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ABSTRACT

We present the results of theoretical investigations of nonhomogeneous fluctuations in submicron active regions of many-valley semiconductors with equivalent valleys (Ge, Si-type), where the dimension $2d$ of the region is comparable to or less than the intervalley diffusion relaxation length L_{iv}. It is shown that for arbitrary orientations of the valley axes (the crystal axes) with respect to lateral sample surfaces, the fluctuation spectra depend on the bias voltage applied to the layer in the region of weak nonheating electric fields. The new physical phenomenon is reported: the fluctuation spectra depend on the sample thickness, with $2d < L_{iv}$ the suppression of fluctuations arises for fluctuation frequencies $\omega \ll \tau_{iv}^{-1}$, τ_{iv}^{-1} is the characteristic intervalley relaxation time.

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1 Introduction

Fluctuations of electrons in semiconductors have received much attention over several decades because of their fundamental importance and applied aspects. For a long time the low-frequency electron fluctuations, such as the flicker and generation-recombination noises, were the focus of the study. However, since the high frequency region is used in experimental investigations and new devices, much attention is paid to fluctuations in the region where the main type of fluctuations for systems in electric fields are the noise of the hot carriers [1]- [10] and the intervalley noise in many-valley semiconductors [10]- [19].

Among other important effects inherent in this type of fluctuations, several exceptional cases are outstanding for which the intrinsic mechanisms of the limitation or the suppression of the noise have been discovered recently [20]-[33]. The suppression of the hot-electron noise in bulk-like samples: compensated semiconductors with a strong scattering of electrons by optical phonons (see theory in Ref. [20], experiment in Ref. [21]), many-valley semiconductors with intensive electron-electron scattering (theory in Ref. [17], §7). Micrometer length diod structures (experiment in Refs. [22], [10], theory in Ref. [23]). Thin submicrometre conductive layers, films, etc.: suppression of the Nyquist noise see in Ref. [24], the ambipolar drift noise in Ref. [17], §8, the hot-electron noise in Refs. [25]-[27]. In nanoscale samples and structures there exists another phenomenon of suppression of the shot noise under ballistic and diffusive quantum transport [28]- [33].

The publications cited above show the fundamental ways of controlling the electron fluctuations and the current noise. Most of these works are focused on III-V compounds. Meanwhile, the silicon remains basic material of microelectronics. Recently [34]-[36], the significant progress has been achieved in the technology of submicrometre Si-based structures and devices, these developments aim high speed and low noise applications. Particularly, in Si-SiGe bipolar transistor the frequency to above 50 GHz has been realized [35], [36].

2 Inter-valley fluctuations: influence of interfaces and boundaries

The fluctuations in many-valley semiconductors due to intervalley transitions have been investigated in a number of theoretical and experimental works [10]- [19]. This type of fluctuations is studied in weak and strong (heating) electric fields in semiconductors with equivalent (Ge, Si-type [11]-[17]) and nonequivalent (GaAs-type [18], [19], [22], [10]) valleys. Though every separate valley is characterized by its proper anisotropy (for instance, partial current density of a valley α, $j_i^{(\alpha)} = \sigma_{ik}^{(\alpha)} E_k$), the total conductivity in the range of nonheating fields and correlation function of the Nyquist fluctuations in many-valley semiconductors with cubic crystal symmetry are isotropic: $\sigma_{ik} = \sum_\alpha^\nu = \sigma\delta_{ik}$, $< \delta J_i \delta J_k >= \delta j^2 \delta_{ik}$, ν is the total number of the valleys. Since in the region of validity of Ohm's low the excess current noise is proportional to the square of the applied external electric field $\boldsymbol{E}$, the intensity of the fluctuations is described by a tensor of fourth rank, which, in general, does not reduce to a scalar value due to the cubic crystal symmetry. Thus, in contrast to the Nyquist noise, the excess intervalley current noise is anisotropic, even if the crystal has cubic symmetry and the heating of the carriers in the electric field is absent. One can obtain the following comparative estimate for the amplitudes of the intervalley (iv) and intravalley (i) noises

$$\frac{(\delta j^2)_\omega^{iv}}{(\delta j^2)_\omega^{i}} \approx \frac{v_d^2 \tau_{iv}}{v^2 \tau_p (1+\omega^2 \tau_{iv}^2)} \approx \frac{E_x^2}{E_0^2 (1+\omega^2\tau_{iv}^2)} \quad . \tag{1}$$

Here τ_{iv}, τ_p are the intervalley and the electron momentum $\vec{p}$ relaxation times, v_d and v are the drift and thermal velocities, respectively; $E_0 = k_0 T / e L_{iv}$ is the diffusion field, $L_{iv} = (D\tau_{iv})^{1/2}$ is the intervalley relaxation length, D is characteristic diffusion coefficient, k_0 is Boltzmann constant, e is the electron charge. According to (1), the Nyquist noise are already dominated by the intervalley current fluctuations in the region of the nonheating fields

$$E_0 \ll E_x \ll E_\epsilon \quad , \tag{2}$$

where E_ϵ is the diffusion heating field, τ_ϵ is the characteristic time of electron energy ϵ relaxation.

