + \frac{3B^2(A^2+4B^2)}{\Delta^4(2c_4+2c_2+1)} > \\ &= < F_{4\alpha}^{(2)}(A,B) > \end{aligned} \tag{15}$$

Spin 5/2

$$\begin{aligned} m_{1z} &= < \frac{A(5s_5+3s_3+s_1)}{2\Delta(c_5+c_3+c_1)} > \\ &= < F_{1\alpha}^{(\frac{5}{2})}(A,B) > \end{aligned} \tag{16}$$

$$\begin{aligned} m_{2z} &= < \frac{5(5A^2+B^2)c_5+(9A^2+13B^2)c_3}{4\Delta^2(c_5+c_3+c_1)} \\ &\quad + \frac{(A^2+17B^2)c_1}{4\Delta^2(c_5+c_3+c_1)} > \\ &= < F_{2\alpha}^{(\frac{5}{2})}(A,B) > \end{aligned} \tag{17}$$

$$\begin{aligned} m_{3z} &= < \frac{5(25A^2+13B^2)s_5+3(9A^2+37B^2)s_3}{8\Delta^3(c_5+c_3+c_1)} \\ &\quad + \frac{(A^2+49B^2)s_1}{8\Delta^3(c_5+c_3+c_1)} > \\ &= < F_{3\alpha}^{(\frac{5}{2})}(A,B) > \end{aligned} \tag{18}$$

$$m_{4z} = < \frac{5(125A^4+114A^2B^2+13B^4)c_5}{16\Delta^4(c_5+c_3+c_1)}$$

$$
\begin{aligned}
& +\frac{(81A^4+682A^2B^2+241B^4)c_3}{16\Delta^4(c_5+c_3+c_1)} \\
& +\frac{(A^4+162A^2B^2+401B^4)c_1}{16\Delta^4(c_5+c_3+c_1)} > \\
= \; & < F_{4\alpha}^{(\frac{5}{2})}(A,B) > \qquad (19)
\end{aligned}
$$

$$
\begin{aligned}
m_{5z} = \; & < \frac{5(625A^4+842A^2B^2+241B^4)s_5}{32\Delta^5(c_5+c_3+c_1)} \\
& +\frac{3(81A^4+1242A^2B^2+961B^4)s_3}{32\Delta^5(c_5+c_3+c_1)} \\
& +\frac{(A^4+482A^2B^2+1681B^4)s_1}{32\Delta^5(c_5+c_3+c_1)} > \\
= \; & < F_{5\alpha}^{(\frac{5}{2})}(A,B) > \qquad (20)
\end{aligned}
$$

where

$$m_{pz} = < S_{0z}^p > \qquad (21)$$

$$\Delta = \sqrt{(A^2+B^2)} \qquad (22)$$

$$s_n = sinh(\frac{n\beta\Delta}{2}) \qquad (23)$$

$$c_n = cosh(\frac{n\beta\Delta}{2}) \qquad (24)$$

The corresponding results for the transverse components, $m_{px} =< S_{0x}^p >$ may be obtained from the longitudinal components by interchanging A and B. The quantities of interest are the moments $m_{p\alpha}$. To perform the thermal averaging on the right-hand side of eqs. (5-19), one now follows the general approach described by Elkouraychi et al. [10]. If this is done one obtains

Spin 1/2

$$
\begin{aligned}
m_{p\alpha} = \; & \sum_{i=0}^{N} [2^{-N} C_i^N (1-2m_{1z})^i (1+2m_{1z})^{N-i} \\
& F_{p\alpha}^{(\frac{1}{2})}(\frac{N-2i}{2}, \Omega)] \qquad (25)
\end{aligned}
$$

where C_n^m are the binomial coefficients $\frac{m!}{n!(m-n)!}$ and N is the coordination number.

