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UTILISATION DE LA METHODE NODALE NEM EN CINETIQUE NEUTRONIQUE

NON-LINEAR ITERATIVE STRATEGY FOR NEM REFINEMENT AND EXTENSION

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ENGRAND P.R.
MALDONADO G.I.
AL-CHALABI R.
TURINSKY P.J.

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SYNTHÈSE :

Cette note concerne la méthode nodale NEM ('Nodal Expansion Method') associée au processus de résolution non linéaire itératif développé par K. Smith à Studsvik of America, dans SIMULATE code de calcul de cœur par l'équation de la diffusion. Originellement, cette méthode ne s'appliquait qu'à la résolution des équations de la diffusion neutronique en régime permanent. On propose ici une amélioration de la méthode existante, ainsi qu'une proposition de généralisation efficace dans le cas de la cinétique neutronique. Ces travaux ont été effectués dans le code NESTLE, développé à North Carolina State University.

On rappelle que le principe de la méthode de Smith aboutit à la résolution des équations sur un ensemble de géométries élémentaires constituées de deux mailles consécutives. Le système matriciel issu de la discrétisation de ce problème à deux mailles possède une structure fixe et sa taille est 16×16 (dans le cas à 2 groupes d'énergie). On propose un moyen de réduire ce système en 2 sous-systèmes équivalents de tailles 4×4 et 8×8. La résolution du problème sur la géométrie complète du cœur nécessitant un très grand nombre de résolutions de ces systèmes élémentaires, le gain en temps CPU apporté par cette méthode s'élève à 45 % sur la partie du code relative à la résolution par la méthode NEM.

L'idée de l'adaptation de la méthode de K. Smith en cinétique consiste à rendre la résolution des équations formellement identique au cas permanent. Pour cela on propose d'approximer les termes de l'équation correspondant au pas de temps précédent et aux précurseurs par un polynome du second degré et de traiter celui-ci comme un terme source. On présente ici de quelle façon cette approximation doit être mise en service pour qu'elle soit consistante et précise.

EXECUTIVE SUMMARY :

The following work is related to the non-linear iterative strategy developed by K. Smith to solve the Nodal Expansion Method (NEM) representation of the neutron diffusion equations. We show how to improve this strategy and how to adapt it to time dependant problems. This work has been done in the NESTLE code, developed at North Carolina State University.

When using Smith's strategy, one ends up with a two-node problem which corresponds to a matrix with a fixed structure and a size of 16×16 (for a 2 group representation). We show how to reduce this matrix into 2 equivalent systems which sizes are 4×4 and 8×8. The whole problem needs many of these 2 node problems solution. Therefore the gain in CPU time reaches 45 % in the nodal part of the code.

To adapt Smith's strategy to time dependent problems, the idea is to get the same structure of the 2 node problem system as in steady-state calculation.

To achieve this, one has to approximate the values of the past time-step and of the previous by a second order polynomial and to treat it as a source term. We show here how to make this approximation consistent and accurate.

Non-Linear Iterative Strategy for NEM Refinement and Extension

Philippe R. ENGRAND

Electricité de France

Guillermo I. MALDONADO, Rifat AL-CHALABI, Paul J. TURINSKY

Department of Nuclear Engineering - NCSU

Introduction

The following work is related to the non-linear iterative strategy developed by Smith [1] to solve the Nodal Expansion Method (NEM) representation of the neutron diffusion equations. We show how it is possible to save computation time by taking advantage of the reducibility of the matrices that have to be inverted when employing this strategy. Additionally, we present how this strategy can be adapted, in an easy and efficient manner, to time dependent problems.

Reducibility

NEM, when applied to the neutron diffusion equation involves typically quartic expansions of the flux in each spatial direction over the node. This produces a total of 13 unknowns per energy group per node for three-dimensional Cartesian geometry. The constraint equations for these unknowns are as follows : 0th moment balance equation, transverse integral 1st and 2nd moment balance equations, and surface flux (with ADFs) and current continuity conditions. The non-linear iterative strategy solves this coupled set of equations by first decoupling the transverse integral equations by using past iterative values of the transverse leakages ; next reducing the node coupling in a given direction to two-node coupling by not requiring surface flux or current continuity on the two-node problem's outer surfaces ; and finally solving the CMFD type representation of the diffusion equation for node average fluxes using coupling coefficients corrected to reproduce the surface currents obtained from the two-node problems. This solution of the CMFD equation is done employing a conventional outer-inner iterative approach. The non-linear iterative strategy results in computer memory savings since expansion coefficients do not need to be saved, and operation count savings since the three-dimensional coupled problem is solved for node average fluxes versus partial surface currents as in the response matrix formulation, a factor of three reduction in unknowns.

For the two-node problem the node average flux is assumed known and obtained from the CMFD equations solution for the previous iteration. This implies 8 unknowns exist for each energy group. Imposing moment balances and continuity on the interior surface provides the required number of equations. Table 1 presents the matrix structure for a two-node problem. We have rearranged the system of equations such that the even expansion coefficients of any node are independant from those of its neighbor. Therefore, it is only necessary to solve for the even expansion coefficients for node $(l+1)$ for the [node (l) - node $(l+1)$] problem. This follows since node (l) even expansion coefficients have already been solved

for when the [node $(l-1)$ - node (l)] problem was solved. Thus, instead of having to solve a 16 by 16 system for every two-node problem, we can solve one 4 by 4 system, and one 8 by 8 system which has the previously calculated even expansion coefficients on the right-hand side (RHS). The saving in CPU cycles is proportional to approximately the total number of nodes. For our standard 3D steady-state case, about 45% and 10% reductions of CPU time were achieved in solving the two-node problems and in the total code, respectively.

