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BOHM'S THEORY VERSUS DYNAMICAL REDUCTION[1]

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ABSTRACT

This essay begins with a comparison between Bohm's theory and the dynamical reduction program. While there are similarities (e.g., the preferred basis), there are also important differences (e.g., the type of nonlocality or of Lorentz invariance). In particular, it is made plausible that theories which exhibit parameter dependence effects cannot be "genuinely Lorentz invariant". For the two approaches under consideration, this analysis provides a comparison that can produce a richer understanding both of the pilot wave and of the dynamical reduction mechanism.

1. Introduction.

We share today's widespread opinion that Standard Quantum Mechanics (SQM), in spite of its enormous successes, has failed in giving a satisfactory picture of the world, as we perceive it. The difficulties about the conceptual foundations of the theory arising, as is well known, from the so-called objectification problem, have stimulated various attempts to overcome them. Among these one should mention the search for a deterministic completion of the theory, the many worlds and many minds intepretations, the so called environment induced superselection rules, the quantum histories approach and the dynamical reduction program. In this paper we will focus our attention on the only available and precisely formulated examples of a deterministic completion and of a stochastic and nonlinear modification of SQM, i.e., Bohm's theory and the spontaneous reduction models, respectively. It is useful to stress that while the first theory is fully equivalent, from a predictive point of view, to SQM, the second one qualifies itself as a rival of SQM but with empirical divergence so small that it can claim all the same experimental support. Accordingly, they represent explicit answers to the conclusion reached by Bell (Bell 1987, 41) that *either the wavefunction as given by the Schrödinger equation is not everything or it is not right*.

The specific purpose of this work is to compare the two just mentioned theoretical schemes with reference to their formal structure, their conceptual implications and the position they allow one to take about physical reality. As we will see, while there are similarities (typically the privileged role assigned to positions) there are relevant differences (particularly with reference to the non local characteristics) between the two theories.

Section 2 is devoted to a sketchy description of the hidden variable program and of Bohm's pilot wave theory. Section 3 recalls the main

features of the dynamical reduction models. In Section 4 we begin comparing the two theories with reference to their formal structure. In Section 5 we will be mainly concerned with the problem of describing physical reality. Section 6 deals with the classical and nonclassical features of the theories under examination, while Section 7 is devoted to discuss their nonlocal aspects and the ensuing implications for a relativistic program.

2. Hidden Variables and Bohm's Theory.

The hidden variables program consists in an attempt to make epistemic the nonepistemic probabilities of SQM, by introducing, to describe individual systems, parameters which have to be added to or which replace the wave function in such a way that the resulting theory is fully deterministic. As is well known any theory of this type meets some difficulties in connection with the problem of attributing objective properties to individual physical systems due to the unavoidable contextual nature of some physical quantities.

The basic principles of the pilot wave theory (Bohm 1952, 165, 180; Bohm and Hiley, 1984, 255) can be summarized as follows:

1. The wave function does not give a complete characterization of the state of a physical system. In Bohmian mechanics one adds to it further variables i.e. the positions of all the particles of the system under consideration. Such parameters (which are the hidden variables of the theory) correspond to properties actually possessed by the constituents. Thus, any particle has a definite position at any time even though the wavefunction is not a position eigenstate and particles associated to the same wavefunction can *have* different positions.

2. The evolution of the wave function is governed by the linear Schrödinger equation in all instances, in particular collapses never occur.

3. Position variables evolve according to a deterministic evolution equation, the dynamics at a given point and at a given time depending on the wave function evaluated at the considered point.

The ontology of the theory is quite simple: there are particles moving along precise trajectories $\mathbf{q}_i(t)$ and, in addition to them, there is a real[1] entity, the wave function $\Psi(\mathbf{q}_1,\dots,\mathbf{q}_N,t)$. The evolution is governed

by two fundamental equations. First of all, as already stated, there is the Schrödinger equation

$$i\hbar\frac{\partial\Psi}{\partial t}=[-\sum_i\frac{\hbar^2}{2m_i}\Delta_i+V]\Psi \tag{2.1}$$

and secondly, there is Bohm's equation for the position variables:

$$\frac{d\mathbf{q}_i}{dt}=\frac{\hbar}{2m_i i}[\frac{\Psi^*(\mathbf{q}_1,\dots,\mathbf{q}_N,t)\nabla_i\Psi(\mathbf{q}_1,\dots,\mathbf{q}_N,t)-\Psi(\mathbf{q}_1,\dots,\mathbf{q}_N,t)\nabla_i\Psi^*(\mathbf{q}_1,\dots,\mathbf{q}_N,t)}{|\Psi(\mathbf{q}_1,\dots,\mathbf{q}_N,t)|^2}]. \tag{2.2}$$

The right hand side of this equation is the ratio between the probability current and the probability density of SQM, evaluated at the point $q=(\mathbf{q}_1,\dots,\mathbf{q}_N)$ of the 3N-dimensional configuration space. The specification of the initial conditions requires to assign the wavefunction $\Psi(\mathbf{q}_1,\dots,\mathbf{q}_N,0)$, and the positions of all particles $\mathbf{q}_i(0)=\mathbf{q}_i^0$, $(i=1,\dots,N)$ at time t=0. The trajectories of the particles are obtained by first solving Eq.(2.1), inserting the solution in Eqs. (2.2), and solving these equations with the assigned initial values.

Obviously, when we perform a preparation procedure of the SQM-type (such procedures are the only ones which can be performed), we have no way to control the positions of the particles. It is assumed that after the preparation such positions turn out to be distributed[2] in accordance with the SQM probability density $|\Psi(\mathbf{q}_1,\dots,\mathbf{q}_N,0)|^2$. The very nice feature of the formalism is that, as a consequence of the evolution equation, the probability density of the positions of the particles at time t turns out to coincide with $|\Psi(\mathbf{q}_1,\dots,\mathbf{q}_N,t)|^2$. This is the precise sense in which Bohmian mechanics can be considered as empirically equivalent to SQM. The measurement processes are described within the scheme without resorting to any additional assumption, and the pointer of the apparatus ends up in a definite position identifying the outcome obtained.

3. The Dynamical Reduction Program.

As already mentioned, genuinely Hilbert space models have recently been presented which, by considering nonlinear and stochastic modifications of Schrödinger's dynamics imply, without entailing any violation of established experimental facts, wave packet reduction with fixed pointer positions in measurement processes and, more generally,

forbid the persistence of linear superpositions of macroscopically distinguishable states.

3.a. Quantum Mechanics with Spontaneous Localizations.

We begin by discussing a slightly modified version (in which mass has a privileged role) of the first and simplest model of this kind (Ghirardi, Rimini and Weber 1986, 470), Quantum Mechanics with Spontaneous Localization (QMSL). The model is based on the assumption that, besides the standard evolution, physical systems are subjected to spontaneous localizations occurring at random times and affecting their elementary constituents. Such processes, which we will call "hittings", are formally described in the following way.

We consider a system of N particles. We suppose that when the i-th particle of the system suffers a localization the wave function changes according to

$$\Psi(\mathbf{r}_1,\ldots,\mathbf{r}_N) \to \Psi_{\mathbf{x}}(\mathbf{r}_1,\ldots,\mathbf{r}_N) = \Phi_{\mathbf{x}}(\mathbf{r}_1,\ldots,\mathbf{r}_N)/\|\Phi_{\mathbf{x}}\|$$
$$\Phi_{\mathbf{x}}(\mathbf{r}_1,\ldots,\mathbf{r}_N) = (\alpha/\pi)^{3/4} e^{-\frac{\alpha}{2}(\mathbf{r}_i-\mathbf{x})^2}\Psi(\mathbf{r}_1,\ldots,\mathbf{r}_N) \qquad (3.1)$$

The probability density of the process occurring at point $\mathbf{x}$ is given by $\|\Phi_{\mathbf{x}}\|^2$. For what concerns the temporal features of the processes we assume that the hittings for the various particles occur independently and at randomly distributed times with a mean frequency λ_m which depends on the mass of the considered particle. We choose $\lambda_m = \frac{m}{m_0}\lambda$, where m is the mass of the particle, m_0 is the nucleon mass and λ is of the order of 10^{-16} sec^{-1}. The localization parameter $1/\sqrt{\alpha}$ is assumed to take the value 10^{-5} cm.