Size dependence of the intervalley fluctuation spectra has been found in short-length $n^+ - n - n^+$ structures of $GaAs$, where longitudinal dimension of the structure L_x in direction of the applied electric field E_x is the smallest size of the structure [22], [10]. In [24]-[27], the electron fluctuation spectra have been calculated under transverse size effects where sample thickness $2d$ is the smallest size (in the y direction) of the structure: $2d < L_\epsilon$ [25]-[27], $2d < L_p$ [24], L_ϵ and L_p are the electron energy and momentum relaxation lengths. The influence of sample thickness and boundary conditions on the intervalley fluctuation spectra has not been considered.

In this work we investigate theoretically the fluctuations in size restricted crystals of many- valley semiconductors with equivalent valleys. We show that for thin samples with $2d < L_{iv}$ the intensities and characteristic frequency of the fluctuations are essentially modified in comparison with those in an infinite many-valley crystal.

For theoretical description of the problem we use the Langevin's procedure introducing into the kinetic equation for fluctuation of one-particle distribution function the microscopic stochastic forces, which correspond to the intravalley (i) and intervalley (iv) scattering of electrons.

Qualitative distinction of fluctuations in size restricted and infinite samples is as follows. In the restricted sample the fluctuations are nonhomogeneous because of the difference between the volume and surface intervalley relaxation rates. The fluctuation spectra have to be determined with taking into account the transverse electric fields, which arise in the conditions involved, and the Langevin surface sources of fluctuations in addition to the volume ones. Besides, the relative part of different volume sources into spectral densities of fluctuations is to be varied depending on sample thickness $2d$. Actually, the intravalley stochastic sources generate such the fluctuations, which are corresponded the fluctuations of the local electron density in individual valley $\delta n_\alpha(\boldsymbol{r},t)$ related to random space flow of electrons $\tilde{\boldsymbol{I}}_\alpha^i(\boldsymbol{r},t)$. For those we can write $\delta n_\alpha^i \sim \tau_{iv}(d\tilde{I}_{\alpha y}^i/dy)$.

Quite the contrary, the fluctuations due to intervalley transitions δn_α^{iv} are generated by the appropriate stochastic source $\tilde{I}_\alpha^{iv}$ homogeneously (in average over sample volume) and therefore those are to be proportional to the value $\tau_{iv}\tilde{I}_\alpha^{iv}$. Relative contribution from both sources into spectral densities of fluctuations averaged over the volume V_0 of the sample is determined by

$$\gamma = \frac{(\delta n_\alpha^2)_\omega^i}{(\delta n_\alpha^2)_\omega^{iv}} \quad . \tag{3}$$

As the appropriate correlation functions $< \tilde{I}_{\alpha k}^i \tilde{I}_{\alpha l}^i > \sim D_{kl}^{(\alpha)}$, $< \tilde{I}_\alpha^{iv} \tilde{I}_\alpha^{iv} > \sim \tau_{iv}^{-1}$, we can estimate the parameter γ in (3) putting $\tilde{I}_{\alpha y}^i \sim D^{1/2}$, $\tilde{I}_\alpha^{iv} \sim \tau_{iv}^{-1/2}$, $(d\tilde{I}_{\alpha y}^i/dy) \sim (\tilde{I}_{\alpha y}^i/l)$, where l denotes the characteristic scale of the fluctuations. Then, from (3) we get $\gamma \approx L_{iv}^2/l^2$. It is evident that for thin crystal ($2d \ll L_{iv}$) we have $l \approx 2d$. If $2d \gg L_{iv}$, the fluctuations δn_α^i generated by the random space electron flows are to be influenced by intervalley transitions for the length scale greater than L_{iv}. As a result, for thick sample the characteristic length scale of fluctuations are determined by the effective "outlined" length $l = (2dL_{iv})^{1/2}$. From this estimate it follows that the fluctuations in the thick sample are mainly due to the intervalley random sources $\tilde{I}_\alpha^{iv}$ because, in accordance with (3), the parameter $\gamma \approx L_{iv}/2d \ll 1$. For thin sample $\gamma \approx L_{iv}^2/(2d)^2 \gg 1$, thus, in this case the fluctuations are mainly due to the intravalley stochastic sources $\tilde{\boldsymbol{I}}_\alpha^i$. With $2d \approx L_{iv}$, both types of the random sources are of the same order of value, hence, they have to be involved simultaneously in the theory. Note that the contribution into intervalley fluctuations from intravalley stochastic sources has been usually ignored in theoretical works.

Another interesting peculiarity of intervalley fluctuations in restricted samples is modification of the characteristic frequency and the spectral intensities of fluctuations with the decreasing of the sample thickness. If the intervalley surface scattering rate S is sufficiently strong $S \gg D/d$, the redistribution of the intensity of fluctuations occurs over the spectrum frequencies, i.e. the decreasing for low frequencies $\omega\tau_{iv} \ll 1$ and the increasing in the range of more higher frequencies.

All the mentioned qualitative features of intervalley fluctuations in size restricted samples are entirely related to sample boundaries and strong surface intervalley relaxation rate. These features are characteristic of the size restricted samples and do not have to be manifested itself in a bulk-like crystal, where the surface does not practically influence the electrophysical and fluctuative properties of the crystal.

