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ARE PENGUINS BLACK-and-WHITE ?

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Abstract

Contributions of low-energy "eye" and "figure-eight" quark diagrams to the $K \to \pi$ weak transitions are studied in a hadron- level phenomenological approach. It is shown that these contributions may be estimated by considering meson-cloud effects. If all intermediate mesons under consideration are degenerate only the "eye" (low-energy penguin) diagram is nonvanishing. When allowance is made for smaller mass of pseudoscalar mesons, the contribution of "figure-eight" diagrams turns out to enhance the $\Delta I = \frac{1}{2}$ (suppress the $\Delta I = \frac{3}{2}$) amplitudes naturally. The overall long-distance-induced enhancement of the ratio of the $\Delta I = \frac{1}{2}$ amplitudes over the $\Delta I = \frac{3}{2}$ amplitudes is estimated at around 4-8.

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1 Introduction

After almost 40 years since the discovery of the $\Delta I = 1/2$ rule in strangeness-changing weak hadronic decays, its origin still eludes our understanding (for a recent review see ref.[1]). Dominance of the $\Delta I = 1/2$ amplitudes over those with $\Delta I = 3/2$ requires a significant enhancement of the former and/or suppression of the latter. While for nonleptonic baryon decays at least part of the effect stems from the Pati-Woo theorem [2], according to which the symmetry of baryon wave functions ensures vanishing of the $\Delta I = 3/2$ amplitude, no such symmetry-based mechanism is available for kaon decays.

The required effects can be obtained to some extent from perturbative QCD. Short-distance QCD corrections modify the effective weak Hamiltonian and lead to an enhancement of the $\Delta I = 1/2$ (suppression of the $\Delta I = 3/2$) operators [3]. In addition, a new purely $\Delta I = 1/2$ mechanism - the so-called penguin operator - appears. Its contributions add constructively to those of standard $\Delta I = 1/2$ operators. Detailed studies [4] show, however, that the original claim of a large penguin contribution is incorrect. This contribution remains small even if one takes into account the increase, over the value quoted in ref.[5], of the real part of the penguin Wilson coefficient due to the incomplete GIM cancellation above the charm quark mass [6].

Dropping the so-called Fierz contributions (which has been argued to be justified in the 1/N expansion, [7]) does help a little, but a large discrepancy still remains [1]. In fact, for consistency with the spirit of the $1/N$ expansion, the Fierz terms should be considered along with nonfactorizable terms of the same order. Starting from an effective chiral Lagrangian, such subleading $1/N$ contributions have been calculated in ref.[8] as nonfactorizable pseudoscalar meson loop corrections to $K \to 2\pi$. Their contribution has been found to be of the same order as that of the genuine factorizable terms. In ref.[1] the following effects are mentioned as contributing to the subleading terms of the $1/N$ approach: the Fierz-transformed contributions, final state-interactions, low energy "eye" graphs, and soft gluon exchanges between two quark loops in "figure-eight" graphs. Because of the long-distance nature of the last three mechanisms, their evaluation from the first principles of QCD is possible on the lattice only. In practice it is the $K \to \pi$ matrix elements that are more amenable to such calculations. From these, the $K \to 2\pi$ amplitudes are then obtained by means of current algebra. Within very large statistical and systematic uncertainties the lattice calculations[9] support the $\Delta I = 1/2$ enhancement and indicate that the purely $\Delta I = 1/2$ "eye" graphs dominate over the "figure-eight" graphs.

The contribution from the "eye" and "figure-eight" graphs of the quark-level description must be contained in the meson-cloud (or unitarity) effects of the hadron-level (as these include all confinement effects, see also ref.[10]). In fact, it has been found repeatedly by many authors that such meson cloud effects are very important in many areas of hadron physics, improving the predictions of the standard quark model. For a unitarity-oriented view of hadron spectroscopy see refs.[11, 12, 13]. Meson-cloud effects have also been found instrumental in several other places [14, 15]. Consideration of their effects in weak nonleptonic hyperon decays yields an explanation of the deviation of the f/d ratio from the naive valence quark model value of -1 to its observed values of about -2 [16]. It is therefore of great interest to perform a similar phenomenological analysis of meson cloud effects in $K \to 2\pi$ decays to see whether and how they may help to explaine relative sizes of the relevant $\Delta I = 1/2$ and $\Delta I = 3/2$ amplitudes. In this paper an analysis of this type is carried out. We study the $K \to \pi$ transition matrix elements and show in detail how hadron-level effects from various two-meson intermediate states contributing to these transitions build up the "eye" (low-energy penguin) and the "figure-eight" diagrams of the quark level. An estimate of the relative and absolute sizes of the "eye" and "figure-eight" diagrams is also given.