Let a macroscopic system be in a state $|\Psi>=|\Psi_1>+|\Psi_2>$ which is a superposition of two states $|\Psi_1>$ and $|\Psi_2>$ in which a certain number of particles are in different positions. When one of these particles suffers a hitting that localizes it in the position corresponding to the state $|\Psi_1>$ ($|\Psi_2>$), the other term of the superposition is exponentially suppressed. Therefore, the macroscopic system jumps either to the state $|\Psi_1>$ or to $|\Psi_2>$ every time one of the particles differently located in the two states suffers a hitting. This implies that the frequency leading to the suppression of the coherence between the two states increases proportionally to the masses which are displaced.

The QMSL mechanism does not respect the symmetry properties of the wave function in the case of identical constituents. Its generalization satisfying such a requirement, the Continuous Spontaneous Localizations Model (CSL), has been presented and discussed in various papers (Pearle 1989, 2277; Ghirardi, Pearle and Rimini 1990, 78; Ghirardi and Rimini 1990, 167).

3.b. The CSL Model.

The model is based on a linear stochastic evolution equation for the statevector. The evolution does not preserve the norm but only the average value of the squared norm. The equation is:

$$\frac{d|\Psi_w(t)>}{dt} = [-\frac{i}{\hbar}H + \sum_i A_i w_i(t) - \gamma\sum_i A_i^2]|\Psi_w(t)>. \qquad (3.2)$$

In eq. (3.2), the quantities $w_i(t)$ are c-number Gaussian stochastic processes satisfying

$$<<w_i(t)>>=0, \quad <<w_i(t)w_j(t')>>= \gamma\delta_{ij}\delta(t-t'), \qquad (3.3)$$

while the quantities A_i are commuting self-adjoint operators. Let us assume, for the moment, that these operators have a purely discrete spectrum and let us denote by P_σ the projection operators on their common eigenmanifolds.

The physical meaning of the model is made precise by the following prescription: if a homogeneous ensemble (pure case) at the initial time t=0 is associated to the statevector $|\Psi,0>$, then the ensemble at time t is the union of homogeneous ensembles associated with the normalized vectors $|\Psi_w(t)>/\||\Psi_w(t)>\|$, where $|\Psi_w(t)>$ is the solution of eq.(3.2) with the assigned initial conditions and for the specific stochastic process w which has occurred in the interval (0,t). The probability density for such a subensemble is the "cooked one", i.e. it is given by:

$$P_{Cook}[w] = P_{Raw}[w]\||\Psi_w(t)>\|^2, \qquad (3.4)$$

where we have denoted by $P_{Raw}[w]$ the "raw" probability density associated, according to eq. (3.3), to the Gaussian process w in the interval (0,t), i.e.,:

$$P_{Raw}[w] = \frac{1}{N} e^{-\frac{1}{2\gamma}\sum_i \int_0^t dw_i^2(\tau)}, \qquad (3.5)$$

N being a normalization factor.

One can prove that the map from the initial ensemble to the final ensemble obeys the forward time translation semigroup composition law. It is also easy to prove that the evolution, at the ensemble level, is governed by the dynamical equation for the statistical operator

$$\frac{d\rho(t)}{dt} = \frac{i}{\hbar}[\rho(t), H] + \gamma \sum_i A_i \rho(t) A_i - \frac{\gamma}{2}\{\sum_i A_i^2, \rho(t)\}, \qquad (3.6)$$

from which one immediately sees that, if one disregards the Hamiltonian evolution, the *off-diagonal elements* $P_\sigma \rho(t) P_\tau$ $(\sigma \neq \tau)$ *of the statistical* operator are exponentially damped.

For our concerns, the relevant feature of the dynamical process (3.2) with the prescription (3.4) is that it drives the statevector of each individual member of the ensemble into one of the common eigenmanifolds of the operators A_i, with the appropriate probability. To make this clear, we consider (Pearle 1989, 2277) a simplified case in which only one operator A appears in eq. (3.2). The solution of this equation corresponding to the particular initial condition (involving only two eigenmanifolds of A with eigenvalues α, β)

$$|\Psi,0> = P_\alpha |\Psi,0> + P_\beta |\Psi,0>, \qquad (3.7)$$

when the Hamiltonian is disregarded, is[3] :

$$|\Psi_B(t)> = e^{\alpha B(t) - \alpha^2 \gamma t} P_\alpha |\Psi,0> + e^{\beta B(t) - \beta^2 \gamma t} P_\beta |\Psi,0>. \qquad (3.8)$$

Here B(t) is the Brownian process

$$B(t) = \int_0^t d\tau w(\tau). \qquad (3.9)$$

Taking into account eq.(3.8) and the cooking prescription, one gets the cooked probability density for the value B(t) of the Brownian process at time t:

$$P_{Cook}[B(t)] = P_{Raw}[B(t)] \| \Psi_B(t)> \|^2 = \|P_\alpha|\Psi,0>\|^2 \frac{1}{\sqrt{2\pi\gamma t}} e^{-\frac{1}{2\gamma t}(B(t) - 2\alpha\gamma t)^2}$$
$$+ \|P_\beta|\Psi,0>\|^2 \frac{1}{\sqrt{2\pi\gamma t}} e^{-\frac{1}{2\gamma t}(B(t) - 2\beta\gamma t)^2}. \qquad (3.10)$$

From (3.10) it is evident that for $t \to \infty$, the Brownian process B(t) can assume only values belonging to an interval of width $\sqrt{\gamma t}$ around[4] either the value $2\alpha\gamma t$ or the value $2\beta\gamma t$. The corresponding probabilities are $\|P_\alpha|\Psi,0>\|^2$ and $\|P_\beta|\Psi,0>\|^2$, respectively. The occurrence of a value near to $2\alpha\gamma t$ for the random variable B(t) leads, according to eq.(3.8), to a state vector that, for $t \to \infty$, lies in the eigenmanifold corresponding to the eigenvalue a of A. In fact, one gets:

$$\frac{\|P_\beta|\Psi_B(t)>\|^2}{\|P_\alpha|\Psi_B(t)>\|^2} \equiv e^{-2\gamma t(\alpha-\beta)^2} \frac{\|P_\beta|\Psi,0>\|^2}{\|P_\alpha|\Psi,0>\|^2} \xrightarrow[t\to\infty]{} 0. \qquad (3.11)$$

Analogously, when the random variable B(t) takes a value near to $2\beta\gamma t$, for $t \to \infty$, the state vector is driven into the eigenmanifold corresponding to the eigenvalue β of A.

It is then clear that the model establishes a one-to-one correspondence between the "outcome" (the final "preferred" eigenmanifold into which an individual statevector is driven) and the specific value (among the only ones having an appreciable probability) taken by B(t) for $t \to \infty$, a correspondence irrespective of what $|\Psi,0>$ is, provided only that it has non-zero amplitudes for the two considered eigenmanifolds. In the general case of several operators A_i, a similar conclusion holds for the "outcomes" α_i of A_i and the corresponding Brownian processes $B_i(t)$.

This concludes the exposition of the general structure of the CSL model. Obviously, to give a physical content to the theory one must choose the so-called preferred basis, i.e. the eigenmanifolds on which reduction takes place or, equivalently, the set of commuting operators A_i. The specific form that has been shown to possess all the desired features has been presented in (Pearle 1989, 2277; Ghirardi, Pearle and Rimini 1990, 78). Recently various considerations have led to a slightly modified version making the mass to play a specific role (Pearle and Squires 1994, 1; Ghirardi, Grassi and Benatti 1995, 5). In this version one identifies the discrete index i and the operators A_i of the above formulae with the continuous index **r** and the operator

$$M(\mathbf{r}) = \sum_k m^{(k)} N^{(k)}(\mathbf{r}), \tag{3.12}$$

where $m^{(k)}$ is the mass of the particles of type k and $N^{(k)}(\mathbf{r})$ are the number density operators giving the average density of particles of type k in a volume of about 10^{-15} cm^3 around the point r:

$$N^{(k)}(\mathbf{r}) = \left[\frac{\alpha}{2\pi}\right]^{\frac{3}{2}} \sum_s \int d\mathbf{q} e^{-\frac{\alpha}{2}(\mathbf{q}-\mathbf{r})^2} a_{(k)}^{+}(\mathbf{q},s) a_{(k)}(\mathbf{q},s). \tag{3.13}$$

Here $a_{(k)}^{+}(\mathbf{q},s)$ and $a_{(k)}(\mathbf{q},s)$ are the creation and annihilation operators of a particle of type (k) at point $\mathbf{q}$ with spin component s, satisfying the canonical commutation or anticommutation relations. Correspondingly one has a continuous family of stochastic Gaussian processes satisfying:

$$<<w(\mathbf{r},t)>>=0, \quad <<w(\mathbf{r},t)w(\mathbf{r}',t')>>= \frac{\gamma}{m_0^2}\delta(\mathbf{r}-\mathbf{r}')\delta(t-t'). \tag{3.14}$$

In the above formula m_0 is the nucleon mass, the parameter α is assumed to take the same value (10^{10} cm^{-2}) as in the case of QMSL and γ is related to the frequency $\lambda=10^{-16}$sec^{-1} of that model according to $\gamma=\lambda(4\pi/\alpha)^{3/2}$. With the above choices the theory exhibits the same features of standard CSL concerning the localization of nucleons. Obviously also other massive particles suffer localizations but with a rate depending on their masses. The nonhamiltonian terms in the dynamical equation lead to the objectification of position variables.