3 The model, basic equations and boundary conditions

3.1 The model and basic equations

Time evolution of spatially inhomogeneous quasineutral fluctuations of the carrier density in individual valleys is determined from the set of coupled stochastic continuity equations

$$\frac{\partial}{\partial t}\delta n_\alpha(\boldsymbol{r},t) + div\delta \boldsymbol{i}^\alpha(\boldsymbol{r},t) = \sum_{\beta\neq\alpha}\left(\frac{\delta n_\alpha(\boldsymbol{r},t)}{\tau_{\alpha\beta}} - \frac{\delta n_\beta(\boldsymbol{r},t)}{\tau_{\beta\alpha}}\right) + \tilde{I}^{iv}_\alpha(\boldsymbol{r},t), \tag{4}$$

where $\alpha = 1 \div \nu$,

$$\delta \boldsymbol{i}^\alpha(\boldsymbol{r},t) = -\mu^\alpha_{kl}(E_l\delta n_\beta(\boldsymbol{r},t) + n_\alpha\delta E_l(\boldsymbol{r},t) - D^\alpha_{kl}\frac{\partial}{\partial x_l}\delta n_\alpha(\boldsymbol{r},t)) + \tilde{\boldsymbol{I}}^i_{\alpha l}(\boldsymbol{r},t) \tag{5}$$

is fluctuation of density of the partial particle flow. Standard symbols for electron transport coefficients are used here. Equations (4), (5) contain different Langevin's sources of fluctuations due to intervalley and intravalley scattering of electrons:

$$\tilde{I}^{iv}_\alpha(\boldsymbol{r},t) = \sum_p \chi^{iv}_{\alpha p}(\boldsymbol{r},t), \tag{6}$$

$$\tilde{\boldsymbol{I}}^i_\alpha(\boldsymbol{r},t) = \sum_p \boldsymbol{v}\tau_p\chi^i_{\alpha p}(\boldsymbol{r},t). \tag{7}$$

Here $\chi^{i,iv}_{\alpha p}$ are the stochastic microscopic forces with known correlation properties arising in the Boltzman-Langevin equation [4], [16], [17]. For these random forces the following

identities take place

$$\sum_p \chi^i_{\alpha p}(\boldsymbol{r},t) = 0, \qquad \sum_{\alpha=1}^{\nu}\sum_p \chi^{iv}_{\alpha p}(\boldsymbol{r},t) = 0, \tag{8}$$

which mean the conservation of partial and total densities of the electrons with respect to intravalley and intervalley scattering, respectively.

The equations (4)-(7) correspond to low-frequency and long-range fluctuations of the hydrodynamic type, which require the validity of the criteria:

$$\omega\tau_p \ll 1, \quad l, L_{iv} \gg L_p. \tag{9}$$

The quasineutrality equations completing the equations (4), (5)

$$\sum_{\alpha=1}^{\nu}\delta n_\alpha(\boldsymbol{r},t) = 0, \qquad \sum_{\alpha=1}^{\nu} n_\alpha = \nu n_0 = N \tag{10}$$

demand the additional limitation, in comparison with (9), for the fluctuation frequency and space scale

$$\omega\tau_p \ll 1, \quad l_D \ll l, L_{iv}, 2d, \quad \tau_M \ll \tau_{iv}, \tag{11}$$

where τ_M is Maxwell's relaxation time, l_D is Debay's screening length, and N is the total carrier density.

We consider the plate-shaped sample of thickness $2d = L_y$ in the y-direction which is the smallest size of the sample. The lateral dimensions of the sample are taken to be considerably large than its thickness $2d < L_{iv} \ll L_x \ll L_z$. The appropriate geometry of the sample [37] - [39] is shown in the figure 1 . The equations (4) can be averaged over the xz plane. After that the problem involved becomes one-dimensional as all the quantities in (4)-(7) depend only on coordinate y. We suppose the external dc electric field to be applied in the x direction. The internal transverse electric field in the y direction arising under the sample geometry considered is to be found from Maxwell's equations

$$rot\ \boldsymbol{E} = 0, \quad div\ \boldsymbol{j} = 0. \tag{12}$$

Assuming the external electric circuit in the y direction to be opened on dc and ac current and taking into account the quasineutrality equation (10), we can find the fluctuative field δE_y from the condition of vanishing the transverse fluctuative current density $\delta i_y = \sum_\alpha \delta i^\alpha_y$:

$$\delta i_y(y,t) = 0. \tag{13}$$

Note that under these assumptions we can also put $\delta E_{x,z} = 0$.

Further it is convenient to use the Fourier transform for the equations (4)-(7), as a result we have

$$-i\omega\delta n_\alpha(y,\omega)+\frac{d}{dy}\delta i_y^\alpha(y,\omega)=-\sum_{\beta\neq\alpha}\left(\frac{\delta n_\alpha(y,\omega)}{\tau_{\alpha\beta}}-\frac{\delta n_\beta(y,\omega)}{\tau_{\beta\alpha}}\right)+\tilde{I}_\alpha^{iv}(y,\omega). \qquad (14)$$

3.2 The boundary conditions

The boundary conditions to equations (14) are

$$\delta i_y^\alpha(y=\pm d,\omega)=\pm\sum_{\beta\neq\alpha}\left(S_{\alpha\beta}^\pm\delta n_\alpha^\pm-S_{\beta\alpha}^\pm\delta n_\beta^\pm\right)\mp\tilde{u}_\alpha^\pm(\omega), \qquad (15)$$

where we introduced the intervalley surface scattering rates S^+ and the Langevin's surface sources of intervalley fluctuations $\tilde{u}_\alpha^\pm(\omega)$:

$$S_{\alpha\beta}^\pm=\pm\int_{\pm(d-\delta)}^{\pm d}\frac{dy}{\tau_{\alpha\beta}(y)},\qquad \tilde{u}_\alpha^\pm(\omega)=\pm\int_{\pm(d-\delta)}^{\pm d}\tilde{I}_\alpha^{iv}(y,\omega)dy. \qquad (16)$$

