IC/95/256

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ABSTRACT

We calculate electric current caused by electron tunneling between two-dimensional layers in the magnetic field applied perpendicular to the layers. An elastic scattering of the electrons is taken into account. Analytical results are obtained for two regimes: i) small magnetic field, when the Landau quantization is suppressed by the scattering and the oscillatory part of the current shows nearly harmonic behavior, ii) high magnetic field, when the Landau levels are well-defined and the conductivity shows series of sharp peaks corresponding to resonant magnetotunneling. In the last case, we used two alternative approaches: self-consistent Born approximation and path integral method, and compared obtained results.

MIRAMARE – TRIESTE
August 1995

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1. Introduction. General formalism.

Tunneling of the electrons between barrier-separated two-dimensional (2D) electron layers is currently under examination in double quantum well systems (DQW's) [1,2]. Application of the magnetic field both perpendicular and parallel to the layers changes the energy spectrum of the electrons and thereby modifies the tunneling. In this paper we present calculation of the tunnel current in DQW's in the perpendicular magnetic field H. The electron spectrum in this case is described by two sets of Landau levels, these sets originate from the size quantization subbands of the left- (l) and right- (r) wells. According to the energy conservation requirements, the probability of tunneling must oscillate with the magnetic field and transverse bias (which controls the splitting energy Δ between the size quantization levels). When the optical phonon-assisted tunneling dominates, the tunneling rate shows giant magnetophonon oscillations [3], with peak positions determined by the condition $\Delta - (n'-n)\hbar\omega_c = \hbar\omega_0$, where ω_0 is the longitudinal optical (LO) phonon frequency, ω_c is the cyclotron frequency, and n, n' are the numbers of Landau levels participating in the transition. The magnetophonon oscillations have been observed in photocurrent through the multiple quantum well system subjected to a high magnetic field [4]. However, in most experimental setups, the electron temperature, level splitting, and difference between the Fermi levels in the wells are smaller than the optical phonon energy and this tunneling mechanism is not important. Instead, an elastic scattering-assisted tunneling takes place. In high enough magnetic field, when the Landau levels are well-defiued, this tunneling has maximum probability in the resonant magnetotunneling conditions:

$$\Delta = \hbar\omega_c(n - n'). \quad (1)$$

The aim of this paper is to calculate the magnetotunneling current in the elastic scattering regime. Coulomb interaction between the electrons is neglected. [This interaction may be important in some cases [5,6], but we think that it is useful to develop the one-particle theory prior to the many-particle one]. We consider weak tunnel coupling, when the value of the tunneling matrix element T is small in comparison with the broadening of the Landau levels due to the scattering. In this case, we can describe each well as a separate sub-system with quasi-equilibrium distribution of the electrons, and the tunnel current between the wells is calculated in the lowest order of T^2. Two existing experimental setups are under consideration. The first implies a contactless (free-standing) DQW's excited by a short laser pulse. The rate of tunneling relaxation between the wells is measured by the time-resolved photoluminescence technique (see Ref.2 for example). General expression of the tunneling rate ν is given by

$$\nu = \frac{2\pi T^2}{\hbar S} \int d\varepsilon f_l(\varepsilon) \int d\mathbf{x} \int d\mathbf{x}' \left\langle G^l_\varepsilon(\mathbf{x},\mathbf{x}') G^r_\varepsilon(\mathbf{x}',\mathbf{x}) \right\rangle \times \left[\int d\varepsilon f_l(\varepsilon) \left\langle G^l_\varepsilon(\mathbf{x},\mathbf{x}) \right\rangle \right]^{-1}, \qquad (2)$$

where $G^j_\varepsilon(\mathbf{x},\mathbf{x}')$ are the Green functions of the electrons in the wells, $f_j(\varepsilon)$ are the distribution functions, $\mathbf{x}$ is the in-plane coordinate, S is the normalization area and $\langle ....\rangle$ means statistical averaging. In Eq.(2) we have assumed that the electrons tunnel from l to r well and neglected the opposite current, supposing that r well is depopulated. As a rule, the electrons in these experiments are nondegenerate and the distribution function can be taken in the form $f_j(\varepsilon) \sim \exp(-\varepsilon/T_e)$, where T_e is the effective electron temperature.

In the second kind of experiments, the DQW's with separate contacts to each well are investigated [1]. This setup allows to measure the density j of tunneling current directly. General expression of the current is

$$j = \frac{4\pi e T^2}{\hbar S^2} \int d\varepsilon [f_l(\varepsilon) - f_r(\varepsilon)] \int\int d\mathbf{x} d\mathbf{x}' \left\langle G^l_\varepsilon(\mathbf{x},\mathbf{x}') G^r_\varepsilon(\mathbf{x}',\mathbf{x}) \right\rangle, \qquad (3)$$

where e is the electric charge. In these experiments, the electron gas is degenerate and the integral over ε must be taken in the interval between the r-well Fermi energy ε_{Fr} and the l-well one ε_{Fl}.