Spin 1

$$
\begin{aligned}
m_{p\alpha} = \; & \sum_{i=0}^{N} \sum_{j=0}^{N-i} [2^{-N+i} C_i^N C_j^{N-i} (1-2m_{2z})^i \\
& (m_{2z}-m_{1z})^j (m_{2z}+m_{1z})^{N-i-j} \\
& F_{p\alpha}^{(1)}(N-i-2j, \Omega)] \qquad (26)
\end{aligned}
$$

Spin 3/2

$$
\begin{aligned}
m_{p\alpha} = \; & \sum_{i=0}^{N} \sum_{j=0}^{N-i} \sum_{k=0}^{N-i-j} [48^{-N} 3^{j+k} C_i^N C_j^{N-i} C_k^{N-i-j} \\
& (-3+2m_{1z}+12m_{2z}-8m_{3z})^i \\
& (9-18m_{1z}-4m_{2z}+8m_{3z})^j \\
& (9+18m_{1z}-4m_{2z}-8m_{3z})^k \\
& (-3-2m_{1z}+12m_{2z}+8m_{3z})^{N-i-j-k} \\
& F_{p\alpha}^{(\frac{3}{2})}(\frac{3N-6i-4j-2k}{2}, \Omega)] \qquad (27)
\end{aligned}
$$

Spin 2

$$
\begin{aligned}
m_{p\alpha} = \; & \sum_{i=0}^{N} \sum_{j=0}^{N-i} \sum_{k=0}^{N-i-j} \sum_{l=0}^{N-i-j-k} [24^{-N+i} 4^{k+l-i} C_i^N C_j^{N-i} C_k^{N-i-j} \\
& C_l^{N-i-j-k} (4-5m_{2z}+4m_{4z})^i \\
& (2m_{1z}-m_{2z}-2m_{3z}+m_{4z})^j \\
& (-4m_{1z}+4m_{2z}+3m_{3z}-4m_{4z})^k \\
& (4m_{1z}+4m_{2z}-3m_{3z}-m_{4z})^l \\
& (-2m_{1z}-2m_{2z}+2m_{3z}+m_{4z})^{N-i-j-k-l} \\
& F_{p\alpha}^{(2)}(2N-2i-4j-3k-l, \Omega)] \qquad (28)
\end{aligned}
$$

Spin 5/2

$$
\begin{aligned}
m_{p\alpha} = \; & \sum_{i=0}^{N} \sum_{j=0}^{N-i} \sum_{k=0}^{N-i-j} \sum_{l=0}^{N-i-j-k} \sum_{m=0}^{N-i-j-k-l} [3840^{-N} 10^{k+l} 5^{j+m} \\
& C_i^N C_j^{N-i} C_k^{N-i-j} C_l^{N-i-j-k} C_m^{N-i-j-k-l} \\
& (45-18m_{1z}-200m_{2z}+80m_{3z}+80m_{4z}-32m_{5z})^i \\
& (-75+50m_{1z}+312m_{2z}-208m_{3z}-48m_{4z}+32m_{5z})^j \\
& (225-450m_{1z}-136m_{2z}+272m_{3z}+16m_{4z}-32m_{5z})^k \\
& (225+450m_{1z}-136m_{2z}-272m_{3z}+16m_{4z}+32m_{5z})^l \\
& (-75-50m_{1z}+312m_{2z}+208m_{3z}-48m_{4z}-32m_{5z})^m \\
& (45+18m_{1z}-200m_{2z}-80m_{3z}+80m_{4z}+32m_{5z})^{N-i-j-k-l-m} \\
& F_{p\alpha}^{(\frac{5}{2})}(\frac{5N-10i-8j-6k-4l-2m}{2}, \Omega)] \qquad (29)
\end{aligned}
$$

3 Susceptibilities

First, we summarize the results for the thermal dependence of the critical temperature against the transverse field obtained earlier [10] for coordination number appropriate to the square lattice $N = 4$ (see Figure 1). The results show that in the spin-S Ising model with a transverse field, at high temperatures the longitudinal S_z components are disordered, and at temperatures below a critical value T_c an ordered phase is set up with $m_{1z} \neq 0$, although in all temperatures there exists an order with $m_{1x} \neq 0$ (see ref. [10]). Therefore it is interesting to study the behaviour of the longitudinal and the transverse susceptibilities with the phase change. The longitudinal and transverse susceptibilities are respectively defined by

$$\chi_{\|} = \frac{\partial m_{1z}}{\partial h})_{h=0} \tag{30}$$

$$\chi_{\perp} = \frac{\partial m_{1x}}{\partial \Omega})_{h=0} \tag{31}$$

By differentiating eqs. (25-29) with h and Ω, we can easily obtain the susceptibilities. The details are not reported here.