Here we used the following designation $f^\pm=F(y=\pm d)$. The problem of the boundary conditions to the kinetic equation for distribution function is a fundamental and extraordinary complex theoretical task. This problem has been discussed, for instance, in [40] - [42], using the quantum microscopic approach. The analogous solution of this problem for the fluctuations is out of the aim of our paper. Here, the most appropriate way to derive the boundary conditions to the equations (14) is as follows. Let us consider the model of a surface layer in the figure 2. We integrate the volume equations (14) over an extremely thing surface layer of thickness δ, where $l_p\ll\delta\ll 2d$. We suppose the intervalley relaxation time within the surface layer involved to be much greater than in the rest volume out of the layer. As a result, we obtain the equations (15) and the expressions (16) which are the boundary conditions to the equations (14).By using the correlation relations [4], [16] for the stochastic sources (6), (7), one can obtain for two valleys

$$(\tilde{u}_1\tilde{u}_2)_\omega^\pm=-\frac{4dn_0S^\pm}{V_0}. \qquad (17)$$

Thus, the spectral density of the Langevin's surface sources can be expressed in terms of the surface rates of intervalley relaxation, which are the parameters of the stationary problem. Modification of the fluctuation spectra has to be the most significant for strong intervalley surface scattering of the electrons

$$S^\pm\gg\frac{D}{d}. \qquad (18)$$

Then we can apply to the equations (15) iteration procedure after that the equations are reduced to

$$\delta n_\alpha(y=\pm d,\ \omega)\ =\ 0. \qquad (19)$$

Boundary conditions of the form of (19) have been used in [43]. Note that criterion (18) is directly opposite to the criterion of Refs. [37], [38]: $S^\pm=0$. It makes possible to neglect here the nonhomogeneity of stationary distribution of the carriers in the valleys (the domains), which can arise in thin samples.

3.3 Solution of stochastic continuity equations

Let us consider the two-valley model which is illustrated in figure 2. Using this model, we can write for the relative fluctuation $\delta f=\delta n_1/n_0$ the following equations

$$\hat{\mathcal{L}}[\delta f(\zeta,\omega)]=\tilde{\Phi}(\zeta,\omega), \qquad (20)$$

$$\delta f(\zeta=\zeta_0,\omega)=0 \qquad (21)$$

Here the operator in the left-hand-side and the function in the right-hand-side of the equation (20) are

$$\hat{\mathcal{L}}\equiv i\omega\tau+\frac{d^2}{d\zeta^2}+a_1(\vartheta)\mathcal{E}\ \frac{d}{d\zeta}-1, \qquad (22)$$

$$\tilde{\Phi}(\zeta,\omega)\equiv\alpha[L_{iv}^{-1}\frac{d}{d\zeta}\ \tilde{I}^i(\zeta,\omega)-\tilde{I}^{iv}(\zeta,\omega)]. \qquad (23)$$

The effective stochastic sources of the fluctuations are determined by the expressions

$$\tilde{I}^{iv}(\zeta,\omega)\equiv\tilde{I}_1^{iv}(\zeta,\omega), \qquad (24)$$

$$\tilde{I}^i(\zeta,\omega)=\tilde{I}_y^-(\zeta,\omega)+a\sin 2\vartheta\tilde{I}_y^+(\zeta,\omega),\quad \tilde{I}_k^\pm=\frac{1}{2}(\tilde{I}_{1k}^i\pm\tilde{I}_{2k}^i). \qquad (25)$$

In (22)-(25) the following designations are used

$$a_1(\vartheta)=\frac{a^2}{2}\sin 4\vartheta, \qquad (26)$$

$\zeta=y/L_{iv},\quad \mathcal{E}=E_x/E_0,\quad \alpha=L_{iv}^2/n_0D,\quad D=D_{xx}^{(1)}(1-a^2\sin^2 2\vartheta),\quad a=D_{xy}^{(1)}/D_{xx}^{(1)},$ $\tau=\tau_{12}/2,\quad \tau_{12}=\tau_{21}$.

The solution of equations (20), (21) can be written as

$$\delta f(\zeta,\omega)=\int_{-\zeta_0}^{\zeta_0}\tilde{\Phi}(\zeta,\omega)\ G_\omega(\zeta,\zeta')\ d\zeta', \qquad (27)$$

where $G_\omega(\zeta,\zeta')$ is the Green's function of the operator (22) with zero boundary conditions. To find $G_\omega(\zeta,\zeta')$ we consider two different regions $\zeta'>\zeta$ and $\zeta'<\zeta$:

$$G_\omega^>(\zeta,\zeta')\equiv G_\omega(\zeta>\zeta',\zeta'),\quad G_\omega^<(\zeta,\zeta)\equiv G_\omega(\zeta<\zeta',\zeta'). \qquad (28)$$

Substituting (28) into formula (27) we rewrite it in the form

$$\delta f(\zeta,\omega)=\int_{-\zeta_0}^{\zeta}\tilde\Phi(\zeta',\omega)\;G^{>}_{\omega}(\zeta,\zeta')d\zeta'+\int_{\zeta}^{\zeta_0}\tilde\Phi(\zeta',\omega)\;G^{<}_{\omega}(\zeta,\zeta')d\zeta'. \tag{29}$$