4. A first comparison of the two approaches.

In this and in the following section we will be interested in comparing the two theories, in particular by taking into account their formal structure, their achievements and the position they allow one to take with respect to the description of the world around us. As we will see there are deep and interesting analogies as well as remarkable differences concerning all the above mentioned points.

Bohm's theory, as already mentioned, assigns an absolutely prominent role to the positions of the particles of which the physical system under consideration is made up. These positions evolve in time according to the precise equations (2.1) and (2.2) which uniquely

determine, once the initial configuration is specified by the assignement of $\mathbf{q}_i^0$ and $\Psi(\mathbf{q}_i^0,0)$, the positions at all subsequent times. The quantum mechanical wave function, evolving according to the Schrödinger equation at all times and under all circumstances (so, no reduction ever takes place), enters the game only to the extent that it determines the particle trajectories. Such trajectories are such as to reproduce at all times the quantum distribution of the outcomes of position measurements on the particles themselves. Thus Bohm's theory can be considered in a very specific sense as "predictively equivalent" to SQM since it yields the same statistical predictions. On the other hand, it is a different theory with a precise conceptual status quite at variance with the one of SQM in all its interpretations. In particular, the theory is completely deterministic and fully reversible at the fundamental level. The positions of all particles of the universe and its wave function are uniquely determined at any time given their values at an earlier time.

The quantum probabilities characterizing SQM which within such a scheme have a nonepistemic nature, acquire an epistemic status: they derive from lack of knowledge, ignorance, and not from any irreducible element of chance in the laws of nature. Obviously, such ignorance plays a fundamental role and has to be accepted as a matter of principle. In fact, would the epistemic uncertainty turn out to be controllable, one could easily prepare statistical ensembles violating the statistical predictions of SQM. But what we cannot control and thus we are doomed to ignore concerns a certain intelligible and precise feature of the world and not an unspecified element of chance characterizing it.

On the contrary, dynamical reduction theories, which assume that the wave function represents the most exhaustive, the complete specification of an individual physical system, accept that genuine chance rules natural processes. The wave function does not evolve according to the linear, deterministic and reversible laws of SQM but obeys a modified equation obtained by including stochastic and nonlinear terms yielding a fundamentally irreversible behaviour. As a consequence the statistical predictions of dynamical reduction theories differ from those of SQM. Obviously the discrepancies are such not to contradict any known fact about physical processes. The irreversible nature of the evolution implies an energy increase even for isolated systems, so that the theory has an arrow of time built in its very grounds.

It has to be stressed that, in spite of the fact that in the theory, as well as in SQM, *there is nothing but the wave function* (Bell 1987, 41), the

nonlinear and stochastic modifications of the standard evolution assign to positions an absolutely privileged role. This is a feature which is shared by both theories we are comparing. As it has been discussed many times, and as it should be obvious from a general point of view, if one tries to induce the objectification of macroscopic properties in a dynamical way, one has to make a precise choice about which macroproperties have to become objective (the so called problem of the preferred basis). At the macrolevel the choice of the positions is quite a natural one (we recall Einstein's statement (Born 1971, 223) that *a macro-body must always have a quasi-sharply defined position in the objective description of reality*). Detailed investigations (Ghirardi, Rimini and Weber 1986, 470) have shown that, within a conceptual framework like the one of the dynamical reduction theories, the only way of inducing precisely such a type of macro-objectification by a fundamental universal dynamics, consists in assuming that the same variables, i.e. positions, are, with extremely low probabilities, objectified also at the microscopic level. Moreover it has also been possible to prove (Benatti, Ghirardi, Rimini and Weber 1988, 333) that processes objectifying other microproperties do not lead to the desired amplification mechanism.

5. How physical reality is described.

We take a very simple position about science. A scientific theory is a conceptual scheme dealing with formal elements which are supposed to embody some information about something *existing* and to account for the physical processes *which take place* "out there". Among these processes there are experiments performed by observers and also what have been called measurement-like events which *we are obliged to admit are going on more or less all the time, more or less everywhere* (Bell 1990, 17).

To be something more than a simple list of facts and to allow one to go beyond a purely instrumentalistic position, i.e. *to explain* rather than simply *to describe*, a theory must be supplemented with some sort of interpretation. We are interested in interpretations which allow one to make claims about properties related to specific variables, possessed by individual physical systems. With reference to this crucial point, which is the core of the debate about the foundations of quantum mechanics, we start by making precise the statement that such properties are objective.

Definition: we will claim that a property corresponding to a value (a range of values) of a certain variable in a given theory is *objectively possessed* or *accessible* when, according to the predictions of that theory,

there is an experiment (or a physical process) yielding reliable information about the variable that would, if performed (or taking place), give an *outcome* corresponding to the claimed value[5]. Thus, the crucial feature characterizing *accessibility* (as far as statements about individual physical systems are concerned) is the matching of the claims and the *outcomes* of physical processes testing the claims.

5.a. Hidden variables and Bohm theory.

Historically, the search for hidden variable theories has been motivated by the desire of getting a *classical picture* of the world compatible with what had been discovered about microscopic systems. In this spirit it was required that in the case of an individual physical system to any *observable* of the theory corresponds an objectively possessed property in the above sense. The Gleason theorem (Gleason 1957, 885) and the Kochen and Specker analysis (Kochen and Specker 1967, 59) have made clear that such a program cannot be consistently pursued. In particular, it has been shown that there exist at least one (actually infinitely many) non maximal observable A which turns out to be a function, in the Dirac sense, of two maximal noncommuting observables (A=f(B), A=g(C)), such that f(b)≠g(c), where b and c are the values assigned to the observables B and C, respectively, given the state of the system.

In spite of the above remarks one can build up deterministic hidden variable theories predictively equivalent to SQM provided one accepts the contextual nature of some observables i.e. the fact that the complete specification of the state assigns a definite truth value to a proposition only relative to a specified context. In particular, as stressed by Bell, if one follows this line, one has to accept that the outcome of the measurement of a nonmaximal observable might depend on which other observable compatible with it is actually measured on the system (Bell 1966, 447). From the previous discussion it should be obvious that, in general, properties referring to contextual observables cannot be objectively possessed[6]. After these general considerations we can analyze this problem within Bohm's theory.

Suppose one wants to claim that to any observable a definite property corresponds at all times. Let us consider a particle in one dimension and its momentum variable p.

a). If one would try to take a "classical" attitude, one could be tempted to identify the value of the momentum with $m\dot{q}$. This cannot be consistently

done. In fact, let the system be described, at t=0, by the non normalizable state corresponding to the wave function:

$$\Psi(q,0)=\frac{1}{2\sqrt{\pi\hbar}}(e^{i\frac{p_0 q}{\hbar}}+e^{-i\frac{p_0 q}{\hbar}}). \tag{5.1}$$

Since $\Psi(q,0)$ is real, according to Eq.(2.2) $\dot{q}(0)=0$ (independently from its actual position), for any particle of the ensemble associated to the considered state. Then, at t=0, the momentum of all particles would have the value $p(0)=m\dot{q}(0)=0$. On the other hand, if one would perform a measurement of the momentum of the particles of the ensemble, one would find, according to the predictions of the theory, either the value $+p_0$ or the value $-p_0$. Therefore, one can conclude that there is no measurement procedure of the momentum leading to a distribution of outcomes agreeing with the one of $m\dot{q}$ as given by Bohm's theory.