In the following we consider a simple case when the scattering potentials of the layers are uncorrelated. Then we have

$$\int\int d\mathbf{x} d\mathbf{x}' \left\langle G^l_\varepsilon(\mathbf{x},\mathbf{x}') G^r_\varepsilon(\mathbf{x}',\mathbf{x}) \right\rangle = S \int d\mathbf{x} G^l_\varepsilon(\mathbf{x}) G^r_\varepsilon(\mathbf{x}), \qquad (4)$$

where $G^j_\varepsilon(\mathbf{x})$ is the translationaly invariant part of the casual one-particle Green function of electrons in j-th well. It can be written through the Green function $G^j_\varepsilon(n)$ of the Landau level representation as

$$G^j_\varepsilon(\mathbf{x}) = \sum_n g_n(|\mathbf{x}|) G^j_\varepsilon(n), \qquad (5)$$

$$g_n(x) = \frac{1}{2\pi a_H^2} \exp\left(-\frac{x^2}{4a_H^2}\right) L_n^0\left(\frac{x^2}{2a_H^2}\right), \qquad (6)$$

where L^0_n are the Laguerre polynomials and $a_H = \sqrt{\hbar c/eH}$ is the magnetic length. Using well-known properties of L^0_n, we also obtain

$$\int d\mathbf{x} G^l_\varepsilon(\mathbf{x}) G^r_\varepsilon(\mathbf{x}) = \frac{S}{2\pi a_H^2} \sum_{n=0}^{\infty} G^l_\varepsilon(n) G^r_\varepsilon(n), \qquad (7)$$

When there is also a magnetic field $H_{\|}$ applied parallel to the layers, the right-hand side of Eq.(7) is modified as

$$\frac{S}{2\pi a_H^2} \sum_{n=0}^{\infty} \sum_{n'=0}^{\infty} 2^{n'-n} \frac{n!}{n'!} e^{-2\beta^2} \beta^{2(n'-n)} \left[L_n^{n'-n}(2\beta^2) \right]^2 G^l_\varepsilon(n) G^r_\varepsilon(n'), \qquad (8)$$

where $\beta = H_{\|} Z/(2H a_H)$ is the dimensionless parameter associated with the parallel magnetic field and Z is the distance between the centers of the wave functions in the wells. We can see that the parallel field mixes Landau levels of different wells.

In spite of already done approximations, calculation of the tunneling current still remains a difficult problem, because it requires knowledge of the electron Green functions in the magnetic field. In the following sections we examine limits of high and low magnetic fields, when calculation of the tunneling current can be carried out analytically and present a brief discussion of the obtained results.

2. Case of low magnetic fields.

In this section we assume that the cyclotron energy is small in comparison with the Fermi energy (or effective temperature) of the electrons, so that many Landau levels participate in the tunneling transitions:

$$\hbar\omega_c \ll \varepsilon_{Fl},\ T_e. \qquad (9)$$

We look for the casual Green functions in the form

$$G^l_\varepsilon(n) = \frac{1}{\pi} \frac{\Gamma_l(\varepsilon)}{\Gamma_l(\varepsilon)^2 + (\varepsilon - \varepsilon_n)^2}, \qquad (10)$$

$$G^r_\varepsilon(n) = \frac{1}{\pi} \frac{\Gamma_r(\varepsilon)}{\Gamma_r(\varepsilon)^2 + (\varepsilon + \Delta - \varepsilon_n)^2}, \qquad (11)$$

where $\varepsilon_n = \hbar\omega_c(n + 1/2)$, and Γ_j are the imaginary parts of the left- and right-well self-energies, which are found self-consistently from the equations [7]

$$\Gamma_l = \frac{\hbar}{2\tau_l} \frac{\sinh(2\pi\Gamma_l/\hbar\omega_c)}{\cosh(2\pi\Gamma_l/\hbar\omega_c) + \cos(2\pi\varepsilon/\hbar\omega_c)}, \qquad (12)$$

$$\Gamma_r = \frac{\hbar}{2\tau_r} \frac{\sinh(2\pi\Gamma_r/\hbar\omega_c)}{\cosh(2\pi\Gamma_r/\hbar\omega_c) + \cos[2\pi(\varepsilon+\Delta)/\hbar\omega_c]}, \tag{13}$$

where τ_l and τ_r are the scattering times in the wells in the absence of the magnetic field. In Eqs.(10)-(13) we have neglected dependence of Γ_j on the Landau level number, which is correct in two situations: the first is the case of short-range correlated scattering (when this dependence is really vanishes), and the second is realized when the difference between the Fermi energies is small enough and the electrons are tunneling in a narrow interval of energy. In this last situation we should treat the imaginary part of the self-energy as corresponding to a Landau level near the Fermi level, and τ_j as the scattering times in the Fermi surface. Also, we have neglected the real parts of the self-energies because they lead only to an unimportant renormalization of the zero-point of energy.