In Figs. (2-4) the numerical results of the longitudinal and transverse susceptibilities are depicted for the system with N=4 (square lattice), for the value of the transverse field $\Omega = 0$.

In Fig. 2, the thermal dependence of the longitudinal susceptibility is presented for $\Omega = 0$. We see clearly a sharp cusp near the critical temperature which corresponds to the divergence of $\chi_{\|}$. The thermal behaviour of the inverse longitudinal susceptibility is shown in Fig. 3. Above the transition, the inverse longitudinal susceptibility changes almost linearly with the temperature, except in a very narrow region near the critical temperature independently of the value of the magnitude of the spin.

The thermal behaviour of the transverse susceptibility is shown in Fig. 4. Below the transition temperature, the transverse susceptibility $\chi_{\perp}$ has constant value $\chi_{\perp}(T = 0) = \frac{1}{N} = \frac{1}{4}$ and the $\chi_{\perp}$ rises slightly with increase in temperature, passes through a cusp at the second order transition temperature and then fall off rapidly. This characteristic feature of the perpendicular susceptibility of the transverse spin-S Ising model has been shown by Thorpe et al. [12] for the system with $N = 2$ using the transfer matrix technique.

4 Acknowledgments

The authors would like to thank the International Centre for Theoretical Physics, Trieste, for hospitality.

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5 Figure captions

Figure 1 : The longitudinal ordering temperature as function of transverse field strength for the square lattice (N=4). The number accompanying each curve denotes the value of S.

Figure 2 : The thermal dependence of the longitudinal susceptibility for the square lattice (N=4). The number accompanying each curve denotes the value of S.

Figure 3 : The thermal dependence of the inverse of the longitudinal susceptibility for the square lattice (N=4). The number accompanying each curve denotes the value of S.

Figure 4 : The thermal dependence of the transverse susceptibility for the square lattice (N=4). The number accompanying each curve denotes the value of S.