b). It is important to stress that, within the theory we are discussing, there are various measurement procedures of the momentum whose outcomes reproduce the quantum predictions. With reference to the previous situation, let us take into account a measurement like process in which the particle under consideration interacts with another particle acting as a measurement device for the momentum. This second particle (the pointer of the apparatus), whose position variable will be denoted as Q, before the interaction takes place is associated to a wave function which is very well localized around Q=0 (the "ready state" of the pointer). Accordingly, the state of the system of the two particles at the initial time t=0 is:

$$\Psi(q,Q,0)=\frac{1}{2\sqrt{\pi\hbar}}\left(\frac{\eta}{\pi}\right)^{1/4}(e^{i\frac{p_0 q}{\hbar}}+e^{-i\frac{p_0 q}{\hbar}})e^{-\frac{\eta}{2}Q^2}. \tag{5.2}$$

The evolution during the measurement time interval $(0,\bar{t})$ is governed by the interaction hamiltonian

$$H=cpP \tag{5.3}$$

where c is an appropriate coupling constant which can take positive or negative values at the experimenter's whim, and P is the momentum

conjugate to Q. Disregarding the kinetic energy terms, the wavefunction associated to the composite system at time $t\in(0,\bar{t})$ is:

$$\Psi(q,Q,t)=\frac{1}{2\sqrt{\pi\hbar}}\left(\frac{\eta}{\pi}\right)^{1/4}(e^{i\frac{p_0 q}{\hbar}}e^{-\frac{\eta}{2}(Q-cp_0 t)^2}+e^{-i\frac{p_0 q}{\hbar}}e^{-\frac{\eta}{2}(Q+cp_0 t)^2}). \tag{5.4}$$

The final pointer position will be centered around either $+cp_0t$ or $-cp_0t$. In order that the measurement outcome be reliable the condition $\eta(cp_0\bar{t})^2 >> 1$ must be satisfied. Obviously, when c>0, we will attibute to the momentum of the measured particle the value $+p_0$ if the pointer particle is, at the end of the process, on the positive real Q-semiaxis, while we will attribute the value $-p_0$ if it is on the negative semiaxis. If the experimenter changes the sign of the coupling constant c, the opposite association of the outcomes of the measurement to the position of the pointer must be made. No matter which choice is made, the considered measurement procedure gives a distribution of outcomes agreeing with the quantum mechanical one.

c). It is important to stress and easy to show that for any definite initial position of the measured particle (i.e. for any given value of the hidden variable) one can get different outcomes depending on the specific measurement procedure which is used. To see this, with reference to our case one has simply to take into account that simmetry considerations imply that the "pointer particle" cannot cross the origin. Thus, if it is on the positive semiaxis at t=0, it will be shifted by a positive quantity of the order of $|c|p_0\bar{t}$ independently from the sign of c. This shows that the outcome of the momentum measurement on the tested particle depends on the measurement set-up[7]. Therefore, in Bohm's theory, the momentum variable is contextual and, consequently, no objectively possessed property corresponding to such variable can be attributed to the particle.

To conclude this subsection we remark that the only consistent attitude one can take about properties within Bohmian mechanics is that of limiting his considerations to what Bell (Bell 1981, 611) has called the *exposed beables* of the theory, i.e. the position variables. Since such variables are noncontextual, their values are accessible according to our previous definition. Obviously, within such a scheme, one has to give the status of a beable of the theory also to the wave function since it determines the evolution of the noncontextual exposed beables.

5.b. CSL, preliminary considerations.

We now discuss an analogous problem within the CSL theory. To this purpose it is useful to devote some time to present a recently proposed interpretation for such a theory (Ghirardi, Grassi and Benatti 1995, 5) and to expouse the reasons which have motivated such a choice. We start by pointing out the inappropriateness of the Hilbert space topology to describe the concept of similarity or difference of two macroscopic states. In fact, suppose a physical system S is an almost rigid body and let us consider the following three states: $|\varphi^A>$, $|\varphi^B>$ and $|\tilde{\varphi}^A>$. The state $|\varphi^A>$ corresponds to a definite internal state of S and to its centre of mass being well localized around A, the state $|\varphi^B>$ is simply the displaced of $|\varphi^A>$ so that it is well localized in a far region B, the state $|\tilde{\varphi}^A>$ differs from $|\varphi^A>$ simply by the fact that one or a microscopic number of its "constituents" are in states which are orthogonal to the corresponding ones in $|\varphi^A>$.

It is obvious that, under any reasonable assumption about similarity or difference of the states of the universe, $|\tilde{\varphi}^A>$ must be considered very close (identical) to $|\varphi^A>$ while $|\varphi^B>$ must be considered very different from $|\varphi^A>$. On the other hand, according to the Hilbert space topology

$$\| (|\varphi^A> - |\tilde{\varphi}^A>) \| = \| (|\varphi^A> - |\varphi^B>) \| = \sqrt{2}. \tag{5.5}$$

This shows with striking evidence that the Hilbert space topology is totally inadequate for the description of the macroscopic world. Within CSL, as we will show in Subsection 5d, taking advantage of its dynamical features it becomes possible to overcome the now mentioned difficulty by introducing an appropriate topology.

To pursue our analysis, let S be a physical system which constitutes our universe, and let H(S) be its associated Hilbert space. Let us make an attempt to describe the physical reality of *what exists out there* by giving the status of exposed beables to the mean values of the average (over the characteristic volume of CSL) mass density operator (3.12):

$$\mathcal{M}(\mathbf{r},t) = <\Psi(t)\,|\,M(\mathbf{r})\,|\,\Psi(t)>. \tag{5.6}$$

If one assumes, as one can consistently do within a non relativistic quantum framework, that the system S contains a fixed and finite number of particles, Eq. (5.6) establishes, for a given t, a mapping of the unit sphere of $H^{(S)}$ into the space of positive and bounded functions of $\mathbf{r}$.

A question naturally arises: within the conceptual framework we are interested in does $\mathcal{M}(\mathbf{r},t)$ represent an objectively possessed property of physical systems? Before answering this question a digression and a comparison with the case of SQM are appropriate.

The map (5.6) is obviously many to one. To better focus on this point as well as for future purposes let us consider a physical system consisting of a very large number N of particles, two space regions A and B with spherical shape and radius R and two possible statevectors $|\Psi^{\oplus}>$ and $|\Psi^{\otimes}>$ of its Hilbert space. The state $|\Psi^{\oplus}>$ is the linear superposition, with equal amplitudes, of two states $|\Psi^A_N>$ and $|\Psi^B_N>$ in which the N particles are well localized with respect to the characteristic length (10^{-5} cm) of the model and uniformly distributed in regions A and B, respectively, in such a way that the density turns out to be of the order of 1 gr/cm^3. On the other hand, $|\Psi'^{\otimes}>$ is the tensor product of two states $|\Phi^A_{N/2}>$ and $|\Phi^B_{N/2}>$ corresponding to N/2 particles being uniformly distributed in region A and N/2 in region B, respectively:

$$|\Psi^{\oplus}> = \frac{1}{\sqrt{2}}\{|\Psi^A_N> + |\Psi^B_N>\}, \quad |\Psi'^{\otimes}> = |\Phi^A_{N/2}> \otimes |\Phi^B_{N/2}>. \tag{5.7}$$

It is trivially seen that the two considered states give rise to the same function $\mathcal{M}(\mathbf{r})$.

Let us now investigate the status of the quantity $\mathcal{M}(\mathbf{r})$ within SQM. It is quite obvious that in the case of $|\Psi^{\oplus}>$, $\mathcal{M}(\mathbf{r})$ cannot be considered as an "accessible" mass density function. To see this, let us suppose that one can use quantum mechanics to describe the gravitational interaction between massive bodies and let us consider the following gedanken experiment: a test mass is sent through the middle point of the line joining the centres of regions A and B with its momentum orthogonal to it. In the case of the state $|\Psi^{\otimes}>$ for the system of the N particles quantum mechanics predicts that the test particle will not be deflected. On the other hand, if the same test is performed when the state is $|\Psi^A_N>$ ($|\Psi^B_N>$), quantum mechanics predicts an upward (downward) deviation of the test particle. Due to the linear nature of the theory this implies that if one would be able to prepare the state $|\Psi^{\oplus}>$ the final state would be

$$|\phi> = \frac{1}{\sqrt{2}}\{|\Psi^A_N> \otimes |\phi^{UP}> + |\Psi^B_N> \otimes |\phi^{DOWN}>\}, \tag{5.8}$$

with obvious meaning of the symbols. If one includes the test particle into the "universe" and considers the mass density operator in regions corresponding to the wave packets $|\phi^{UP}>$ and $|\phi^{DOWN}>$, one discovers that nowhere in the universe is there a density corresponding to the density of the test particle. In a sense, if one would insist in giving a meaning to the density function he would be led to conclude that the particle has been split by the interaction into two pieces of half its density so that the test has not a definite outcome. According to our criterion this proves clearly that the mass density distribution $\mathcal{M}(\mathbf{r})$ in the case of the state $|\Psi^{\oplus}>$ is manifestly nonaccessible.