After substitution of (10) and (11) in Eq.(7), we can calculate the sum over n, extending the lower limit of summation to $-\infty$, which is correct under condition (9). In the same way as in description of magnetooscillatory phenomena in solids, we use the Poisson's rule of summation and obtain the current as follows:

$$j = \frac{2eT^2m}{\hbar^4\pi}\int_{\varepsilon_{Fl}}^{\varepsilon_{Fr}} d\varepsilon \frac{2\Delta^2\Gamma_l\Gamma_r(\tau_l+\tau_r)}{[\Delta^2+(\Gamma_l+\Gamma_r)^2][\Delta^2+(\Gamma_l-\Gamma_r)^2]}\left\{1+\frac{\tau_l-\tau_r}{\tau_l+\tau_r}\frac{\Gamma_r^2-\Gamma_l^2}{\Delta^2}\right.$$
$$\left.+\frac{2\tau_l\Gamma_l}{(\tau_l+\tau_r)\Delta}\frac{\sin(2\pi\varepsilon/\hbar\omega_c)}{\sinh(2\pi\Gamma_l/\hbar\omega_c)} - \frac{2\tau_r\Gamma_r}{(\tau_l+\tau_r)\Delta}\frac{\sin[2\pi(\varepsilon+\Delta)/\hbar\omega_c]}{\sinh(2\pi\Gamma_r/\hbar\omega_c)}\right\} \tag{14}$$

(m is the effective electron mass). This equation describes the oscillating current, which depends on the magnetic field H, level splitting Δ, positions of the Fermi levels in the wells, and on the characteristics of the scattering. It is important to notice that this current is also sensitive to the scattering asymmetry (it means $\tau_l \neq \tau_r$), while at $H = 0$ this dependence vanishes.

In order to make an analytical calculation of the integral over energy in Eq.(14), we consider the case $\exp[-\pi/(\omega_c\tau_j)] \ll 1$, i.e., small magnetic fields. In this case, the oscillating part of the current appears as a small additional contribution to the background current. We also assume symmetric scattering ($\tau_l = \tau_r = \tau$). The current is calculated as

$$j = j_0 + j_0\exp\left(-\frac{\pi}{\omega_c\tau}\right)\frac{\hbar\omega_c}{2\pi(\varepsilon_{Fl}-\varepsilon_{Fr})}\left\{\delta^{-1}\left(\cos\left[\frac{2\pi(\varepsilon_{Fl}+\Delta)}{\hbar\omega_c}\right]\right.\right.$$
$$\left. - \cos\left[\frac{2\pi(\varepsilon_{Fr}+\Delta)}{\hbar\omega_c}\right] - \cos\left[\frac{2\pi\varepsilon_{Fl}}{\hbar\omega_c}\right] + \cos\left[\frac{2\pi\varepsilon_{Fr}}{\hbar\omega_c}\right]\right) - \frac{2}{1+\delta^{-2}}\left(\sin\left[\frac{2\pi(\varepsilon_{Fl}+\Delta)}{\hbar\omega_c}\right]\right.$$

$$\left.\left. - \sin\left[\frac{2\pi(\varepsilon_{Fr}+\Delta)}{\hbar\omega_c}\right] + \sin\left[\frac{2\pi\varepsilon_{Fl}}{\hbar\omega_c}\right] - \sin\left[\frac{2\pi\varepsilon_{Fr}}{\hbar\omega_c}\right]\right)\right\} \tag{15}$$

where $\delta = \Delta\tau/\hbar$ and j_0 is the current at $H = 0$, which can be expressed through the 2D density of states $\rho_{2D} = m/(2\pi\hbar^2)$ and tunneling rate ν_0 at $H = 0$:

$$j_0 = 2e\rho_{2D}\nu_0(\varepsilon_{Fl}-\varepsilon_{Fr}), \quad \nu_0 = \frac{2T^2\tau}{\hbar^2}\frac{1}{1+\delta^2} \tag{16}$$

(note that ν_0 and j_0 show a resonance at $\Delta = 0$ corresponding to the resonant tunneling). An expression similar to (15) can be derived for the tunneling rate ν in nondegenerate case. Neglecting higher-orger contributions on $(\hbar\omega_c/T_e)$, we obtain

$$\nu = \nu_0 + \nu_0\exp\left(-\frac{\pi}{\omega_c\tau}\right)\frac{\hbar\omega_c}{2\pi T_e}\left\{\delta^{-1}\left(1-\cos\left[\frac{2\pi\Delta}{\hbar\omega_c}\right]\right) + \frac{2}{1+\delta^{-2}}\sin\left[\frac{2\pi\Delta}{\hbar\omega_c}\right]\right\}. \tag{17}$$

This rate shows oscillations with Δ and $\hbar\omega_c$. The phase of these oscillations (in similar way as in Eq.(15)) depends on the level splitting Δ.