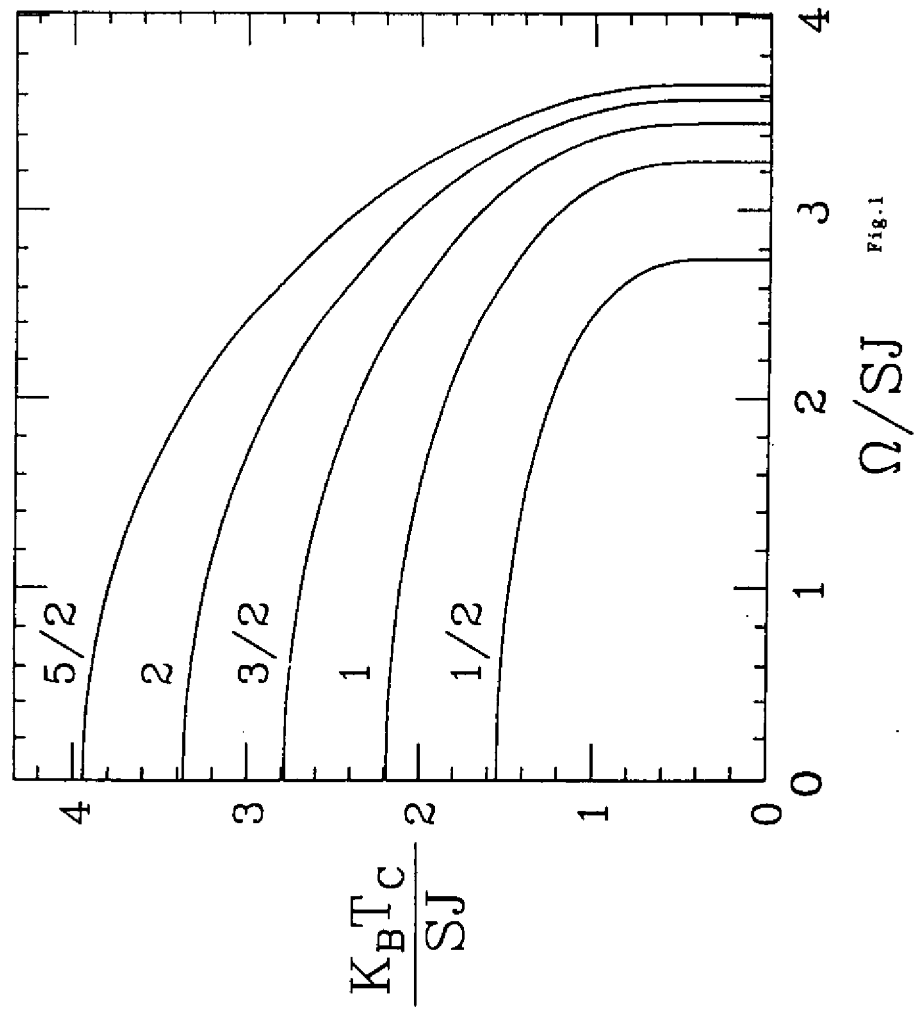
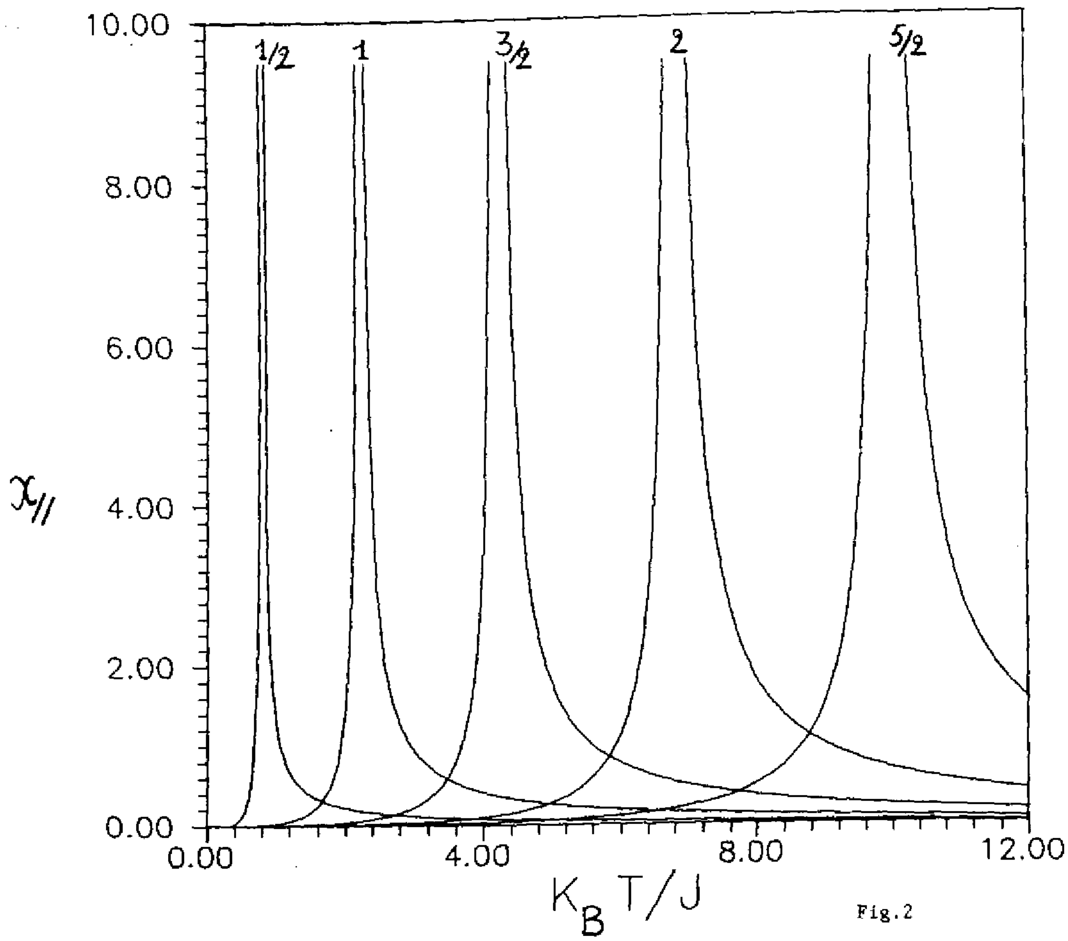