2 Hadronic loop contributions to the $K \to \pi$ transitions

The effect of pseudoscalar meson loop contributions to $K \to 2\pi$ was studied in dispersion relation framework [14], and in chiral approach [8, 20]. In more phenomenological way such meson rescattering FSI effects are discussed in ref.[22]. In this paper we are concerned with meson loop (hadron sea) effects in $K \to \pi$ transitions themselves (see Fig.1). If only ground-state mesons are permitted in the loop, at least one of them must be a vector meson (the allowed intermediate states are PV+VP and VV (P-pseudoscalar, V-vector mesons). Although all these two-particle states are much heavier than the PP ones that were considered in refs.[8, 14, 20, 22], their contribution is expected to be significant as evidenced by estimates of their effects in hadron spectroscopy[11, 13]. A transparent way to include both pseudoscalar and vector mesons in the intermediate state is to use general ideas of the unitarized quark model of ref.[11].

What we want to estimate here is, in essence, the contribution from virtual two-meson continuum states admixed into the wave functions of the standard quark model. We shall disregard the virtual states composed of charmed mesons as such states lie much higher (by about 2 GeV) than those built of light flavours. In the approach of ref.[11] the admixture probability $|c_{M_1M_2}|^2$ of the $| M_1M_2 >$ two-particle state relative to the "pure" quark-model state for meson M is given by [21]

$$|c_{M_1M_2}|^2 = L(M \to M_1M_2)[Tr(F_M^\dagger F_{M_1}F_{M_2}) + C_M C_{M_1} C_{M_2} Tr(F_M^\dagger F_{M_2} F_{M_1})]^2 \tag{1}$$

where, for ground-state mesons M_1M_2, we have

$$\begin{aligned} L(M \to M_1M_2) &= S(M \to M_1M_2)\, I \\ &\equiv S(M \to M_1M_2)\, \frac{1}{\pi}\frac{f^2}{\pi} \int_{thr}^{\infty} \frac{\frac{k^3}{\sqrt{s}} \exp -(\frac{k}{k_{cutoff}})^2}{(m^2-s)^2} ds \end{aligned} \tag{2}$$

The trace factor in Eq.1 (F_M is the SU(3) matrix corresponding to meson M) gives F- or D- type flavour couplings depending on the sign of $C_M C_{M_1} C_{M_2}$ (where C_M is the charge conjugation quantum number of meson M). The spin-weight factors $S(M \to M_1M_2)$ are equal to $\frac{1}{4}, \frac{1}{4}, \frac{1}{2}$ for (M, M_1M_2) being (P, PV), (P, VP), (P, VV) respectively, and they sum up to 1. The overall size of the two-meson admixture is fixed by the size of the coupling constant $f = f_{\rho NN} = 5.14$ (Eq.2), and by k_{cutoff} which is related to the (harmonic oscillator) meson size by $R_M^2 = \frac{6}{k_{cutoff}^2}$. The size of the integral I depends on the actual positions of thresholds. As a rough estimate of effects under discussion, we evaluate the integral I using for the external (M) meson mass the value $m = \frac{1}{2}(m_\pi + m_K) = 0.32\ GeV$ for two sets of masses of intermediate mesons: 1) the degenerate case with $m_V = m_P = 0.9\ GeV$, and 2) the light pseudoscalar meson case with $m_V = 0.9\ GeV$, $m_P \approx m = 0.32\ GeV$. The obtained values are gathered in Table 1 for $k_{cutoff} = 0.6, 0.7, 0.8\ GeV$ ($R_M =$

$0.80, 0.69, 0.60\ fm$). In the unitarized quark model of ref.[11, 12] the value of $k_{cutoff} = 0.7\ GeV$ gives the best description of meson spectra.

Since, according to Eq.1, admixtures of two-meson $\mid \rho\pi\ >$, $\mid \rho\ \eta\ >$, etc. states to π meson ($\mid \rho\ K\ >$ etc. to K) are all to be considered, we will have to deal with the $K \rightarrow \eta$ transitions as well. With the Fierz terms dropped and small short-distance penguin contributions neglected, standard QCD-corrected short-distance calculations give the following predictions for the amplitudes:

$$
\begin{aligned}
< \pi^+ \mid H_w \mid K^+ > &= [c_1 - (c_2 + c_3 + c_4)]\ X \\
< \pi^0 \mid H_w \mid K^0 > &= \frac{1}{\sqrt{2}}[c_1 - (c_2 + c_3 - 2c_4)]\ X \\
< \eta_8 \mid H_w \mid K^0 > &= \frac{1}{\sqrt{6}}[c_1 - c_2 + 9c_3]\ X \\
< \eta_1 \mid H_w \mid K^0 > &= \frac{1}{\sqrt{3}}[c_1 + 5c_2]\ X
\end{aligned} \tag{3}
$$