To finish this section, we evaluate the tunneling conductance G in the DQW's with separate contacts. This value is defined as $G = (dj/dV)$, where $V = (\varepsilon_{Fl} - \varepsilon_{Fr})/e$ is the applied voltage. We consider two important cases: a) ohmic conductance, when $V \to 0$, $\varepsilon_{Fl} \simeq \varepsilon_{Fr} = \varepsilon_F$, and b) conductance in symmetric DQW's with matched electron densities [1], when $eV = \Delta$ and $\varepsilon_{Fl} = \varepsilon_{Fr} + \Delta = \varepsilon_F$. The expressions are the following:

$$G = 2e^2\rho_{2D}\nu_0\left\{1+\exp\left(-\frac{\pi}{\omega_c\tau}\right)\left[\delta^{-1}\left(\sin\left[\frac{2\pi\varepsilon_F}{\hbar\omega_c}\right] - \sin\left[\frac{2\pi(\varepsilon_F+\Delta)}{\hbar\omega_c}\right]\right)\right.\right.$$
$$\left.\left. - \frac{2}{1+\delta^{-2}}\left(\cos\left[\frac{2\pi\varepsilon_F}{\hbar\omega_c}\right] + \cos\left[\frac{2\pi(\varepsilon_F+\Delta)}{\hbar\omega_c}\right]\right)\right]\right\} \tag{18}$$

for the case (a) and

$$G = G_0 + G_1, \quad G_0 = 2e^2\rho_{2D}\nu_0(1-\delta^2)/(1+\delta^2),$$

$$G_1 = 2e^2\rho_{2D}\nu_0\exp\left(-\frac{\pi}{\omega_c\tau}\right)\cos\left(\frac{2\pi\varepsilon_F}{\hbar\omega_c}\right)\left\{-\frac{2}{\delta}\sin\left(\frac{2\pi\Delta}{\hbar\omega_c}\right) - \frac{4}{1+\delta^{-2}}\cos\left(\frac{2\pi\Delta}{\hbar\omega_c}\right)\right.$$
$$\left. + \frac{4\hbar\omega_c}{\pi\Delta}\sin\left(\frac{2\pi\Delta}{\hbar\omega_c}\right)\frac{1-\delta^{-2}}{(1+\delta^{-2})^2} - \frac{\hbar\omega_c}{\pi\Delta\delta}\left[1-\cos\left(\frac{2\pi\Delta}{\hbar\omega_c}\right)\right]\frac{3+\delta^{-2}}{1+\delta^{-2}}\right\} \tag{19}$$

for the case (b). In both cases value of G depends on the position of the Fermi energy ε_F, because it determines the density of states for the tunneling electrons. Fig.1 shows dependence of the conductance on the applied voltage in the case (b), calculated for $H = 0.2$ T, $\hbar/\tau$ =0.17 meV (conditions of the experiment described in [1]), and the Fermi energy is chosen in the way to give $\cos(2\pi\varepsilon_F/\hbar\omega_c) = -1$, i.e. maximum amplitude of the oscillations. A transition from the non-oscillating ($H = 0$, dashed line) to the oscillating ($H \neq 0$, solid) regime is clear. We note that behavior G versus V is very similar to the one experimentally observed in [1] and [8].

3. Case of high magnetic field.

In this section we consider the case when the Landau levels are well-defined, i.e. the Landau level broadening is small in comparison with the cyclotron energy. Since it requires rather high magnetic fields (of order 10 T), we will assume that the electrons are all situated in the lowest Landau level of the left well. In order to avoid many-body effects, such as in the fractional Hall regime, we assume that the temperature or Fermi energy are large enough so that the lowest Landau level is nearly uniformly occupied. We also assume that the right well ("collector") is unpopulated. The current j in these conditions is equal to $en\nu$, where n_l is the electron concentration in the left well. So, we will consider the tunneling rate only.

Description of the one-particle Green function in the high magnetic field can be based on two different approaches. The first is path- integral approach, which has been used in Ref.9 (see also [10]). The second is the self-consistent Born approximation (SCBA) developed in application to 2D systems by Ando (see references in [7,10]). Below we calculate the tunnel current using these approaches and compare the obtained results.