Time Dependent Problems

For NEM based space-time kinetics, treating the temporal component implicitly and solving the precursor equations analytically assuming a linear variation of fission rate over the time-step, which allows the precursor number densities to be eliminated as unknowns [2], the following equation is obtained for each node (node indexing has been suppressed) :

$$-D_g^{(n)}\frac{d^2}{du^2}\Phi_g^{(n)}(u)+\Sigma_{t_g}\Phi_g^{(n)}(u)=\left[\sum_{g'}\Sigma_{g'g}^{(n)}\Phi_{g'}^{(n)}(u)+\chi_g^{(n)}\Psi^{(n)}(u)-L_{g_v}^{(n)}(u)-L_{g_w}^{(n)}(u)+S_{ext_g}^{(n)}(u)\right]$$

$$+\left[\chi_g^{(n)}\left((A^{(n)}-\beta)\Psi^{(n)}(u)+B^{(n)}\Psi^{(n-1)}(u)\right)+C^{(n-1)}f_g\left(\{C_i^{(n-1)}(u)\}\right)+\frac{1}{v_g\Delta t_n}\left(\phi_g^{(n-1)}(u)-\phi_g^{(n)}(u)\right)\right] \quad (1)$$

$$\text{for } 1\le g\le G \quad \text{and} \quad u,v,w\in(x,y,z)$$

where Ψ is the prompt fission source, L_{g_v} and L_{g_w} are the transverse leakages, superscripts (n) and $(n-1)$ refer to current and past time step quantities, and the coefficients A, B and C result from the temporal treatment of the precursors equations.

Straight application of the non-linear iterative strategy to the kinetic problem would involve solving the moment equations obtained from Eq. (1) for each two-node problem. Examination of the RHS of Eq. (1) indicates that time-step $(n-1)$ values of the expansion coefficients for the flux and precursors densities would appear in these equations. This could be reduced to a single set of expansion coefficients by combining all past time quantities on the RHS ; however, this still requires saving expansion coefficients which nullifies one of the advantages of the non-linear iterative strategy.

To address this problem, we use a lower order polynomial for the approximation of all quantities that do not appear in the steady-state equations, these quantities contained within the second bracketed term of Eq. (1) RHS. Those quantities are approximated just like transverse leakages, using a quadratic polynomial fitted to node average values obtained from previous time-step values or for current time-step quantities from previous iterative values. Note that this assures that the kinetic solution when at equilibrium conditions reproduces the steady-state solution. This also assures no drift when using the steady-state solution as the initial condition for a kinetic calculation. This method does introduce a lower order error in the within-node spatial treatment when under true transient conditions, but it is confined to delayed neutron and flux time derivative terms denoted in the Eq. (1) RHS second bracket, whose magnitude should be small versus the first bracket's magnitude. Transient test cases predictions using this method demonstrated solution convergence with decreasing time-step size, and establishment of new equilibrium conditions which reproduce the steady-state solutions.

References

[1] K. S. SMITH, "Nodal Method Storage Reduction by Non-Linear Iteration," Transactions of the American Nuclear Society, June 12-16, 1983, Detroit, Michigan.

[2] B. R. BANDINI, "A three-dimensional transient neutronics routine for the TRAC-PF1 reactor thermal hydraulic computer code," PhD Thesis, Pennsylvania State University (1990).

Table 1 : Non Zero Values in the 16x16 matrix of the NEM two-node problem

Equation	Group	Node	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p
0th Moment	Fast	l	x		x													
0th Moment	Thermal	l		x		x												
2nd Moment	Fast	l	x	x	x	x												
2nd Moment	Thermal	l	x	x	x	x												
0th Moment	Fast	l+1					x		x									
0th Moment	Thermal	l+1						x		x								
2nd Moment	Fast	l+1					x	x	x	x								
2nd Moment	Thermal	l+1					x	x	x	x								
1st Moment	Fast	l									x	x	x	x				
1st Moment	Thermal	l									x	x	x	x				
1st Moment	Fast	l+1													x	x	x	x
1st Moment	Thermal	l+1													x	x	x	x
Cont. of Current	Fast		x		x		x		x		x		x		x		x	
Cont. of Current	Thermal			x		x		x		x		x		x		x		x
Disc. of Flux	Fast		x				x				x				x			
Disc. of Flux	Thermal			x				x				x				x		

unknown	node	group	exp. coef.
a	l	Fast	2
b	l	Th	2
c	l	Fast	4
d	l	Th	4
e	l+1	Fast	2
f	l+1	Th	2
g	l+1	Fast	4
h	l+1	Th	4
i	l	Fast	1
j	l	Th	1
k	l	Fast	3
l	l	Th	3
m	l+1	Fast	1
n	l+1	Th	1
o	l+1	Fast	3
p	l+1	Th	3

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