where c_i are Wilson coefficients and

$$X = < \pi^+ \mid -(d\bar{u}) \mid 0 >< 0 \mid (u\bar{s}) \mid K^+ > \tag{4}$$

with the notation

$$(q_1\bar{q}_2) \equiv \bar{q}_2\gamma_\mu(1 - \gamma_5)q_2 \tag{5}$$

Let us express the matrix elements of the parity conserving part of weak Hamiltonian between pseudoscalar meson states through amplitudes of definite isospin:

$$
\begin{aligned}
< \pi^+ \mid H_w \mid K^+ > &= \sqrt{\frac{2}{3}}A_{\frac{1}{2}} - \frac{1}{\sqrt{3}}A_{\frac{3}{2}} \\
< \pi^0 \mid H_w \mid K^0 > &= \frac{1}{\sqrt{3}}A_{\frac{1}{2}} + \sqrt{\frac{2}{3}}A_{\frac{3}{2}} \\
< \eta_8 \mid H_w \mid K^0 > &= B \\
< \eta_1 \mid H_w \mid K^0 > &= C
\end{aligned} \tag{6}
$$

Using the Gilman-Wise values [5]:

$$\begin{aligned} c_1 &= -2.11 \\ c_2 &= 0.12 \\ c_3 &= 0.09 \\ c_4 &= 0.45 \end{aligned} \tag{7}$$

for the Wilson coefficients, we obtain from Eqs.3,6

$$\begin{aligned} \frac{A_{\frac{3}{2}}}{A_{\frac{1}{2}}} &= -0.28 \\ \frac{B}{A_{\frac{1}{2}}} &= 0.20 \\ \frac{C}{A_{\frac{1}{2}}} &= 0.31 \end{aligned} \tag{8}$$

The experimental value for $\left|\frac{A_{\frac{1}{2}}}{A_{\frac{3}{2}}}\right|$ is around 22, six times larger than the theoretical value from Eq.8 ($\left|\frac{A_{\frac{1}{2}}}{A_{\frac{3}{2}}}\right| = 3.6$). When short-distance penguin contribution is included (with $c_5 \approx -0.06$) one obtains [1] $|A_{\frac{1}{2}}/A_{\frac{3}{2}}| = 4.3$, i.e.

$$\left(\frac{A_{\frac{1}{2}}}{A_{\frac{3}{2}}}\right)_{out} = 1.2 \left(\frac{A_{\frac{1}{2}}}{A_{\frac{3}{2}}}\right)_{fact} \tag{9}$$

an enhancement factor of 1.2 only. The remaining discrepancy by a factor of around 5 constitutes the $\Delta I = \frac{1}{2}$ puzzle.

The hadron-sea generated corrections to the matrix elements of Eq.6 are due to weak Hamiltonian acting in one of M_1, M_2 mesons. Let the meson in which such a transition occurs be labelled M_1 (see Fig.1). We restrict our considerations to the case when M_1 is in the ground state (i.e. $M_1 = P, V$). For the sake of our discussion this should be a reasonable approximation: Quark-antiquark annihilation into a W-boson is expected weaker for excited mesons. Moreover, the additional contributions arising from weak transition in an intermediate excited meson should (when estimated along lines similar to those presented in this paper) only increase the enhancement/suppression effects herein discussed (this should become understandable later, after the discussion of the case $M_1 = P, V$).

On the other hand, both ground-state and excited M_2 mesons will be considered. In fact, in strong virtual decays $M \to M_1 M_2$ the p-wave (that must appear somewhere to ensure parity conservation in the production of $q\bar{q}$-pair out of the vacuum) may reside either between mesons $M_1 M_2$ or within meson M_2. The contributions from these two possibilities should be comparable. The relative size of the two terms may be fixed by requiring their mutual cancellation in Zweig-rule-forbidden strong amplitudes [11]. This relative size may also be obtained under some additional assumptions through explicit calculations in the $^3P_0$-model [23]. The spin-flavour factors $([Tr(F_M^\dagger F_{M_1} F_{M_2}) + C_M C_{M_1} C_{M_2} Tr(F_M^\dagger F_{M_2} F_{M_1})]^2 * S(P \to M_1 M_2)$ corresponding to the total contribution from all possible intermediate states under consideration are gathered in Table 2.