I. The path-integral method

This method means exact expression of the Green function as

$$G^l_\varepsilon(\mathbf{x}) = \frac{1}{\hbar}\int_0^\infty dt \exp\left(\frac{i}{\hbar}\varepsilon t\right)\int_{\mathbf{x}_0=0}^{\mathbf{x}_t=\mathbf{x}} \mathcal{D}[\mathbf{x}_\tau]\exp\left\{\frac{i}{\hbar}\int_0^t d\tau L(\mathbf{x}_\tau, \dot{\mathbf{x}}_\tau) - \frac{1}{2\hbar^2}\int_0^t\int_0^t d\tau d\tau' W_l(|\mathbf{x}_\tau - \mathbf{x}_{\tau'}|)\right\} + c.c., \tag{20}$$

$$L(\mathbf{x}, \dot{\mathbf{x}}) = \frac{m}{2}\dot{\mathbf{x}}^2 + \frac{e}{2}\mathbf{H}[\mathbf{x}\times\dot{\mathbf{x}}],$$

where $\mathcal{D}[\mathbf{x}_\tau]$ implies functional integral over all the paths coming from the point $\mathbf{x}_0 = 0$ to $\mathbf{x}_t = \mathbf{x}$, and the random potential correlator is defined as $W_j(|\mathbf{x}|) = \langle U_j(\mathbf{x})U_j(0)\rangle$. [Expression for $G^r_\varepsilon(\mathbf{x})$ differs by index change $l \to r$ and $\varepsilon \to \varepsilon + \Delta$ in Eq.(20), as before].

However, further evaluation of this expression may be done only in some special cases. In the following, we consider one of such cases, when the characteristic scales of the disorder potential (correlation lengths l_l and l_r) are large in comparison with the Landau orbit radius:

$$l_l,\ l_r \gg a_H \tag{21}$$

In the lowest order on the disorder smoothness, when the potential correlators $W_j(|\mathbf{x}|)$ in the exponent is just replaced by the constant $W_j = W_j(0)$, Green function $G^l_\varepsilon(n)$ is given by

$$G^l_\varepsilon(n) = \frac{1}{\sqrt{2\pi W_l}}\exp\left[-\frac{(\varepsilon - \varepsilon_n)^2}{2W_l}\right], \tag{22}$$

($G^r_\varepsilon(n)$ may be written in similar way). However, to describe tunneling between the Landau levels with different numbers, we should search for $G^r_\varepsilon(n)$ in a more elaborate way. We expand the potential part of the exponential term under the path integral from in series on $\left[\int_0^t d\tau \int_0^t d\tau W_r(|\mathbf{x}_\tau - \mathbf{x}_{\tau'}|) - W_r t^2\right]/2\hbar^2$ up to the first order. After such a transformation, the path integral can be exactly calculated in the way similar to the one described in [11] for the 3D systems at $\mathbf{H} = 0$. Substituting a Gaussian correlator $W_r(x) = W_r\exp(-x^2/l_r^2)$, we perform successive calculation of the integrals over coordinate $\mathbf{x}$ and time t and obtain

$$\int d\mathbf{x}\exp\left(-\frac{x^2}{4a_H^2}\right)G^r_\varepsilon(\mathbf{x}) \simeq \frac{1}{\sqrt{2\pi W_r}} \times\left\{\exp\left[-\frac{(\varepsilon + \Delta - \hbar\omega_c/2)^2}{2W_r}\right] + \frac{W_r}{(\hbar\omega_c)^2}\sum_{n=1}^\infty \frac{1}{n^2} \times\left(2\frac{a_H^2}{l_r^2}\right)^n \exp\left[-\frac{(\varepsilon + \Delta - \hbar\omega_c(n+1/2))^2}{2W_r}\right]\right\}. \tag{23}$$

In this equation we have neglected small values of order $2a_H^2/l_r^2$ everywhere except in the last term, which contains contribution from all $r-$well Landau levels. This term describes coupling between the lowest Landau level in the $l-$well [it is situated at the energy $\varepsilon \simeq \hbar\omega_c/2$] and the $n-th$ ($n \geq 1$) Landau level in the $r-$well. Such coupling occurs under the resonant magnetotunneling conditions, see Eq.(1). Now we can calculate the tunneling rate [12]:

$$\nu = \nu_0\left(\frac{\Delta}{\hbar\omega_c}\right)^2\sum_{n=1}^\infty \exp\left[-\frac{(\Delta - \hbar\omega_c n)^2}{2(W_l + W_r)}\right]$$

$$\times\frac{1}{n^2}\left(2\frac{a_H^2}{l_r^2}\right)^n, \quad \nu_0 = \frac{\sqrt{2\pi}W_rT^2}{\hbar\sqrt{W_l+W_r}\Delta^2}, \tag{24}$$

where we have neglected the exponentially small contribution arisen from the first term in the right-hand part of (23). Remaining sum describes series of the symmetrical peaks caused by the resonant magnetotunneling.

