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MATHEMATICS IN COMPUTED TOMOGRAPHY AND RELATED TECHNIQUES

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Computed Tomography (CT) is a nondestructive imaging technique that produces cross-sectional views of an object. Nondestructive evaluation of an object determines its internal structure or property from measurements made outside of it without damaging or altering it in any way. The imaging technique allows the internal structure or property to be presented in the form of an image or a map that presents object morphology in its correct geometry.

The mathematical basis of tomographic techniques was formulated in 1917 by Radon (1). His theorem states that the 2-D function f(x,y) can be determined at all points from a complete set of its line integrals $m_l = \int f(x,y)dl$. In practical implementation, f(x,y) is a physical property and a line integral is a result of an interaction between radiation used for imaging and the substance of which the object is composed. A set of line integrals measured at one angle is called a projection. Tomography can be practiced using different types of radiation, but X- or γ-ray and NMR tomography are the techniques most used until now. In X- or γ-ray transmission tomography, line integrals of photon attenuation in the object under investigation are measured, and a computed CT image is a map of attenuation coefficients in the measured cross section, which can be transformed to a map of density variations.

The current excitement of tomographic imaging started in the early 1970's (Nobel prize for Hounsfield and Cormack in 1972), when Hounsfield constructed an X-ray scanner and obtained the first useful CT image. The breakthrough was to show that, although projection data do not strictly satisfy theoretical models, and very many measurements with fairly complex mathematical operations are needed to construct a CT image, using the efficiently implemented reconstruction algorithms one can get incredibly accurate images. CT images have been dramatically improved since then owing to developments in reconstruction algorithms, and various versions of scanners have been constructed. This can be considered a second phase of CT development (the first was the formulation of the theorem by Radon).

We are presently in the third phase of CT: CT is a standard diagnostic technique in medicine (most hospitals have CT scanners), the technique is spreading in scientific laboratories and industry, and further developments are being made in experimental techniques, equipment construction and mathematical methods. The experimental techniques progress from, on the one hand, constructing relatively inexpensive scanners for a variety of uses, and on the other, to building sophisticated equipment. The latter includes very high resolution detectors (for materials analysis), employing synchrotron radiation sources (for 3-D tomography of small objects), very fast scanning (to image dynamical processes), and complex machines that incorporate such extra features as scanning objects under high pressure or radioactive materials. In both cases, high reconstruction accuracy is required and mathematical/computational techniques have to follow. In the first case, there is a switch to the use of personal computers; in the second case, dedicated fast processors are usually employed. CT techniques also progress towards very high resolution coupled with very high accuracy (i.e., high contrast) imaging, 3-D imaging, and tomography with truncated projection data; this requires not only fast computation techniques capable of handling large amounts of data, but also more precision in calculating CT images and new or improved mathematical approaches. CT using diffracting sources (for example for acoustic and electromagnetic refractive index measurements), an alternative to straight-ray (X- or -ray) tomography, is also being developed.

Modern methods of image reconstruction include three approaches (2-4): (i) ART (algebraic reconstruction technique) with modifications SIRT (simultaneous iterative reconstruction technique) and SART (simultaneous algebraic reconstruction technique), (ii) convolution back-projection, and (iii) the Fourier transform method. There is no one best approach. There is always a compromise between how accurate reconstruction can be and how fast it can be done, and how well experimental data can be approximated by mathematical models. Most scanners employ convolution back-projection methods, using various convolution functions (or so-called convolution kernels that filter the projection data before the back-projection process, in order to obtain a true representation).

The image reconstruction algorithm must be formulated in the geometry of the beam/detector configuration, which is defined by scanner geometry. There are four basic types of scanner geometry, which classify the scanners into four generations. First-generation scanners measure projections consisting of sets of parallel, pencil-beam rays. The use of divergent ray-beams speeds up the process of data collection, and this geometry is used in scanners of higher generations. Second- to fourth-generation scanners employ a fan beam geometry, with various modifications in the source-detector movement. Independent of the scanner geometry, 2-D images created from sets of 2-D data measured in parallel planes can be used for a 3-D reconstruction. A novel approach in 3-D imaging has been proposed (5) that uses a cone beam geometry coupled to a 2-D detector and a single-axis rotational stage, and permits a direct 3-D reconstruction from the data collected in this way; this can be considered fifth-generation scanning.

Because the experimental data do not strictly satisfy theoretical models, a number of effects have to be taken into account, which require mathematical solutions and add to the complexity of the problem. In particular, the problems of beam geometry, finite beam dimensions and distribution (causing partial volume effects), beam scattering, and the radiation source spectrum (multienergy sources causing beam hardening effects) have to be addressed. In high-accuracy, high-resolution CT imaging, the problems of data accuracy, image noise, detectability limits for various types of features, imaging of inhomogeneously distributed multielement materials, etc., also have to be addressed.

Tomography with truncated data is of interest, when it is impossible or undesirable to collect the complete set of data required by the Radon transform of the object. This arises in a variety of cases and forms a field in itself. Mathematical approximations are used to compensate for the unmeasured projection data; otherwise, images are strongly distorted and a clear understanding of the limitations of the reconstructed images is needed. Examples of the use of truncated data tomography are mapping of underground resources via cross-borehole imaging and region-of-interest tomography (i.e., high-resolution imaging of a portion of an object). CT images reconstructed using incomplete data present features in the object with varying degrees of accuracy, depending on which data were missing, and the accuracy of the approximations used to compensate for the unmeasured projection data. To avoid image distortion, when reconstructing CT images from truncated data, one must use some approximations to compensate for the unmeasured projection data. The techniques of incomplete data tomography have recently been applied to calculate laminographic images (6). Laminography provides a tomographic-type image in one plane, using a series of about a dozen radiographic images. Traditionally, the image is measured with film as the imaging medium: the film is exposed to a series of radiographic shots while moving the source and detector in a correlated way, to obtain an image focused in one plane and thus separate overlying features. Computed laminography provides digital laminographic images that are more exact and can be calculated in various planes in the object from one series of digital radiographs.

Finally, because CT data are obtained as numerically measured images, mathematical techniques in image processing and data analysis are extensively used, such as image data filtering, algebraic operations on images, data profiling and statistical calculations.

The CT laboratory at AECL's Chalk River Laboratories practices CT for a variety of nonmedical applications (7, 8, and references therein). The experimental bases are two first-generation and one multidetector (a hybrid of second and third generation) scanners, with Co-60 and Ir-192 used as radiation sources (7, 9). The reconstruction algorithm used for image reconstruction employs a filtered back-projection technique in a pencil-beam geometry. Presently used algorithms have to be adapted to faster computers and a greater amount of data (larger images). Mathematical techniques have been developed to incorporate beam hardening effects for Ir-192 sources (10), and to employ truncated data approximations in cases of high resolution imaging of central parts of objects (region-of-interest tomography, 11). Issues such as detectability limits for detecting small ("point") defects and low-amplitude density gradients, depending on the scan parameters, have been addressed to some extent (12, 13), but these issues require more thorough approaches. Our goal is the construction of a multidetector scanner facility, based on the newest detection technology, with a 2-D imaging detector. Image reconstruction procedures must be developed for the divergent beam and a multienergy spectrum of an X-ray tube. Because we are aiming for CT imaging of high accuracy, such issues of technique limitations as sensitivity, resolution and defect detectability have to be further addressed, and in general should be considered for particular types of scanner configurations, beam conditions and reconstruction algorithms. Image processing is currently performed for 2-D images, using satisfactory image analysis programs. However, the capability to handle large data matrices will be needed for high-resolution tomography, and 3-D image processing is required.

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