In the normalization of Eq.1 the contributions from the $M_1 M_2 = \underline{P}V$ two-meson states (the meson undergoing weak transition underlined for clarity) are easily calculable to be:

$$A_{\frac{3}{2},loop} = -2L(P \to PV)A_{\frac{3}{2}}$$

(27) $$\frac{1}{\sqrt{10}}(A_{\frac{1}{2},loop} - 3B_{loop}) = -2L(P \to PV)\frac{1}{\sqrt{10}}(A_{\frac{1}{2}} - 3B)$$

(8) $$\frac{1}{\sqrt{10}}(3A_{\frac{1}{2},loop} + B_{loop}) = +3L(P \to PV)\frac{1}{\sqrt{10}}(3A_{\frac{1}{2}} + B)$$

$$C_{loop} = 0 \qquad (10)$$

(Where such an assignment is not obvious, the SU(3) classification of the amplitude is given on the left.) When the p-wave excitation resides in the M_2 meson, total contribution from the S- and D- wave two-meson states $\underline{P}V^*$ (V^* $=S$(scalar, $J^{PC}=0^{++}$), A(axial, 1^{++}), T(tensor, 2^{++}) mesons) is:

$$\begin{aligned}
A_{\frac{3}{2},loop} &= +2L(P\to PV^*)A_{\frac{3}{2}} \\
\frac{1}{\sqrt{10}}(A_{\frac{1}{2},loop}-3B_{loop}) &= +2L(P\to PV^*)\frac{1}{\sqrt{10}}(A_{\frac{1}{2}}-3B) \\
\frac{1}{\sqrt{10}}(3A_{\frac{1}{2},loop}+B_{loop}) &= +\frac{1}{3}L(P\to PV^*)(\frac{1}{\sqrt{10}}\left(3A_{\frac{1}{2}}+B\right)-4\sqrt{5}C\Big) \\
C_{loop} &= +\frac{1}{3}L(P\to PV^*)\left(-4\sqrt{5}\frac{1}{\sqrt{10}}(3A_{\frac{1}{2}}+B)+8C\right)
\end{aligned} \tag{11}$$

In writing Eq.11 we summed the contributions from the S- and D-waves by assuming that they are equal apart from their difference in weight (see Table 1). This should be a reasonable assumption since, at small values of m ($\approx m_\pi$ or m_K), we are away from thresholds where such differences might be important. In the $^3P_0$ model, factors $L(P\to PV^*)$ are given by a formula similar to Eq.2.

Meson M_1 need not be a pseudoscalar meson. It may be a vector meson as well. For weak transitions in vector mesons we introduce notation analogous to that of Eq.6: the $K^*\to\rho$ transitions are described by amplitudes $A^V_{\frac{1}{2}}$,$A^V_{\frac{3}{2}}$ of definite isospin etc. When $M_1=V$ we have contributions from $\underline{V}P$ and $\underline{V}P^*$ ($P^*=B$ (axial $J^{PC}=1^{+-}$) meson) two-meson states. They are, respectively:

a) for $\underline{V}P$ loops:

$$\begin{aligned}
A_{\frac{3}{2},loop} &= -2L(P\to VP)A^V_{\frac{3}{2}} \\
\frac{1}{\sqrt{10}}(A_{\frac{1}{2},loop}-3B_{loop}) &= -2L(P\to VP)\frac{1}{\sqrt{10}}(A^V_{\frac{1}{2}}-3B^V) \\
\frac{1}{\sqrt{10}}(3A_{\frac{1}{2},loop}+B_{loop}) &= +3L(P\to PV)\frac{1}{\sqrt{10}}(3A^V_{\frac{1}{2}}+B^V) \\
C_{loop} &= 0
\end{aligned} \tag{12}$$

b) for $\underline{V}P^*$ loops:

$$\begin{aligned}
A_{\frac{3}{2},loop} &= +2L(P \to VP^*)A^V_{\frac{3}{2}} \\
\frac{1}{\sqrt{10}}(A_{\frac{1}{2},loop} - 3B_{loop}) &= +2L(P \to VP^*)\frac{1}{\sqrt{10}}(A^V_{\frac{1}{2}} - 3B^V) \\
\frac{1}{\sqrt{10}}(3A_{\frac{1}{2},loop} + B_{loop}) &= +\frac{1}{3}L(P \to VP^*)\left(\frac{1}{\sqrt{10}}(3A^V_{\frac{1}{2}} + B^V) - 4\sqrt{5}C^V\right) \\
C_{loop} &= +\frac{1}{3}L(P \to VP^*)\left(-4\sqrt{5}\frac{1}{\sqrt{10}}(3A^V_{\frac{1}{2}} + B^V) + 8C^V\right)
\end{aligned} \tag{13}$$