Fig.2 illustrates dependence of ν/ν_0 on the dimensionless cyclotron energy $\hbar\omega_c/\Delta$, given by Eq.(24). Calculation is done at $W_r = W_l$ with use of the following dimensionless parameters: $2\hbar^2/(ml_r^2\Delta)$ =0.05, $\sqrt{W_r}/\Delta$ =0.03. Several peaks shown here correspond to the resonances described by Eq.(1) with n =0, n' =1,2,3,4,5. The amplitudes of the peaks decrease with the increasing n' rather fast, because the transitions between the Landau levels in conditions (21) are rapidly suppressed with the increase of $n'-n$. The relaxation rate ν also shows peaks as a function of the splitting energy Δ.

II. The self-consistent Born approximation

The SCBA implies solution of the Dyson diagrammatic equation, which is written in the Landau level representation as

$$G_\varepsilon^{j\ R,A}(n) = G_\varepsilon^{j(0)}(n) + G_\varepsilon^{j(0)}(n)G_\varepsilon^{j\ R,A}(n)\sum_{n'}\Phi^j_{n,n'}G_\varepsilon^{j\ R,A}(n'), \tag{25}$$

where $G^{l(0)}(n) = (\varepsilon-\varepsilon_n)^{-1}$ and $G^{r(0)}(n) = (\varepsilon+\Delta-\varepsilon_n)^{-1}$ are the Green functions of free motion, R, A means retarded or advanced Green functions,

$$\Phi^j_{nn'} = \int\frac{d\mathbf{q}}{(2\pi)^2}|Q_{nn'}(q)|^2W_j(q), \tag{26}$$

$$|Q_{nn'}(q)|^2 = \frac{n!}{n'!}\exp[-(qa_H)^2/2]\left[\frac{(qa_H)^2}{2}\right]^{n'-n}\left[L_n^{n'-n}\left(\frac{(qa_H)^2}{2}\right)\right]^2, \tag{27}$$

and $W_j(q)$ are the Fourier transforms of the scattering potential correlators $W_j(|\mathbf{x}|)$. The Dyson equation is obtained as a result of a partial summation of the diagrams, and it is not exact. However, it is rather easy to solve this equation in case of high H considered here. If we do not take into account "interaction" between the different Landau levels, we obtain $G^j_\varepsilon(n)$ as semi-elliptical peaks [10] centered near the positions of proper Landau levels. In order to describe the tunneling between the Landau levels with different numbers, we should calculate $G^j_\varepsilon(n)$ taking into account all terms in the sum of n in (25). This

calculation is made by iterations on the small parameters of order $\Phi^j_{nn'}/\hbar\omega_c$. The result is given by (here $\Phi_n \equiv \Phi_{nn}$)

$$G^l_\varepsilon(n) = \frac{1}{\pi}\theta(2\sqrt{\Phi^l_n}-|\varepsilon_n-\varepsilon|)\left\{\sqrt{\frac{1}{\Phi^l_n}-\frac{(\varepsilon-\varepsilon_n)^2}{4\Phi^{l2}_n}}+\frac{1}{2\Phi^l_n}\frac{\varepsilon_n-\varepsilon}{\sqrt{4\Phi^l_n-(\varepsilon_n-\varepsilon)^2}}\right.$$
$$\left.\times\sum_{n'}\frac{\Phi^l_{nn'}}{\varepsilon_{n'}-\varepsilon_n}\right\}+\frac{1}{\pi}\sum_{n'\neq n}\frac{\Phi^l_{nn'}}{(\varepsilon_n-\varepsilon_{n'})^2}\sqrt{\frac{1}{\Phi^l_{n'}}-\frac{(\varepsilon-\varepsilon_{n'})^2}{4\Phi^{l2}_{n'}}}\theta(2\sqrt{\Phi^l_{n'}}-|\varepsilon_{n'}-\varepsilon|), \tag{28}$$

($G^r_\varepsilon(n)$ is described in analogous way) and we see that a number of smaller semi-elliptical peaks arises in addition to the main peak. The tunneling rate is expressed as

$$\nu = \frac{2T^2}{\hbar^3\omega_c^2}\sum_{n=1}^{\infty}\frac{\Phi^r_{0n}}{\sqrt{\Phi^r_n n^2}}\int d\varepsilon\sqrt{1-\frac{\varepsilon^2}{4\Phi^l_0}}\sqrt{1-\frac{(\varepsilon+\Delta-n\hbar\omega_c)^2}{4\Phi^r_n}}$$
$$\times\left[\int d\varepsilon\sqrt{1-\frac{\varepsilon^2}{4\Phi^l_0}}\right]^{-1} \tag{29}$$