Fig.1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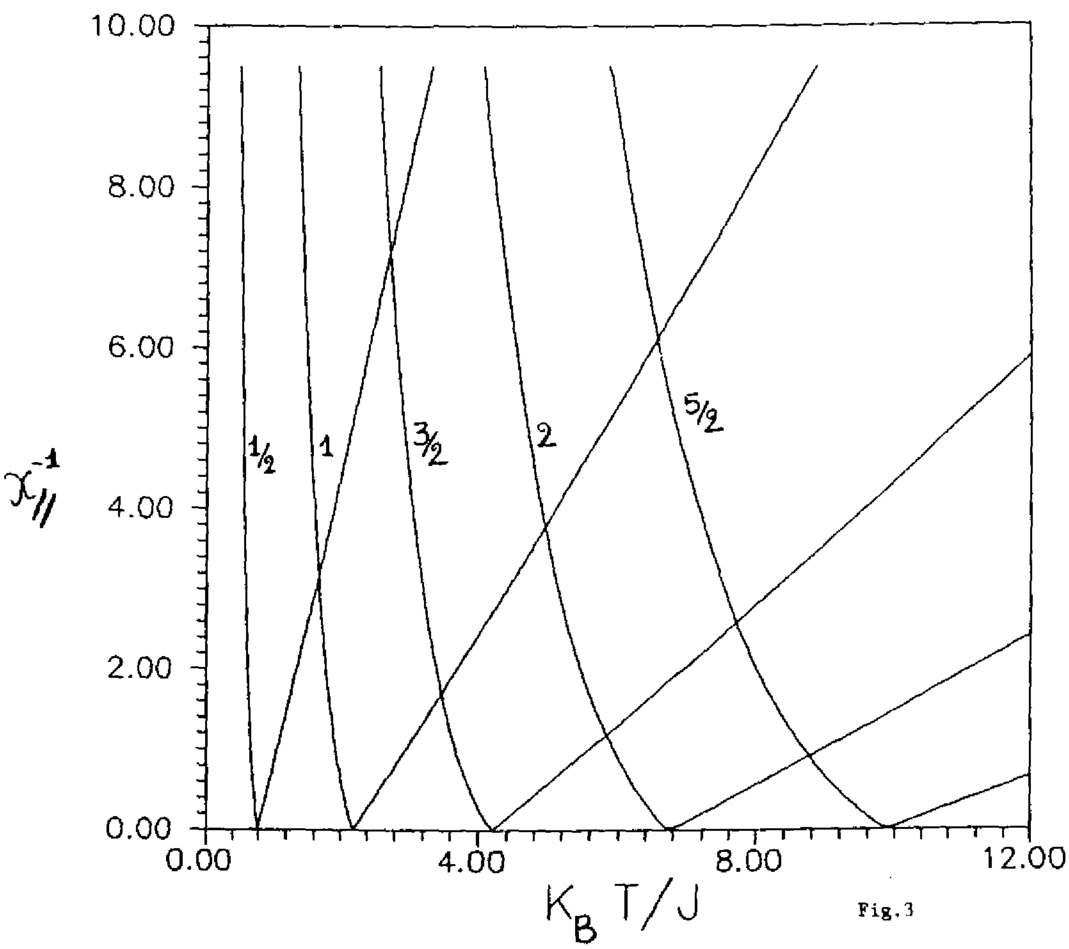
8.00
6.00
4.00
2.00
0.00
0.00
4.00
8.00
12.00
$\chi_{//}$
$K_B T/J$
1/2
1
3/2
2
5/2