For Green's functions (28) we obtain the expressions:

$$G^{>}_{\omega}(\zeta,\zeta')=\frac{\exp\left[k_{(+)}(\zeta-\zeta')\right]}{k_{(-)}\sinh\left[2k_{(-)}\zeta_0\right]}\sinh\left[k_{(-)}(\zeta-\zeta_0)\right]\;\sinh\left[k_{(-)}(\zeta'+\zeta_0)\right], \tag{30}$$

$$G^{<}_{\omega}(\zeta,\zeta')=\frac{\exp\left[k_{(+)}(\zeta-\zeta')\right]}{k_{(-)}\sinh\left[2k_{(-)}\zeta_0\right]}\sinh\left[k_{(-)}(\zeta+\zeta_0)\right]\;\sinh\left[k_{(-)}(\zeta'-\zeta_0)\right], \tag{31}$$

where $k_{(\pm)}=(k_1\pm k_2)/2$,

$$k_{(+)}=-\frac{1}{2}a_1(\vartheta)\mathcal{E},\qquad k_{(-)}=\sqrt{\frac{a_1^2(\vartheta)}{4}\mathcal{E}^2+1-i\omega\tau}. \tag{32}$$

Finally, we can write

$$\delta f(\zeta,\omega)=k_{(-)}^{-1}[\mathcal{F}(\zeta,\zeta_0)+\mathcal{F}(\zeta,-\zeta_0)], \tag{33}$$

where for the function $\mathcal{F}(\zeta,\zeta_0)$ we have the following expression:

$$\mathcal{F}(\zeta,\zeta_0)=\frac{\sinh\left[k_{(-)}(\zeta-\zeta_0)\right]}{\sinh\left[2k_{(-)}\zeta_0\right]}\int_{-\zeta_0}^{\zeta}\tilde\Phi(\zeta',\omega)\exp\left[k_{(+)}(\zeta-\zeta')\right]\sinh\left[k_{(-)}(\zeta'+\zeta_0)\right]. \tag{34}$$

In order to compare the contributions of different Langevin's sources (6) and (7) in the spectral densities of fluctuations, next we will consider those separately. As a result, we have

$$\delta f(\zeta,\omega)=\delta f^{i}(\zeta,\omega)+\delta f^{iv}(\zeta,\omega). \tag{35}$$

For these two terms we find the following expressions:

$$\delta f^{i,iv}(\zeta,\omega)=\frac{exp\left[k_{(+)}\zeta\right]}{k_{(-)}}\Big[\int_{\zeta_0}^{\zeta}\tilde I^{i,iv}(\zeta',\omega)\;F^{i,iv}(-\zeta,\zeta',\omega)d\zeta'+ \tag{36}$$

$$\frac{\sinh\left[k_{(-)}(\zeta_0-\zeta)\right]}{\sinh\left[2k_{(-)}\zeta_0\right]}\int_{-\zeta_0}^{\zeta_0}\tilde I^{i,iv}(\zeta',\omega)F^{i,iv}(\zeta_0,\zeta',\omega)d\zeta'\Big],$$

where the direct calculation of the function $F^{i,iv}$ gives

$$F^{iv}(\zeta,\zeta',\omega)=exp\left[-k_{(+)}\zeta'\right]\sinh\left[k_{(-)}(\zeta+\zeta')\right], \tag{37}$$

$$F^{i}(\zeta,\zeta',\omega)=\frac{\partial}{\partial\zeta'}F^{iv}(\zeta,\zeta',\omega).$$

¿From these results we can calculate, by using the expressions (35)-(37), the spectral densities of fluctuations of the valley carrier density, current density, transverse voltage in the sample and determine those frequency, field and size dependences.

4 Intervalley near-equilibrium fluctuations

One of the peculiarities of fluctuations in restricted samples is spatial inhomogeneity of this fluctuations. Therefore, it is necessary to average the spectral densities of the fluctuating quantities over the sample thickness

$$(\delta j_i\delta j_k)_\omega=\frac{1}{(2d)^2}\int_{-d}^{d}dy_1\int_{-d}^{d}dy_2\;(\delta j_i(y_1)\delta j_k(y_2))_\omega. \tag{38}$$

Using the following expressions for the current density

$$\delta j_x=-2e\Big[\tilde I_x^{+}-an_0D(\mathcal{E}\sin(2\vartheta)+cos(2\vartheta)\frac{d}{d\zeta})\delta f\Big] \tag{39}$$

and the fluctuative transverse electric field

$$\delta\mathcal{E}_y=\frac{L_{iv}}{n_0D}\tilde I_y^{+}+a(\sin(2\vartheta)\frac{d}{d(\zeta)}-cos(2\vartheta)\mathcal{E})\delta f, \tag{40}$$

we find the expression for the spectral density of fluctuations of the current density in the restricted sample

$$S_j(\omega,E_x)=1+a^2\sin^2(2\vartheta)\mathcal{E}^2S_f(\omega,E_x). \tag{41}$$

Here we introduce the adimensional quantities

$$S_j(\omega,E_x)=\frac{(\delta j_x\delta j_x)_\omega(E_x)}{(\delta j^2)^{\infty}_{\omega=0}},\quad S_f(\omega,E_x)=\frac{(\delta f\delta f)_\omega(E_x)}{(\delta f^2)^{\infty}_{\omega=0}}, \tag{42}$$

where $(\delta f^2)^{\infty}_{\omega=0}=\tau/NV_0$, and $(\delta j^2)^{\infty}_{\omega=0}=2e^2ND/V_0$ are the spectral densities of equilibrium fluctuations of the valley carrier density and the current density for infinite crystal, respectively.