Finally, contributions from the $\underline{V}V$ and $\underline{V}V^*$ diagrams are:

a) for the $\underline{V}V$ loops:

$$\begin{aligned}
A_{\frac{3}{2},loop} &= +2L(P \to VV)A^V_{\frac{3}{2}} \\
\frac{1}{\sqrt{10}}(A_{\frac{1}{2},loop} - 3B_{loop}) &= +2L(P \to VV)\frac{1}{\sqrt{10}}(A^V_{\frac{1}{2}} - 3B^V) \\
\frac{1}{\sqrt{10}}(3A_{\frac{1}{2},loop} + B_{loop}) &= +\frac{1}{3}L(P \to VV)\left(\frac{1}{\sqrt{10}}(3A^V_{\frac{1}{2}} + B^V) - 4\sqrt{5}C^V\right) \\
C_{loop} &= +\frac{1}{3}L(P \to VV)\left(-4\sqrt{5}\frac{1}{\sqrt{10}}(3A^V_{\frac{1}{2}} + B^V) + 8C^V\right)
\end{aligned} \tag{14}$$

b) for the $\underline{V}V^*$ loops:

$$\begin{aligned}
A_{\frac{3}{2},loop} &= -2L(P \to VV^*)A^V_{\frac{3}{2}} \\
\frac{1}{\sqrt{10}}(A_{\frac{1}{2},loop} - 3B_{loop}) &= -2L(P \to VV^*)\frac{1}{\sqrt{10}}(A^V_{\frac{1}{2}} - 3B^V) \\
\frac{1}{\sqrt{10}}(3A_{\frac{1}{2},loop} + B_{loop}) &= +3L(P \to VV^*)\frac{1}{\sqrt{10}}(3A^V_{\frac{1}{2}} + B^V) \\
C_{loop} &= 0
\end{aligned} \tag{15}$$

As already discussed, the relative size of contributions from $M_2 = P,V$ and $M_2 = P^*,V^*$ is fixed when the validity of Zweig's rule is ensured by cancellation of contributions from intermediate states involving mesons of opposite

C-parity. This amounts to putting $L(P \to PV^*) = L(P \to PV) = L(P \to VP) = L(P \to V^*P)(= \frac{1}{4}I)$ and $L(P \to VV^*) = L(P \to VV)(= \frac{1}{2}I)$. Summing the contributions from all intermediate states considered we obtain:

$$\begin{aligned}
A_{\frac{3}{2},loop} &= 0 \\
\frac{1}{\sqrt{10}}(A_{\frac{1}{2},loop} - 3B_{loop}) &= 0 \\
\frac{2}{3\sqrt{10}}(3A_{\frac{1}{2},loop} + B_{loop}) + \frac{\sqrt{5}}{3}C &= 0 \\
\frac{1}{3\sqrt{2}}(3A_{\frac{1}{2},loop} + B_{loop}) - \frac{2}{3}C_{loop} &= \\
= 2 * 6 * \frac{1}{4}I \left\{ \frac{1}{3\sqrt{2}}(3A_{\frac{1}{2}} + B) - \frac{2}{3}C + 3\left(\frac{1}{3\sqrt{2}}(3A^V_{\frac{1}{2}} + B^V) - \frac{2}{3}C^V\right) \right\}
\end{aligned} \tag{16}$$

In the last of equations in (16) the overall factor of "2" on the r.h.s. stems from the fact that weak interaction may occur in either one of the two intermediate mesons. From Eqs.(10-16) we see that after summing over the charge-conjugated $M_2 = V, V^*(P, P^*)$ meson states, virtual two-meson states give no contribution to the **27**-plet $\Delta I = \frac{1}{2}$ and $\frac{3}{2}$ transition amplitudes. Furthermore, only one of the two combinations of octet ($\Delta I = \frac{1}{2}$) transition amplitudes receives contributions from such states. We shall estimate the loop contribution to this transition by using short-distance QCD-modified factorization approximation (with Fierz-transformed terms dropped) for the $\Delta S = 1$ transition occurring in meson M_1. This gives

$$\frac{1}{3\sqrt{2}}(3A_{\frac{1}{2},loop} + B_{loop}) - \frac{2}{3}C_{loop} = 2 * 6 * I * \frac{1}{\sqrt{3}}(c_1 - 5c_2)\left[\frac{X + 3X^V}{4}\right] \tag{17}$$

wherein X^V is the factorization contribution from weak transition in intermediate vector meson

$$X^V = < \rho^+ \mid -(d\bar{u}) \mid 0 >< 0 \mid (u\bar{s}) \mid K^{*+} > \tag{18}$$

The loop contribution of Eq.17 should be compared with the short distance contribution to this transition amplitude which is

$$\frac{1}{\sqrt{3}}(c_1 - 5c_2)X \tag{19}$$

The matrix elements of currents in Eqs.4,18 are given by

$$\begin{aligned} < \pi^+|A^\mu|0 > &= f_\pi q^\mu \\ < \rho^+|V^\mu|0 > &= f_\rho \varepsilon^\mu \end{aligned} \tag{20}$$

where $f_\pi = 0.13\ GeV$, $f_\rho = 0.17\ GeV^2$. In accordance with the $SU(3)$ symmetry used elsewhere in this paper we assume, for the sake of the order-of-magnitude estimate, that $f_K = f_\pi$, $f_{K^*} = f_\rho$.