Fig.2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1/2
1
3/2
2
5/2

Fig.3

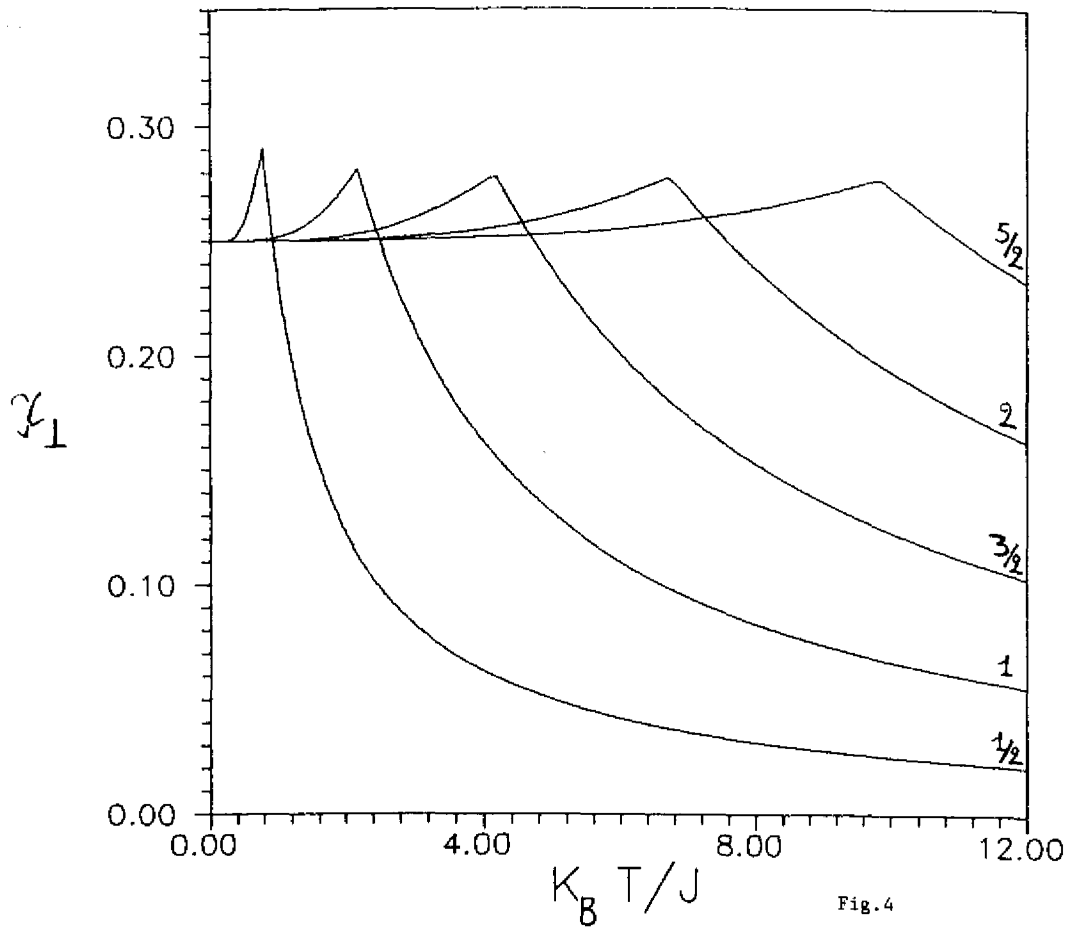

Fig.4