5.c. CSL and accessible reality.

Coming back to CSL, it should be obvious that the situation is radically different from the one in SQM since such a theory does not allow the persistence for more than a split second of states like $|\Psi^{\oplus}>$ which are those giving rise to the measurement problem.

To analyze this point it is useful to introduce a precise mathematical criterion emboding, for the mass density function, the accessibility request put forward at the beginning of this Section. To this purpose, let us consider the mass density variance at $\mathbf{r}$ at time t defined by the following map from the unit sphere of $H^{(S)}$ into $\Re^3$:

$$\mathcal{V}(\mathbf{r},t) = <\Psi(t)|[M(\mathbf{r})-<\Psi(t)|M(\mathbf{r})|\Psi(t)>]^2|\Psi(t)>, \qquad (5.9)$$

We consider the ratio

$$\mathcal{R}^2(\mathbf{r},t)=\mathcal{V}(\mathbf{r},t)/\mathcal{M}^2(\mathbf{r},t), \qquad (5.10)$$

and we claim that at time t the mass density at point $\mathbf{r}$ is accessible if:

$$\mathcal{R}(\mathbf{r},t)<<1. \qquad (5.11)$$

This criterion is clearly reminiscent of the probabilistic interpretation of the statevector in standard quantum mechanics. Actually, within such a theory, Eq.(5.11) corresponds to the fact that the spread of the operator $M(\mathbf{r})$ is much smaller than its mean value. Even though we take a completely different attitude with respect to the mean value of $M(\mathbf{r})$, it turns out to be useful to adopt the above criterion also within the new context. In fact, as we will discuss in what follows, when one has a space

region such that for all its points relation (5.11) holds, all reliable tests aimed to ascertain the mass density value give outcomes corresponding to $\mathcal{M}(\mathbf{r})$.

With reference to the previous example we stress that in the case of $|\Psi^{\otimes}>$ all points within regions A and B are such that criterion (5.11) is very well satisfied. In the case of $|\Psi^{\ominus}>$ for the same points one would have:

$$\mathcal{M}(\mathbf{r})\cong\frac{d}{2}m_0, \qquad \mathcal{V}(\mathbf{r})\cong\frac{d^2}{4}m_0^2, \qquad (5.12)$$

where d represents the average density in a volume of 10^{-15} cm^3 around $\mathbf{r}$. There follows

$$\mathcal{R}(\mathbf{r})\cong 1, \qquad (5.13)$$

so that, as expected, the average mass density in regions A and B is nonobjective.

Obviously for a microsystem, unless its wavefunction is localized better than 10^{-5} cm, its average mass density turns out to be nonaccessible at almost all times since CSL does not forbid superpositions of arbitrarily far apart states. This fact does not constitute a difficulty for the proposed interpretation, on the contrary, it embodies the fact that nature has compelled us to allow *electrons* (in general microsystems) *to enjoy the cloudiness of waves,* while our experience of the world requires *tables and chairs and ourselves, and black marks on photograph to be rather definitely in one place rather than another and to be described in classical terms* (Bell 1986, 11). The possible nonaccessibility of the mass density distribution does not entail that it is not real. Actually even regions in which the mass density is non accessible according to our mathematical criterion may play an important physical role[8] . We point out that the requests of being able to speak, within a context like the one of CSL leading to objectification at the macroscopic level, of objectively possessed properties and of introducing an appropriate topology to describe similarity and difference of macroscopic situations, has led us to consider "two levels of reality". The attribution of a real status to $\mathcal{M}(\mathbf{r})$ even when it is not accessible does not give rise to the difficulties one meets when one tries to interpret the wavefunction as giving the charge or the mass density of the elementary coistituents of nature. Actually this corresponds

to plainly accept the well known fact that *however far the wavefunction has extended, the reaction of a detector ... remains spotty* (Bell 1990, 17).

5.d. The appropriate topology for the CSL Model.

Let us denote by $\mathcal{U}^{(S)}$ the unit sphere in $H^{(S)}$ and let us consider the non linear map $\mathcal{M}$ associating to the element $|\varphi>$ of $\mathcal{U}^{(S)}$ the element $\mathcal{M}(\mathbf{r},|\varphi>)=<\varphi|M(\mathbf{r})|\varphi>$ of $L^{(2)}$, the Hilbert space of the real square integrable functions on $\Re^{(3)}$.

On $\mathcal{U}^{(S)}$ we define a topology by introducing a mapping Δ: $\mathcal{U}^{(S)}\otimes\mathcal{U}^{(S)}\rightarrow\Re^{+}$ according to:

$$\Delta(|\varphi>,|\psi>)=\left\{\int d\mathbf{r}[\mathcal{M}(\mathbf{r},|\varphi>)-\mathcal{M}(\mathbf{r},|\psi>)]^2\right\}^{1/2}. \qquad (5.14)$$

Such a mapping is not a distance since it may happen that $\Delta(|\varphi>,|\psi>)=0$ even though $|\varphi>\neq|\psi>$. However Δ meets all other properties of a distance:

$$\Delta(|\varphi>,|\psi>)=\Delta(|\psi>,|\varphi>)\geq 0,,\ \Delta(|\varphi>,|\psi>)\leq\Delta(|\varphi>,|\chi>)+\Delta(|\chi>,|\psi>), \qquad (5.15)$$

as one can easily prove.

From now on we will limit our considerations to the proper subset $\mathcal{A}^{(S)}$ of $\mathcal{U}^{(S)}$ of those states which are allowed by the CSL dynamics[9]. For any element $|\varphi>$ of $\mathcal{A}^{(S)}$ we then consider the set of states of $\mathcal{A}^{(S)}$ for which $\Delta(|\varphi>,|\psi>)\tilde{<}\varepsilon$. Here the quantity ε has the dimensions of a mass and is chosen of the order of $10^{18}m_0$, with m_0 the nucleon mass. From the properties of the map Δ it follows that:

i. $\{\Delta(|\varphi>,|\psi>)\tilde{<}\varepsilon$ and $\Delta(|\varphi>,|\chi>)\tilde{<}\varepsilon\}$ implies $\Delta(|\chi>,|\psi>)\tilde{<}\varepsilon$.

ii. $\{\Delta(|\varphi>,|\psi>)>>\varepsilon$ and $\Delta(|\varphi>,|\chi>)\tilde{<}\varepsilon\}$ implies $\Delta(|\chi>,|\psi>)>>\varepsilon$.

We have introduced the parameter ε in such a way that it turns out to be sensible to consider similar to each other states whose "distance" Δ is smaller than (or of the order of) ε. More specifically, when

$$\Delta(|\varphi>,|\psi>)\tilde{<}\varepsilon \qquad (5.16)$$

we will say that $|\varphi>$ and $|\psi>$ are "physically equivalent".

We now discuss the "distorted" (with respect to the Hilbert space one) topology associated to the "distance" Δ. First of all we stress that the two states $|\varphi^A>$ and $|\bar{\varphi}^A>$ which are maximally distant in the Hilbert space topology, turn out to be equivalent, i.e. to satisfy condition (5.16) in the new topology. This represents an example showing how such a topology takes more appropriately into account the fact that, under any sensible assumption, the "universes" associated to the considered states are very similar.

Obviously, one problem arises. Criterion (5.16) leads to consider equivalent states which are quite different from a physical point of view, even at the macroscopic level. To clarify this statement we take into account two states $|\varphi>$ and $|\psi>$ corresponding to an almost rigid body located, at $t = 0$, in the same position but with macroscopically different momenta, let us say $P = 0$ and P, respectively. Even though the two states are physically quite different, their distance at $t = 0$ is equal to zero. However, if one waits up to the time in which the state $|\psi>$ has moved away from $|\varphi>$, the "distance" $\Delta(|\varphi(t)>,|\psi(t)>)$ becomes large and the two states are no longer equivalent.

5.e. The CSL ontology.