The intervalley fluctuations can be studied experimentally by investigating besides the excess current noise, the fluctuations of the transverse bias voltage

$$\delta U=\int_{-d}^{d}\delta E_y(y)\;dy. \tag{43}$$

Using (40),(43), we obtain the expression for the appropriate spectral density

$$S_{\mathcal{U}}(\omega,E_x)=\frac{(\delta\mathcal{U}\delta\mathcal{U})_\omega(E_x)}{(\delta f^2)^{\infty}_{\omega=0}},\;S_{\mathcal{U}}(\omega,E_x)=1+a^2\cos^2(2\vartheta)\mathcal{E}^2S_f(\omega,E_x), \tag{44}$$

where $\delta\mathcal{U}\equiv\delta U/2dE_0$ is the adimensional transverse voltage.

Important peculiarity of excess current noise related to decreasing of sample thickness arises due to dependence of the spectral density $S_f(\omega,E_x)$ (the second term in (42)) on the external electric field. This statement follows from the fact that for an arbitrary orientation of the valleys relative to lateral sample surfaces the electric field enters in the equation (20). As a result, the spectral density is decreased with the increasing the electric field because of the drift to the surfaces and destruction of the fluctuations due to strong surface intervalley relaxation. For the valley's orientations corresponding $\vartheta = 0,\ \pi/4$ the electric field drops from equation (20). In these cases the intervalley fluctuations are identical to ones under the thermal equilibrium conditions in the range of electric fields (2). Therefore, this situation has to be analyzed.

Setting $E_x = 0$, we find the equilibrium local spectral density $(\delta f(\zeta_1)\delta f(\zeta_2))_\omega$ and average it over the sample thickness. For the averaged spectral density $S_f(\omega,\zeta_0) \equiv S_f(\omega, E_x = 0)$ we obtain

$$S_f(\omega,\zeta_0) = \mathcal{K}(\omega,\zeta_0) = \mathcal{K}^{iv}(\omega,\zeta_0) + \mathcal{K}^{i}(\omega,\zeta_0). \tag{45}$$

Here functions $\mathcal{K}^{iv,i}(\omega,\zeta_0)$ give the frequency and size dependences of the equilibrium intervalley spectra

$$\mathcal{K}^{iv}(\omega,\zeta_0) = \frac{1}{1+\omega^2\tau^2}\left[1 + \frac{1}{\cosh(\xi_1)+\cos(\xi_2)}\left(\frac{\sinh(\xi_1)}{\xi_1} + \frac{\sin(\xi_2)}{\xi_2} - 4\frac{\xi_1\sinh(\xi_1)+\xi_2\sin(\xi_2)}{\xi_1^2+\xi_2^2}\right)\right], \tag{46}$$

$$\mathcal{K}^{i}(\omega,\zeta_0) = \frac{1}{(1+\omega^2\tau^2)^{1/2}}\frac{1}{\cosh(\xi_1)+\cos(\xi_2)}\left(\frac{\sinh(\xi_1)}{\xi_1} - \frac{\sin(\xi_2)}{\xi_2}\right), \tag{47}$$

where $\xi_1 = 2\zeta_0 Re(k_{(-)})$, $\xi_2 = 2\zeta_0 Im(k_{(-)})$.

Let us analyze, in short, the expression (45). The contribution related to the second term is entirely determined by the sample boundaries and disappears if $d \to \infty$ $(\zeta_0 \to \infty)$: $\mathcal{K}_i \to 0$, $\mathcal{K}_{iv} \to (1+\omega^2\tau^2)^{-1}$. For the range of low frequencies $\omega\tau \ll 1$ we find

$$\mathcal{K}^{iv}(\omega=0,\zeta_0) = \frac{3}{2}\left(1 - \frac{1}{3}\tanh^2(\zeta_0) - \frac{\tanh(\zeta_0)}{\zeta_0}\right), \tag{48}$$

$$\mathcal{K}^{i}(\omega=0,\zeta_0) = \frac{1}{2}\left(\frac{\tanh(\zeta_0)}{\zeta_0} + \tanh^2(\zeta_0) - 1\right).$$

These give the following size dependence of the low-frequency spectrum

$$S_f(\omega=0,\zeta_0) = 1 - \frac{\tanh(\zeta_0)}{\zeta_0}. \tag{49}$$

For $\zeta_0 \gg 1$ the main contribution to the spectral density is due to intervalley scattering because we have $\mathcal{K}^i \approx 1/(2\zeta_0)$, $\mathcal{K}^{iv} \approx 1$, i.e. $\mathcal{K}^{iv} \gg \mathcal{K}^i$. For $\zeta_0 \ll 1$ we obtain that

$\mathcal{K}^{iv} \ll \mathcal{K}^i$, consequently, size dependence of the spectral density is mainly determined by the transverse fluctuative electron flows:

$$S_f(\omega=0,\zeta_0) = \frac{1}{3}\zeta_0^2\,. \tag{50}$$

Thus, frequency dependence of spectral density of fluctuations is determined for $\zeta_0 \gg 1$ by the $\tilde{I}^{iv}$-source and for $\zeta_0 \ll 1$ – by the $\tilde{I}^{i}$-source.