The limits of integrals in this equation are determined by the requirement that the expressions under square roots must be positive. In the same way as Eq.(24), Eq.(29) shows peaks in the resonant magnetotunneling conditions (1). The oscillatory picture is presented in Fig.3. In the calculation we used a model of short-range scattering potentials symmetrically distributed across the DQW's, i.e. $W_j(q) = w$ and $\Phi^j_{nn'} = \Phi = w/(2\pi a_H^2)$. The tunneling rate is expressed in units ν_1,

$$\nu_1 = \frac{2T^2}{\hbar^2}\sqrt{\frac{2wm}{\pi^3\Delta^3}}, \tag{30}$$

and we have taken $\sqrt{2wm/(\pi\hbar^2\Delta)} = 0.07$ for numerical calculation. In contrast to the case shown in Fig.2, the shape of the peaks is not Gaussian. Also, decrease of the peak heights with the decrease of the magnetic fields proceeds slower than in Fig.2, because the scattering model is different. However, if we take result of the SCBA in the limit (21) and compare the ratio of heights for two neighboring peaks, we find that it is the same and equal to $(1/n^2)(a_H/l_r)^{2n}$ for both models. As to the absolute values of the peak heights, it appears that SCBA gives the heights which differ from the heights calculated in path integral formalism by a numerical factor $16/(3\pi^{3/2}) \simeq 0.96$, i.e., very close to 1. We conclude that SCBA may be applied for calculation of peak heights very well. On the other hand, SCBA is not good in description of the edges of the density of states, and the magnetotunneling peaks, following the density of states, show non-physically sharp edges.

In conclusion, we have presented an analytical calculation of the elastic scattering-assisted magnetotunneling between the weakly coupled 2D gases and demonstrated oscillating behavior of the tunneling relaxation rate as a function of the magnetic field, splitting energy, and (in case of low H) of Fermi energy. At low H, when the Landau quantization is suppressed by the scattering, the oscillations appear as a small correction to the background current and can be considered by the methods usually used in description of the other magnetooscillatory phenomena (for example, Shubnikov-de Haas oscillations). The peak positions does not follow condition (1), but are determined also by the other factors, such as position of the Fermi energy. In high H, when the Landau levels are well-defined, peaks appear in the resonant magnetotunneling conditions (1). Electron transitions between the Landau levels with different numbers in this case become possible only due to the scattering. As a result, the peak heights are small according to small parameter: ratio of the Landau level broadening energy to the cyclotron energy. If a sufficiently high parallel magnetic field is applied to the system, the mixing of the Landau levels with different numbers is possible without scattering (see Eq.(8)), and this small parameter disappears. However, the shape of the peaks will remain as one in the absence of the parallel field, because this shape is determined solely by the properties of the electron Green function in the perpendicular magnetic field.

Acknowledgment - This work has been supported in part by the Joint Fund of the Government of Ukraine and International Science Foundation, Grant No U65200.

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Fig.1. Dependence of the tunneling conductance of on the applied bias. Dashes: H=0, solid: H=0.2 T.

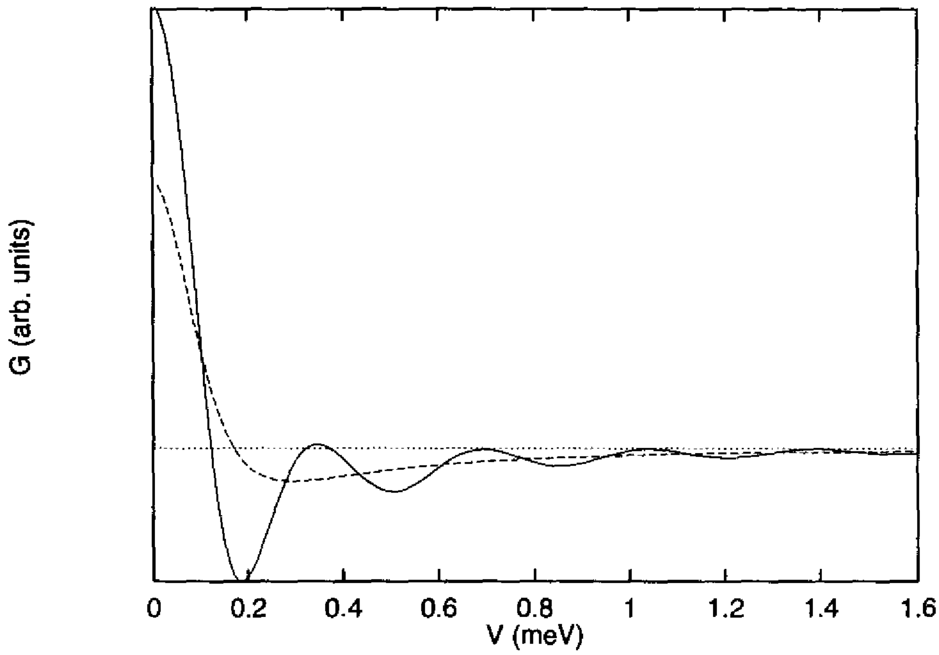


Fig.2. Tunneling rate versus the magnetic field in case of long-range scattering potential.