As we have pointed out at the beginning of this Section, we are interested in describing something "existing and evolving out there" in such a way that the resulting physical picture is compatible with our experience of the world. This requires, as nicely pointed out by Bell (Bell 1989, 1) to make precise what he has called the kinematical and the dynamical aspects of the theory. *The kinematics requires to identify something as being really there* i.e. something *we are going to take seriously*, while the dynamics accounts for the way it changes with time.

For the reader who has followed the above analysis, it should be clear that, within CSL, physical systems are fully described by the statevector which evolves according to equations embodying genuine elements of chance. However, even though *the statevector is everything* (Bell 1987, 41), since the dynamics itself tends to make objective the mass density, it turns out to be appropriate to relate the kinematical elements in Bell's sense to the mass density distribution. This is what we have done in the previous Subsections by putting forward explicit criteria allowing one to recognize its being accessible and by showing the appropriateness of resorting to the mass density function to define the idea of similarity and difference between macroscopic situations.

In a sense we could state that the exposed beables of the theory are the values of the mass density function and that, at the appropriate level, the dynamics allows one to take them seriously. All other properties which emerge as a consequence of the now described process derive from this basic dynamical feature concerning the mass density function. With reference to the case analyzed at the end of the previous subsection we stress that, if one has a superposition of the two considered states, the macroscopic momentum of the system becomes accessible just as a consequence of the objectification of the mass density.

6. Classical and nonclassical features.

From the previous analysis it should be clear to everybody that both theories under discussion are basically non classical. However, they also exhibit some relevant classical features which, in particular, allow one to take a realistic position about nature. The classical aspects of CSL emerge only at the macroscopic level. On the contrary Bohm's theory has a feature in common with classical theories even in the case of microscopic systems, i.e., the fact that particles have definite positions. In spite of this, the status of a beable given to the wave function unavoidably brings into play highly nonclassical features. We recall that, e.g., in the case of one particle whose wavefunction is different from zero in two far apart space regions, in spite of the fact that the particle must be claimed to be *either* in one *or* in the other region, one must attribute a real status also to the branch of the wave function corresponding to the empty region.

To see this let us consider the case (see Fig.1) of a particle whose wavefunction is different from zero practically only along the two paths 1 and 2, and suppose that things are arranged in such a way that one has constructive interference in one of the two final directions and destructive interference in the other. In Bohm's theory the particle follows one of the two paths. However, for particles following path 1, the insertion of an absorber along path 2, allows them to activate, in some cases, the detector correponding to destructive interference.

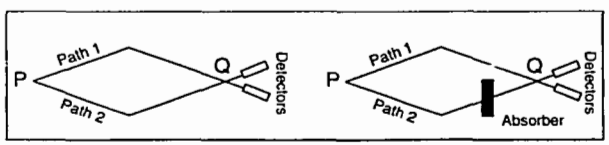

Fig.1. An interference experiment which puts into evidence the nonclassical features of Bohmian mechanics.

Let us compare this situation with the one in which the wavefunction is different from zero practically only along path 1. In such a case, obviously, the insertion of the absorber does not affect, neither within SQM nor in Bohm's theory, the subsequent evolution of the system. The previous statement about the emergence of nonclassical features of the theory should now be clear. When standard quantum mechanics allows to state that the particle follows path 1, the presence of the absorber along path 2 has no effect whatsoever on the particle itself. When the wave function is the superposition of two packets following the two paths, in standard quantum mechanics one cannot even think of the particle as following a definite path. On the contrary, in such a case, in Bohm's theory the particle actually follows one of the two paths. However even for particles following path 1 the presence of the absorber has a considerably invasive effect since it may remarkably change their evolution. This shows that even when the *noncontextual* property of *being* along path 1 is possessed by an individual system, it does not exhibit the feature of a classical particle possessing such a property. It seems to be aware of "being in a superposition".

7. Nonlocal Aspects and Relativistic Invariance.

As is well known, the principal locality assumption needed to prove Bell's theorem for the stochastic case (Bell 1971, 171) is equivalent to the conjunction of two other assumption, viz., in Shimony's terminology, Parameter Independence and Outcome Independence (Suppes and Zanotti 1976, 445; van Fraassen 1982, 25; Jarrett 1984, 569; Shimony 1984, 225). In view of the experimental violation of the Bell inequality, one has to give up either or both of these assumptions. We are interested in comparing the status of the two theories under examination from this point of view.

To start with, let us fix our notation. We will denote by λ all parameters (which may include the quantum mechanical statevector or even reduce to it alone) which provide, within the theoretical scheme one is interested in, the complete specification of the state of an individual physical system. For simplicity we will refer to a standard EPR-Bohm like situation and we will denote by

$$p_\lambda^{LR}(x,y;\mathbf{n},\mathbf{m}) \qquad (7.1)$$

the joint probability of getting the outcome x (x=±1) in a measurement of the spin-component along **n** at the left (L), and y (y=±1) in a measurement of the spin-component along **m** at the right (R) wing of the apparatus.

We assume that the experimenter at L can make a free-will choice of the direction **n** and similarly for the experimenter at R and the direction **m**. Both experimenters can also choose not to perform the measurement.

Bell's locality assumption can be expressed as:

$$p_\lambda^{LR}(x,y;\mathbf{n},\mathbf{m}) = p\lambda^{L}(x;\mathbf{n},*)p_\lambda^{R}(y;*,\mathbf{m}) \qquad (7.2)$$

where the symbol * appearing in the probability distributions at the r.h.s. denotes that the corresponding measurement is not performed.

As already remarked condition (7.2) is equivalent to the conjunction of two logically independent conditions, i.e.:

$$p_\lambda^{L}(x;\mathbf{n},\mathbf{m}) = p_\lambda^{L}(x;\mathbf{n},*)$$

$$p_\lambda^{R}(y;\mathbf{n},\mathbf{m}) = p_\lambda^{R}(y;*,\mathbf{m}) \qquad (7.3.I)$$

and

$$p_\lambda^{LR}(x,y;\mathbf{n},\mathbf{m}) = p_\lambda^{L}(x;\mathbf{n},\mathbf{m})p_\lambda^{R}(y;\mathbf{n},\mathbf{m}) \qquad (7.3.II)$$

where we have denoted, e.g., by the symbol $p_\lambda^{L}(x;\mathbf{n},\mathbf{m})$ the probability of getting, for the given settings **n**,**m**, the outcome x at L. Equations (7.3.I) express *parameter independence*, i.e. the requirement that the probability of getting an outcome at L (R) is independent from the setting chosen at R (L), while Eq. (7.3.II) (*outcome independence*) expresses the requirement that the probability of an outcome at one wing does not depend on the outcome which is obtained at the other wing.

The above "splitting" of the locality requirement into two independent conditions is particularly useful to discuss their different conceptual implications with respect to relativistic requirements. In fact, as proved by Jarrett himself (Jarrett 1984, 569), when conditions (7.3.I) are violated, if one had access to the variables λ which specify completely the state of individual physical systems, one could send faster-than-light signals from R (L) to L (R). On the contrary, if only condition (7.3.II) is violated, then faster-than light signalling cannot be achieved since the stochastic outcome at a wing cannot be controlled by the experimenter.

It is well known that in a deterministic theory (i.e. one for which the range of any one of the above probability distributions is the set {0,1}) there cannot be outcome dependence, so that violation of the locality requirement (7.2) implies parameter dependence. This fact by itself shows that deterministic theories which reproduce quantum predictions meet more serious difficulties with relativity than do stochastic ones.

Before coming to analyze these matters with reference to the two theories we are concerned with we recall that SQM violates Locality by violating only Outcome Independence and that it does not consent faster-than-light signalling between distant observers (Eberhard 1978, 392; Ghirardi, Rimini, andWeber 1980, 293).

7.a. Parameter Dependence and Outcome Dependence in the Two Theories.

Concerning the pilot wave theory we can state without further analysis that it violates parameter independence since, as we have already remarked, any deterministic theory reproducing the quantum mechanical probabilities must do so. Bohmian mechanics is a well defined theory with a precise formalism and conceptual status. If one takes seriously, as we do, the fact that the complete specification of the state of an individual physical system, at a certain time, is given by the positions of its constituents, one has to accept that the theory implies instantaneous effects at-a-distance at its fundamental level[10]. Obviously, since one has not access to the "hidden" positions and, at the ensemble level, the theory gives the same predictions of SQM, one cannot take advantage of this fact to actually send superluminal signals.

Coming to CSL we first of all prove that it satisfies the condition of parameter independence and therefore, since for the experiment under discussion its predictions agree with those of SQM, it must violate the outcome independence requirement.