The most interesting feature of spectra (45)-(48) is the low-frequency spectral density of fluctuations for restricted sample can be much less than for an infinite crystal. That follows directly from the formula (50) and also is illustrated by numerical calculations in figure 3 and figure 4. We also calculate the field dependences of low-frequency spectral density of fluctuations of the current density which are presented in figure 5. As is seen from this figure, the significant decreasing of the fluctuation intensity takes place. In the range of high frequencies $\omega\tau \gg 1$ the dispersion of fluctuation spectra can be varied from $\approx (\omega\tau)^{-2}$ to $\approx (\omega\tau)^{-3/2}$.

5 Conclusion

We have shown that fluctuation features of the electron gas in submicron structures of many-valley semiconductors are significantly different from the ones for infinite crystal. In particular, under Ohm's low conditions the fluctuation spectra of the valley carrier density depend on the applied electric field. The intensity of fluctuations depends on the sample thickness, for thin samples it is significantly suppressed due to strong surface intervalley scattering of electrons. These results indicate the way to control the intervalley current noise in submicron structures in the base of many-valley semiconductors, where the different interfaces play an important role.

6 Acknowledgments

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Figure captions

Figure 1 a, b. (a) Geometry of size restricted sample with $2d = L_y \ll L_x \ll L_z$; (b) Orientation of valleys relative to the lateral sample faces in two-valley model.

Figure 2. The surface layer with a strong rate of intervaley relaxation.

Figure 3. The dependences of spectral density of low-frequency fluctuations of the carrier valley density on the sample thicknesses ζ_0 : 1, $\mathcal{K}_1(\omega,\zeta_0)$; 2, $\mathcal{K}_2(\omega,\zeta_0)$; 3, $S_f(\omega,\zeta_0) = \mathcal{K}_1(\omega,\zeta_0) + \mathcal{K}_1(\omega,\zeta_0)$.

Figure 4. Frequency dependences of spectral density of fluctuations of the carrier valley density for different sample thicknesses ζ_0 : 1, 0.5; 2, 1.0; 3, 2.0; 4, 3.0; 5, 5.0; 6, ∞ .

Figure 5. Field dependences of low-frequency spectral density of fluctuations of the current density for different sample thicknesses ζ_0 : 1, 0.5; 2, 1.0; 3, 2.0; 4, 3.0; 5, 5.0; 6, ∞ .