Calculation of the $K \to \pi,\ \eta$ matrix elements in the vacuum insertion method gives expressions proportional to the four-momentum squared (q^2), (i.e. $< \pi(p)|H_W^{p.c.}|K(q) >= p.q\ g_{\pi K}$ etc.) in agreement with general requirements of chiral symmetry [17, 18, 19, 1]. Such momentum dependence is not manifest in our phenomenological calculations of long-distance effects. It is well supported by lattice calculations, however [1]. Because of the lack of explicit momentum dependence there is a problem here as to what value should be used for the q^2 of the pseudoscalar meson undergoing weak transition in the short-distance factorization contribution with which the loop effect is compared. (The contribution from weak transitions in intermediate pseudoscalar mesons is much smaller than that from corresponding transitions in vector mesons and, consequently, this ambiguity is less important in the estimate of loop effects themselves). As a rough measure we employ $q^2 = \frac{1}{2}(m_K^2 + m_\pi^2) = 0.132\ GeV^2$ (see also ref.[19]). Consequently, the relevant ratio of factorization contributions X^V and X is

$$\frac{X^V}{X} = \frac{f_\rho^2}{f_\pi^2 q^2} \approx 13.0 \tag{21}$$

and the two-meson admixture contributes approximately

$$3\ I\ \left(1 + 3\frac{X^V}{X}\right) \approx 120\ I \tag{22}$$

times more than the original factorization contribution. For $I = 0.022$ (from Table 1 for $m_{M_1} = m_{M_2} = 0.9\ GeV$) we obtain an enhancement factor of 2.6. Clearly, the bulk of the enhancement obtained comes from the contribution of weak transitions in intermediate *vector* meson. The hadron-loop-induced enhancement factor of 2.6 should be compared with the standard short-distance estimates of penguin effects that give a factor of 1.2 (Eq.9 and ref.[1]).

3 Discussion

Let us see what types of quark-level diagrams are generated by hadron-level loops under discussion. Consider $\underline{P}V$ and $\underline{P}V^*$ intermediate states as an example. In the contribution from the $\underline{P}V$ loop (Eq.10), strong vertices are described by F-type flavour factors, while for the $\underline{P}V^*$ loop the corresponding couplings are of D-type (see Eq.1). The flavour structure of these strong vertices may be represented diagrammatically as in Fig.2. The wavy lines symbolize confining strong forces.

The structure of the product of flavour factors corresponding to two strong vertices of the loop is then

a)for $P \rightarrow \underline{P}V \rightarrow P$ loops:

$$Tr(F_M[F^\dagger_{M_1}, F^\dagger_{M_2}])Tr(F^\dagger_{M'}[F_{M'_1}, F_{M_2}]) \tag{23}$$

b)for $P \rightarrow \underline{P}V^* \rightarrow P$ loops:

$$Tr(F_M\{F^\dagger_{M_1}, F^\dagger_{M_2}\})Tr(F^\dagger_{M'}\{F_{M'_1}, F_{M_2}\}) \tag{24}$$

Using the equality $\sum_{M=1\ominus 8} Tr(AM)Tr(AM^\dagger) = Tr(AB)$, summation over all intermediate mesons M_2 may be performed, giving the expression

$$\begin{gathered} Tr(F_{M_1^\dagger}F_MF_{M'^\dagger}F_{M'_1}) + Tr(F_MF_{M_1^\dagger}F_{M'_1}F_{M'^\dagger}) \\ \mp Tr(F_{M_1^\dagger}F_MF_{M'_1}F_{M'^\dagger}) \mp Tr(F_MF_{M_1^\dagger}F_{M'^\dagger}F_{M'_1}) \end{gathered} \tag{25}$$

with $-(+)$ signs for $F(D)$ respectively. Flavour contractions implicit in the first and the second term of Eq.25 are visualised in Fig.3a, while those of the remaining two terms - in Fig.3b. The black blob in Fig.1 is replaced in Fig.3 with boxes marked with dashed lines. Inside the boxes the diagrammatic representation of the genuine factorization prescription is drawn.

Fig.3a represents the familiar low-energy penguin ("eye") diagram, while Fig.3b is easily recognizable as the "figure-eight"-type diagram with soft gluon exchanges between two quark loops. When the internal organization of the weak-interaction box is taken into account, the "figure-eight" diagram of Fig.3b is actually equivalent to the W-exchange diagram with all possible soft gluon exchanges between an initial (anti)quark and a final (anti)quark. When the summation of two contributions from $M_2 = V$ and V^* (Eq.25) is performed with equal weights (which in the previous section was argued to be a reasonable approximation), the "figure-eight" contribution drops out totally from final formulas (Eq.16) and, consequently, expressions in Eq.16 correspond to the low-energy penguin interaction with a u-quark loop.