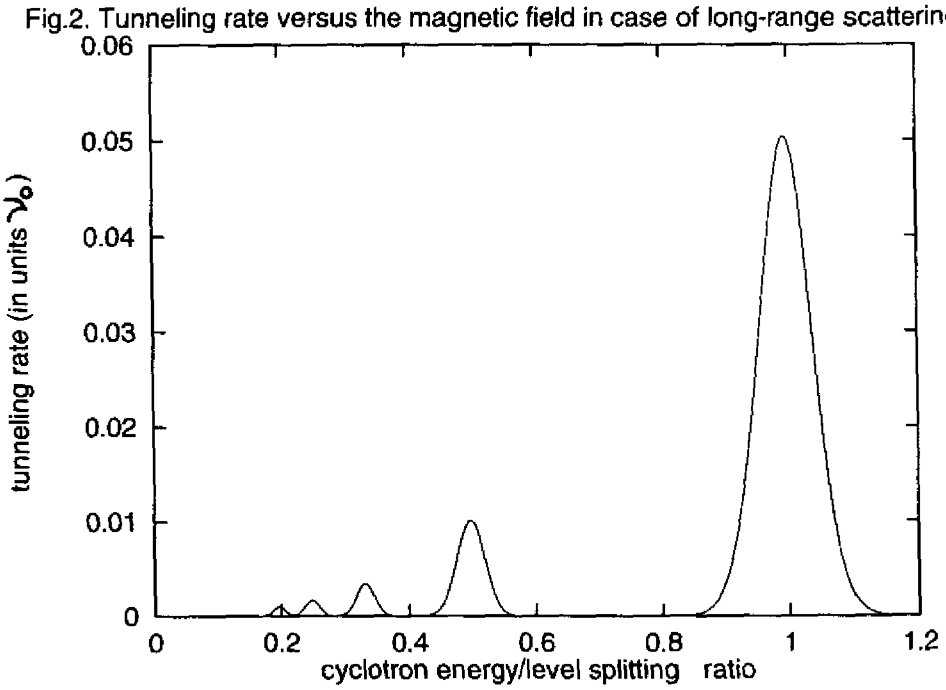
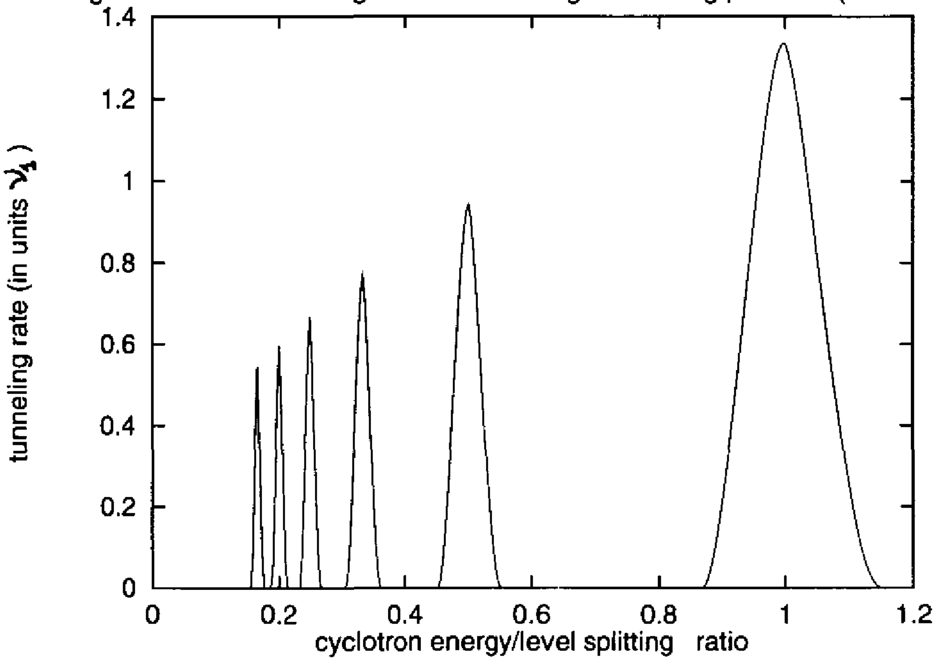


Fig.3. The same as in Fig.2 for a short-range scattering potential (result of SCBA).



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