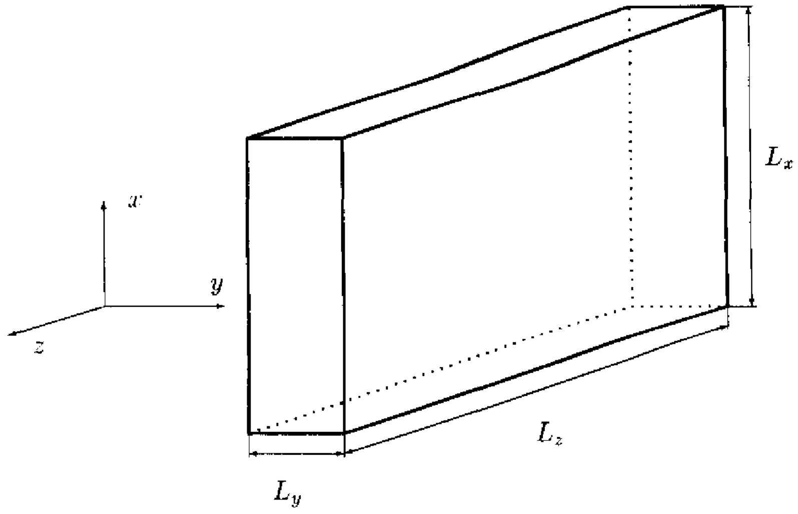

Fig.1a

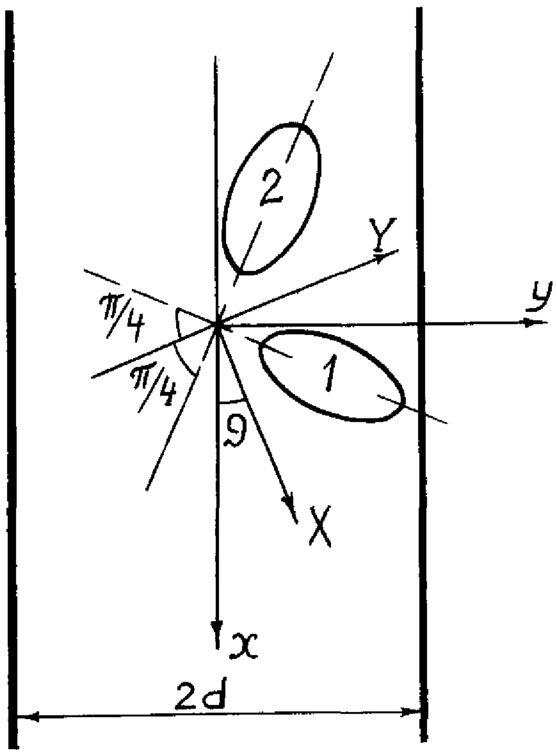

Fig.1b

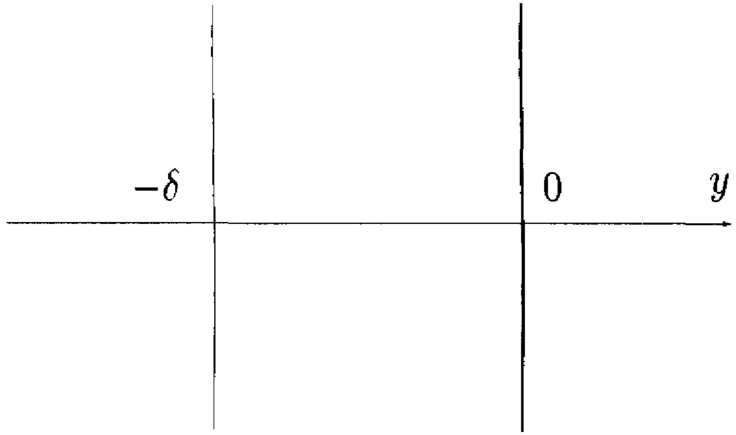

Fig.2

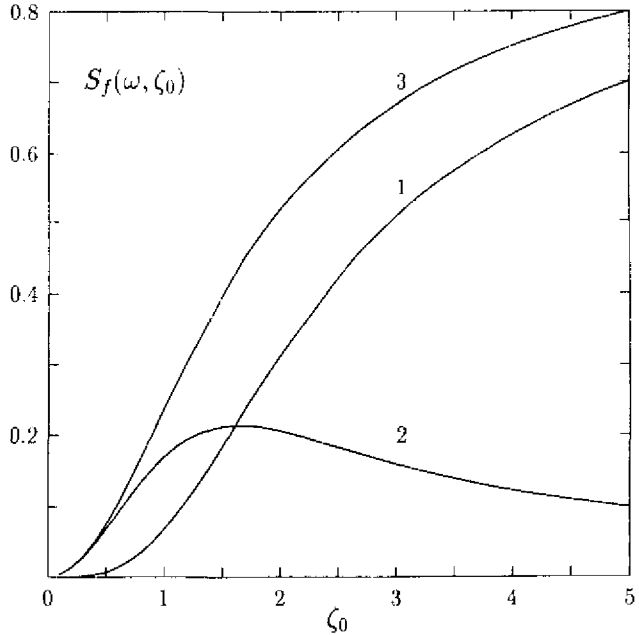

Fig.3

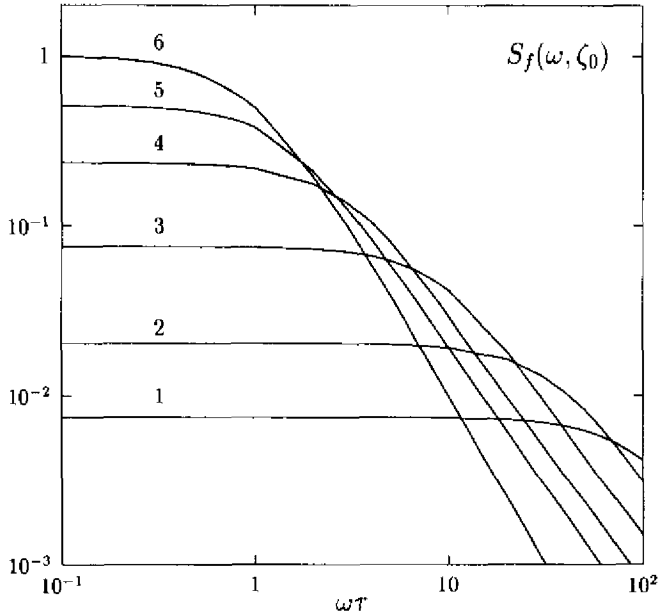
$S_f(\omega, \zeta_0)$
1
10^{-1}
10^{-2}
10^{-3}
10^{-1}
1
10
10^2
$\omega\tau$
6
5
4
3
2
1

Fig.4

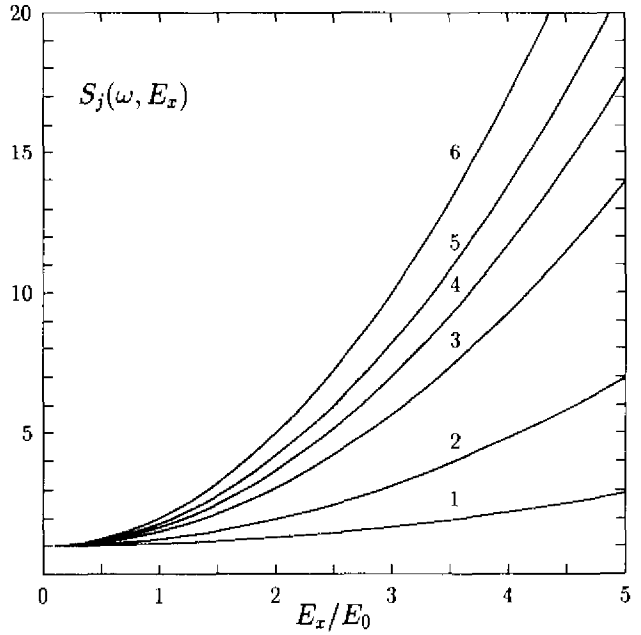
$S_j(\omega, E_x)$
20
15
10
5
0
1
2
3
4
5
E_x/E_0
6
5
4
3
2
1

Fig.5