In the discussion so far we have assumed that the contributions from all loops with different internal mesons are essentially identical, irrespectively of the actual location of the relevant thresholds. In reality, pseudoscalar mesons are much lighter than the remaining scalar, axial and tensor mesons. Consequently, contribution from intermediate states containing a pseudoscalar meson (especially a pion) will be larger. To see what effect such nondegeneracy might have, let us assume - as a very rough approximation - that all pseudoscalar mesons are lighter than the remaining, still approximately degenerate, vector, axial and tensor mesons. This idealization corresponds to the expected dominance of contributions from low-lying thresholds and to small (and thus negligible) differences in the overall scale of contributions from the remaining states.

Using Eqs.(10-15) we derive the following corrections to the fully symmetric expressions of Eq.16:

$$\begin{aligned}
\Delta A_{\frac{3}{2},loop} &= -4\ \Delta L\ A^V_{\frac{3}{2}} \\
\frac{1}{\sqrt{10}}(\Delta A_{\frac{1}{2},loop} - 3\ \Delta B_{loop}) &= -4\ \Delta L\ \frac{1}{\sqrt{10}}(A^V_{\frac{1}{2}} - 3B^V) \\
\frac{1}{\sqrt{10}}(3\ \Delta A_{\frac{1}{2},loop} + \Delta B_{loop}) &= 6\ \Delta L\ \frac{1}{\sqrt{10}}(3A^V_{\frac{1}{2}} + B^V) \\
\Delta C_{loop} &= 0
\end{aligned} \tag{26}$$

where

$$\Delta L \equiv [L(P \to PV) - L(P \to VP^*)] \tag{27}$$

In Eq.26 we have neglected the contribution from weak interaction in intermediate pseudoscalar meson, as they are much smaller than those arising from interaction in intermediate vector meson.

From Eq.26 and the fact that $\Delta L \equiv L(P \to PV) - L(P \to VP^*) > 0$ we see that corrections to **27**-plet amplitudes (both for $\Delta I = \frac{3}{2}$ and $\Delta I = \frac{1}{2}$) are negative, and thus these amplitudes are suppressed. On the other hand, corrections to octet $K \to \pi$ and $K \to \eta_8$ amplitudes are positive and, consequently, these amplitudes are enhanced. The $K \to \eta_1$ amplitude is not modified in the approximation under consideration. By assuming light η_1, η_8 ($I = 0$) mesons we slightly overestimate hadron-level corrections to the $\Delta I = 1/2$ amplitudes. On the other hand, the $\Delta I = 3/2$ amplitudes receive corrections from the $K \to \pi$ in-loop transitions only (the $K \to \eta$ transitions change isospin by 1/2). Thus, the estimate of the relevant suppression factors is not affected by this simplification.

Using Eqs.3,6,7, and $X^V/X = 13.0$ we find from Eqs.16,26 that

$$\begin{aligned} A^{out}_{\frac{3}{2}} &= (1 - 52.0\ \Delta L\ )A^{fact}_{\frac{3}{2}} \\ A^{out}_{\frac{1}{2}} &= (1 + 46.6\ I + 72.6\ \Delta L\ )A^{fact}_{\frac{1}{2}} \end{aligned} \tag{28}$$

From Table 1 we have $I = 0.022$ and $\Delta L = 0.0093$ (for $k_{cutoff} = 0.7\ GeV$). From Eq.28 we then obtain

$$\begin{aligned} A^{out}_{\frac{3}{2}} &= 0.52 A^{fact}_{\frac{3}{2}} \\ A^{out}_{\frac{1}{2}} &= 2.70 A^{fact}_{\frac{1}{2}} \end{aligned}$$

$$\frac{A^{out}_{\frac{1}{2}}}{A^{out}_{\frac{3}{2}}} = 5.2\frac{A^{fact}_{\frac{1}{2}}}{A^{fact}_{\frac{3}{2}}} \tag{29}$$

The total hadron-level enhancement factor of 5.2 should be compared with the number of 1.2 obtained for the case of short-distance penguin contribution (Eq.9). For $k_{cutoff} = 0.6, 0.8\ GeV$ we obtain suppression (enhancement) factors of 0.62,0.40 (2.18,3.33) for the **27**-plet (octet) $K \rightarrow \pi$ amplitudes respectively. Numerically, the hadron-level penguin diagram enhances the octet amplitude more than the "figure-eight" (here: W-exchange [24, 25, 26]) diagram (the I term in Eq.28 is slightly larger than the ΔL term). The suppression of the **27**-plet amplitudes is due to the "figure-eight" diagram. Our numerical estimates indicate that "figure-eight" diagrams enhance the $\frac{A_{\frac{1}{2}}}{A_{\frac{3}{2}}}$ ratio by a factor slightly smaller than do the penguin diagrams. In lattice calculations "figure-eight" contributions were much smaller than those of "eye" diagrams. This difference between our paper and lattice calculations seems to result from breaking of intermediate meson degeneracy, a feature not explicitly considered in lattice calculations.