Since within CSL the stochastic dynamical reduction takes place just as a consequence of the fact that macroscopic objects evolve into a superposition of macroscopically different states, to perform our analysis we must take into account which apparatuses are present and how the triggering mechanism describing the micro-macro interactions works. For simplicity we will assume that such interactions taking place at L and R are governed by appropriate coupling constants g_L and g_R. To investigate whether there is parameter dependence we will then compare situations in which one of the coupling constants is made equal to zero

(corresponding to no measurement being performed) with a situation in which it is non-zero.

We recall that within the CSL model the initial situation is *completely* characterized by the assignment of the initial statevector $|\Psi,0>$. As stated above, the unfolding of the individual physical processes is governed by a linear evolution equation depending on the coupling constants g_L and g_R and on a specific realization $w(\mathbf{x},t)$ of the stochastic process. The probability of occurrence of such processes depends on the overall physical situation and, in particular, on the coupling constants being equal to zero or not. This is due to the fact that the evolution of the statevector $|\Psi,0>$ depends on the coupling constants and the statevector itself enters in determining the cooked probability density of occurrence of the stochastic processes.

We can now come to the specific discussion of the process we are interested in. Strictly speaking, one should take into account the whole physical situation and in particular one should specify the initial state of the system plus the apparatuses and study the subsequent evolution. Nevertheless, it should be clear that, for the analysis we are interested in, one can simplify the description by limiting his considerations to the spin Hilbert space only, provided one correspondingly changes the value of the parameter γ governing the reduction rate in such a way that reduction takes place within the times which are characteristic for a macroscopic measurement device.

Taking such a position, one has, in the case in which both apparatuses are switched on ($g_R \neq 0$ and $g_L \neq 0$) a linear dynamical equation analogous to (3.2):

$$\frac{d|\Psi_{w_L,w_R}(t)>}{dt} = \{[(\sigma^L \cdot \mathbf{n})w_L(t) - \gamma] + [(\sigma^R \cdot \mathbf{m})w_R(t) - \gamma]\}|\Psi_{w_L,w_R}(t)> \tag{7.4}$$

The probability distribution of the stochastic processes is obtained through the cooking procedure prescribed by the theory:

$$P_{Cook}[w_L \& w_R] = P_{Raw}[w_L \& w_R]\,\||\Psi_{w_L \& w_R}(t)>\|^2 \tag{7.5}$$

Where $P_{Raw}[w_L \& w_R]$ is the probability distribution associated to a white noise with

$$<<w_L(t)>>=0; \quad <<w_R(t)>>=0; \quad <<w_L(t)w_R(t')>>= \gamma\delta_{L,R}\ddot{o}(t-t'). \tag{7.6}$$

Obviously the appearance of the two terms at the r.h.s. of (7.4), corresponds to the fact that both apparatuses are switched on and are triggered by the microsystems at L and R, respectively. If one wants to compare this situation with the one in which the observer at right, at free will, switches off the apparatus by making $g_R=0$, one has to consider another stochastic equation, i.e.

$$\frac{d|\Psi_{w_L}(t)>}{dt} = [(\sigma^L \cdot \mathbf{n})w_L(t) - \gamma]|\Psi_{w_L}(t)>. \tag{7.7}$$

The solutions of equations (7.4) and (7.7) at time t for the same initial conditions are[11]

$$|\Psi_{B_L,B_R}(t)> = e^{F_{LB_L}(t)}e^{F_{RB_R}(t)}|\Psi,0> \tag{7.8}$$

and

$$|\Psi_{B_L}(t)> = e^{F_{LB_L}(t)}|\Psi,0>, \tag{7.9}$$

respectively. In eqs.(7.8) and (7.9) we have put:

$$F_{LB_L}(t) = \sigma^L \cdot \mathbf{n}B_L(t) - \gamma t; \qquad F_{RB_R}(t) = \sigma^R \cdot \mathbf{n}B_R(t) - \gamma t, \tag{7.10}$$

where

$$B_L(t) = \int_0^t d\tau w_L(\tau), \qquad B_R(t) = \int_0^t d\tau w_R(\tau). \tag{7.11}$$

We come back now to eq. (7.4) and we evaluate the cooked probability density of occurrence of the Brownian processes $B_L(t)$ and $B_R(t)$ by multiplying the raw probability density by the square of the norm of the statevector (7.8). As usual we have:

$$P_{Cook}[B_L(t) \& B_R(t)] = P_{Raw}[B_L(t) \& B_R(t)]\,\||\Psi_{B_L \& B_R}(t)>\|^2 \tag{7.12}$$

and

$$P_{Raw}[B_L(t)\&B_R(t)] = P_{Raw}[B_L(t)]P_{Raw}[B_R(t)] \tag{7.13}$$

Taking into account eq.(7.8), one then gets from (7.12):

$$P_{Cook}[B_L(t)\&B_R(t)] = P_{Raw}[B_L(t)]P_{Raw}[B_R(t)]\|\Psi_{B_L\&B_R}(t)>\|^2 =$$
$$P_{Raw}[B_L(t)]\|e^{F_{1B_1}(t)}|\Psi,0>\|^2 P_{Raw}[B_R(t)]\left\|\frac{e^{F_{RB_R}(t)}e^{F_{1B_1}(t)}|\Psi,0>}{\|e^{F_{1B_L}(t)}|\Psi,0>\|}\right\|^2. \tag{7.14}$$

Let us consider the marginal cooked probability density of $B_L(t)$:

$$P^{\#}_{Cook}[B_L(t)] = \int d[B_R(t)]P_{Cook}[B_L(t)\&B_R(t)] =$$
$$= P_{Raw}[B_L(t)]\|e^{F_{1B_1}(t)}|\Psi,0>\|^2 \tag{7.15}$$
$$\cdot\int d[B_R(t)]P_{Raw}[B_R(t)]\left\|\frac{e^{F_{RB_R}(t)}e^{F_{1B_1}(t)}|\Psi,0>}{\|e^{F_{1B_1}(t)}|\Psi,0>\|}\right\|^2.$$

Since the equation

$$\frac{d|\Psi_{w_R}(t)>}{dt} = [(\sigma^R\cdot\mathbf{m})w_R(t) - \gamma]|\Psi_{w_R}(t)> \tag{7.16}$$

preserves the stochastic average of the square of the norm of the statevector, the last integral in eq.(7.15) takes the value 1. This means that $P^{\#}_{Cook}[B_L(t)]$ turns out to equal the probability $P_{Cook}[B_L(t);*]$, i.e. the cooked probability density of occurrence of the Brownian process $B_L(t)$ for the same initial condition if the process were described by eq.(7.7) or, equivalently, if the apparatus at R were switched off.

As we have already remarked in Section 3, within CSL it is possible to establish a one-to-one correspondence between the outcome at left (right) and the specific value taken by the Brownian process $B_L(t)$ ($B_R(t)$) for $t\to\infty$. Then, the above proof of the equality between $P^{\#}_{Cook}[B_L(t)]$ and $P_{Cook}[B_L(t);*]$ amounts to having shown that the theory under consideration exhibits parameter independence. The analysis performed by Bell (Bell 1987, 41) of the QMSL model by resorting to a multi-time formalism for two far apart systems shows that also this theory does not present parameter dependence effects.

7.b. Parameter dependence and relativistic invariance requirements.

In (Ghirardi, Grassi, Butterfield and Fleming 1993, 341) the connection between nonlocality and "genuine Lorentz invariance" has been investigated[12]. In particular it has been proved that it is not possible to build a "genuinely Lorentz invariant" theory which exhibits parameter dependence effects and which, in the nonrelativistic limit, does not give rise to backward causation. To analyze this point, let us start by considering, with reference to an EPR-Bohm like situation, the nonrelativistic limit of the theory in a given reference frame O. In this frame we assume that the measurement at right takes place at an earlier time than the one at left, i.e., $t_R<t_L$ and that there exist a set of values of the parameters λ which have a non zero probability of occurrence and are such that:

$$p_\lambda{}^L(x;\mathbf{n},\mathbf{m}) \neq p_\lambda{}^L(x;\mathbf{n},*) \tag{7.17.I}$$

while, the absence of backward causation implies, for all λ's:

$$p_\lambda{}^R(y;\mathbf{n},\mathbf{m}) = p_\lambda{}^R(y;*,\mathbf{m}). \tag{7.17.II}$$

In the case of a Galilean theory, there is a residue, or an analogue of Lorentz invariance which can be taken into account in the case in which one is dealing with two widely separated systems. In particular, if one assumes that the two regions R and L are very far apart from each other, there exist reference frames (say, O') moving even with a very small velocity with respect to O, but for which the temporal order of the space-time events (R,t_R) and (L,t_L) is inverted, i.e., $t'_R>t'_L$. Since the probability of occurrence of an event is an objective fact (i.e. it cannot depend from the considered reference frame) it must hold also for O' that

$$p'_{\lambda'}{}^L(x';\mathbf{n}',\mathbf{m}') = p'_{\lambda'}{}^L(x';\mathbf{n}',*) \tag{7.18}$$

where λ' is the parameter which identifies for O' the systems which are identified by λ for O, and the primes on p, x, n and m have an obvious meaning. It is important to stress that (7.18) is simply the request that both observers can look at the same systems and that they agree on the objective probabilities. However, as already remarked, for O', $t'_R>t'_L$. If the theory were invariant for the considered transformation, an analogous situation would occur also for the observer O for some values of λ. But this

contradicts assumption (7.17.II) expressing that for O the past cannot have a parametric dependence from the future. This argument constitutes therefore a proof that, under the considered assumptions, there cannot be a "genuinely relativistic invariant" theory which exhibits parameter dependence. Here we understand "genuinely relativistic" rather strongly: it excludes theories (Like Lorentz's classical electromagnetic theory, or the quantum field theory version of Bohm's theory (Bohm 1952,180) that have a preferred frame which cannot be discovered by experiments. We favor genuinely relativistic theories. We recall that Bell, despite his deep appreciation of Bohm's theory, believed (Bell 1989,1) that to make the Lorentz group phenomenological in this way *is an incredible position to take - I think it is quite logically consistent, but when one sees the power of the hypothesis of Lorentz invariance in modern physics, I think you just can't believe in it*.

In a recent preprint (Berndl, Dürr, Goldstein and Zanghì 1995) attention has been called to the fact that there seems to be the possibility (Dürr, Goldstein and Zanghì 1990, 374) of a Bohmian quantum field theory *in which a foliation of space-time into space-like hypersurfaces is an additional beable*. Due to this fact the considered theory has no preferred reference frame. It can circumvent our impossibility proof because, in the nonrelativistic limit, it exhibits backward causation effects. The authors assume that the hypersurfaces of the foliation do not intersect and this would be a nice feature of the model since it would guarantee that the backward causation effects implied by it would not give rise to paradoxical consequences (see below). However it seems to us that the model is not so precisely formulated to allow one to be sure that the above assumption is dynamically consistent.

Another attempt one could think of would be to look, e.g., for a Tomonaga-Schwinger type equation for the beables f with no particular foliation (σ). Obviously one could meet serious difficulties: this equation could turn out to be non integrable, etc. However, we think that even if a theory of this type could be formulated, due to the fact that the space-like hypersurfaces cross each other and there are parameter dependence effects (which allow faster than light signalling would one have access to the hidden parameters λ), it would exhibit backward causation effects which would give rise to paradoxical situations. Tipically, one could consider three space-time points A, B, C such that B is space-like with respect to both A and C, while C is in the absolute past of A. Would one have access to the hidden variables there might be situations in which A

could cause an effect at B, and B might correspondingly make C aware of what A has done. Being C in the absolute past of A, he could forbid A to perform the original action.

To conclude this Section we remark that, in our opinion, it is not so obvious that there could be a genuinely Lorentz invariant theory exhibiting parameter dependence effects.

8. Conclusions.

In this paper we have compared Bohm's theory and the dynamical reduction theories with reference to the most relevant conceptual issues about the foundational problems of quantum theory. We have pointed out the pros and cons of both approaches. Our conclusion is that they both deserve to be taken seriously as yielding a possible realistic description of nature.

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Notes.

1. It is useful to point out that at least two different attitudes can be taken about the interpretation of the theory. They have been focused in a recent paper by Bas van Fraassen (van Fraassen 1994, 7) and have been denoted as the Cartesian and the Newtonian construal, respectively. According to the first attitude what is real of the theory are only the positions of the particles. Consequently, since equal positions at a given time can evolve into different positions at a subsequent time, under this construal the theory is not deterministic. In the Newtonian construal, besides the position of the particles, also the wave function is considered as real. The theory is then strictly deterministic. We will take the Newtonian attitude in what follows.

2. Recently, some beautiful results about this point (Dürr, Goldstein and Zanghì 1992, 843) have been derived. The authors show that typical configurations of the universe as a whole imply that the positions of the particles of an ensemble associated to a given wavefunction Ψ are distributed according to $|\Psi|^2$ with overwhelming probability. We cannot discuss this important point here, we refer the reader to the above paper.

3. In eq. (3.8) and following we have changed the notation for the state vector from $|\Psi_w(t)>$ used in eq. (3.2) to $|\Psi_B(t)>$ to stress the fact that, under our assumptions, the state at time t does not depend on the specific sample function $w(\tau)$ in the interval (0,t) but only on its integral B(t) of eq.(3.9).

4. Note that, even though the spread $\sqrt{\gamma t}$ tends to ∞ for $t\to\infty$, its ratio to the distance $2(\alpha-\beta)\gamma t$ between the two considered peaks of the distribution tends to zero.

5. In a previous paper (Ghirardi, Grassi, and Benatti 1995, 5) we have simply used the expression "objective" to denote what we call here "objectively possessed or accessible". We had a vague feeling that the term we were using was not completely pertinent and could give rise to misunderstandings. Prof. S. Goldstein has appropriately called our attention on the fact that both usual meanings of that term, i.e., "real" or "opposite to subjective" do not fit with the sense which emerges for it from our work and has suggested the expression "accessible". In the search for an expression which would embody precisely what we had in mind we have considered also the possibility of resorting to the expression "empirically adequate". However, this term reminds directly the "empirical reality" concept introduced by B. d'Espagnat (d'Espagnat 1990,

1147). Due to this fact the use of such term could seriously mislead the reader. In fact, according to the definition of empirical reality, due to the practical impossibility of distinguishing, at the macroscopic level, a pure state from a statistical mixture, replacing one with the other would be empirically adequate, which is completely at odds with the meaning we want to give to the expression objectively possessed or accessible.

6. We point out that here we assume that the measurement can be chosen at free will. In any case, even if one gives up such an assumption, it turns out to be impossible to attribute to the system considered by itself objective properties, i.e. properties which do not depend on the overall experimental set up. Accordingly, property attribution becomes a relational feature.

7. We note that if the "pointer particle" would be on the negative semiaxis at t=0, it would be shifted by a negative quantity of the same order, independently from the sign of c. Thus, the outcome of the measurement of p, given a complete specification at t=0 of the state of the measured particle, depends both from the sign of the coupling constant which can be choosen at free will by the observer and from the initial position of the "pointer particle" (a parameter which, however, cannot be controlled by the experimenter).

8. A typical example is represented by an intergalactic cloud with a very low mass density and wavefunctions for its particles with a large spread. According to our strict criterion (5.8), the mass density of the cloud is nonobjective, while, if we adopt the physically meaningful criterion put forward at the beginning of this Section and to test it we resort to consider its effects, e.g., on the trajectory of a far asteroid, we would be led to state that our claims about the mass density are objectively true since they agree with the "physical outcome". Here, an important distinction is appropriate. The theory contains a specific dynamical mechanism and specific parameters which allow a precise mathematical formulation of the objectivity criterion. Obviously, when one is interested in a particular class of physical processes the objectivity requirement at the beginning of this section could be satisfied even though the strict mathematical one is not. This does not raise any problem. It finds its counterpart in the fact that to analyze a specific physical situation one can use a coarser graining than the one corresponding to the fundamental characteristic length of the theory. Accordingly, if we adapt the graining to the problem of interest, we can assert that the mass of the cloud is objective. Obviously, while for practical purposes one can change the graining according to the problem

he is interested in, adopting a graining which is finer than the fundamental one of the theory makes no sense because if one takes the theory seriously as describing the laws governing the evolution of the universe, then mass density would almost never become objective for such a graining.

9. For a more precise definition of such a set see (Ghirardi,Grassi and Benatti 1995, 5).

10. This attitude has the same logical status as the assertion that the probabilities concerning physical processes have an epistemic character and are not due to the presence of genuine elements of chance in nature.

11. In eq.(7.8) and following we make, for the same reasons dicussed there, a change of notation analogous to the one made for (3.8).

12. See below for the precise meaning we attribute to this expression.

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