Our estimates involve significant simplifications and cannot be trusted to more than 50% or so. Still, it should be obvious that the contribution from two-meson intermediate states is large and must be responsible for a large part of the $\Delta I = 1/2$ over $\Delta I = 3/2$ enhancement providing an overall enhancement factor of order 4-8. Thus, long-range effects are very important indeed. The author hopes that, in comparison to approaches based on the "first principles", the estimate of these effects in hadron-level phenomenological framework is more realistic and transparent [27].

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Table 1. Dependence of I and ΔL on k_{cutoff} and m_P.

$k_{cutoff}(GeV^2)$	0.6	0.7	0.8
$I(m_P = 0.9\ GeV)$	0.014	0.022	0.032
$I(m_P = 0.32\ GeV)$	0.041	0.059	0.078
ΔL	0.0073	0.0093	0.0115

Table 2. Spin-flavour factors for $P \to M_1M_2$ loops (summed over flavour)

PV	VP	VV		PS	PA	PT	VB	VS	VA	VT
$\frac{3}{2}$	$\frac{3}{2}$	3	S-wave	$\frac{1}{2}$	0	0	$\frac{1}{2}$	0	1	0
			D-wave	0	0	1	1	0	$\frac{1}{2}$	$\frac{3}{2}$

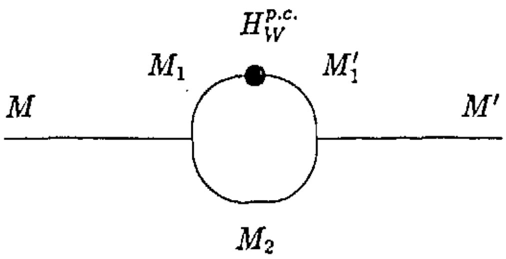


Fig.1 Weak transition in hadronic loop.

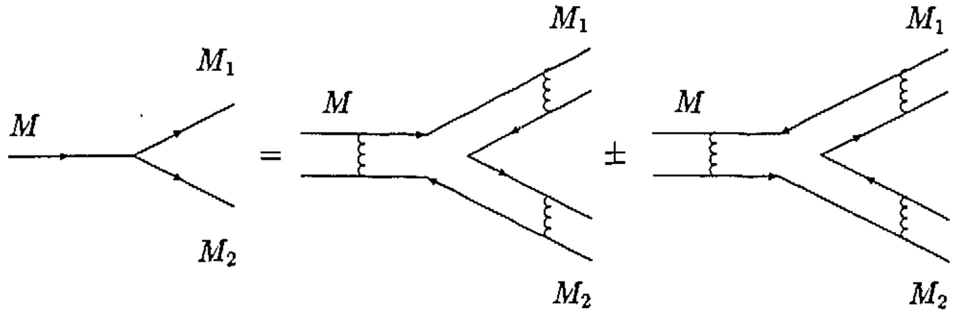
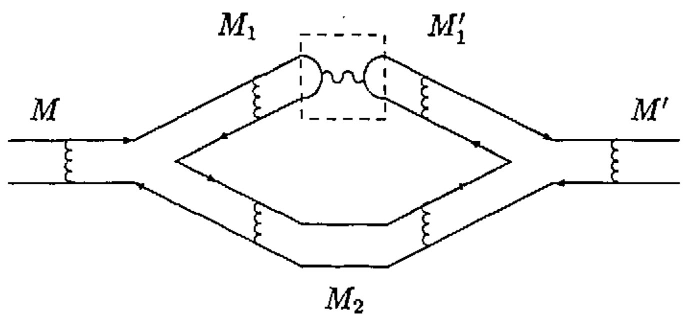
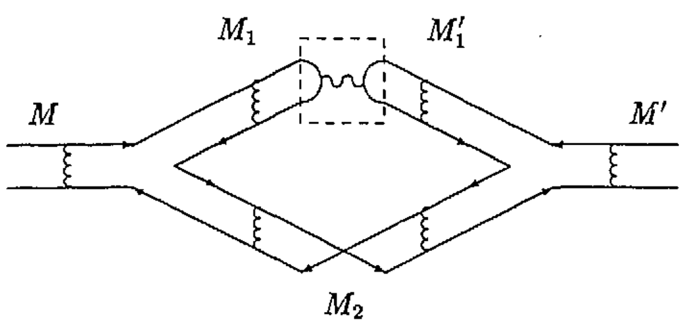


Fig.2 Diagrammatic representation of F- and D-type strong vertices.



(a)



(b)

Fig.3. Quark-level diagrams generated by hadronic loops of Fig.1:
(a) "eye" (low-energy penguin), (b) "figure-eight".