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**INTERNATIONAL CENTRE FOR
THEORETICAL PHYSICS**

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IN A DUSTY PLASMA**

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**INTERNATIONAL
ATOMIC ENERGY
AGENCY**



**UNITED NATIONS
EDUCATIONAL,
SCIENTIFIC
AND CULTURAL
ORGANIZATION**

VOL 27 № 07

MIRAMARE-TRIESTE

International Atomic Energy Agency
and
United Nations Educational Scientific and Cultural Organization
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

LOW-FREQUENCY DUST-LOWER-HYBRID MODES
IN A DUSTY PLASMA

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ABSTRACT

The existence of low-frequency dust-lower-hybrid modes in a magnetized dusty plasma has been examined. These modes arise on account of the inequalities of charge and number densities of electrons, ions, and dust particles, and finite Larmor radius effects in a dusty plasma.

MIRAMARE - TRIESTE

October 1995

Wave propagation in dusty plasmas¹⁻⁶ is a subject of much interest in recent years because of its relevance in space and astrophysical plasmas, interstellar gas, supernova remnants, proto-star and proto-planetary environments, planetary magnetospheres as well as in laboratory plasmas. A number of workers⁷⁻¹² have studied the wave propagation properties, damping, absorption, wave scattering, new modes and instabilities, etc. in dusty plasmas either by considering the dust dynamics with fluid equations or by Vlasov-kinetic theory with finite dust correlation for the static random distribution of the highly charged ($Z_d/e = 10^3 - 10^4$) and massive ($m_d/m_{proton} \sim 10^6 - 10^{18}$) dust grains with or without magnetic field and temperature. Since the mass of a dust grain can be many orders higher than that of the ions, the finite Larmor radius effect must become significant in a hot magnetized dusty plasma.

In this Letter, we study the low-frequency modes in a hot and uniformly magnetized dusty plasma by considering dust dynamics with Vlasov-kinetic theory. In particular, we investigate the low-frequency electrostatic modes propagating perpendicular to the magnetic field and having frequencies below the ion-cyclotron frequency of ions.

For propagation of any electrostatic mode perpendicular to external magnetic field and using Vlasov equation expressed in guiding center coordinate¹³, we obtain the following dispersion relation

$$k_{\perp}^2 = \sum_{\alpha} \sum_{n=1}^{\infty} \frac{4n^2 \omega_{p\alpha}^2 \omega_{c\alpha}^2}{V_{th\alpha}^2 (\omega^2 - n^2 \omega_{c\alpha}^2)} I_n(b_{\alpha}) \exp(-b_{\alpha}), \quad (1)$$

where α denotes electrons, ions or dust particles, $b_{\alpha} = k_{\perp}^2 V_{th\alpha}^2 / \omega_{c\alpha}^2$, $\omega_{p\alpha}^2 = 4\pi q_{\alpha}^2 n_{\alpha} / m_{\alpha}$, $\omega_{c\alpha} = q_{\alpha} B_0 / m_{\alpha} c$, $V_{th\alpha}^2 = 2k_B T_{\alpha} / m_{\alpha}$; q_{α} , m_{α} , B_0 and c are the charge and mass of particle α , external magnetic field and the velocity of light in a vacuum, respectively.

We now solve Eq.(1) explicitly for the dispersion relation for perpendicular electrostatic waves for the following two conditions :

I. **Low-temperature limit** : $\omega_{cd} < \omega < \omega_{ci} < \omega_{ce}$ and $\frac{k_{\perp} V_{thc}}{\omega_{ce}} < \frac{k_{\perp} V_{thi}}{\omega_{ci}} < \frac{k_{\perp} V_{thd}}{\omega_{cd}} < 1$

The dispersion relation with small temperature correction is obtained from Eq.(1) by

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expanding with $n = 0, +1$ as

$$\omega^2 = \omega_{cd}\omega_{ci} \left(\frac{q_d}{q_i} \right) \left(\frac{\bar{n}_{od}}{\bar{n}_{oi}} \right) \left(1 + \frac{\bar{n}_{oe} m_e}{\bar{n}_{oi} m_i} \right)^{-1} \left(1 - \frac{k_{\perp}^2 V_{thd}^2}{\omega_{cd}^2} \right), \quad (2)$$

which for $\bar{n}_{oi} > \bar{n}_{oe}$ and $T_d = 0$ yields

$$\omega^2 = \omega_{cd}\omega_{ci} \left(\frac{q_d}{q_i} \right) \left(\frac{\bar{n}_{od}}{\bar{n}_{oi}} \right). \quad (3)$$

This may be called the "dust lower-hybrid" frequency because it involves the hybrid dynamics of dust grains and ions and $\bar{n}_{od} \neq \bar{n}_{oi} \neq \bar{n}_{ei}$ in dusty plasmas. The motion of electrons is not important in this frequency regime. In ordinary electron-ion plasma, an analogous mode is the lower-hybrid oscillations, $\omega = \sqrt{\omega_{ci}\omega_{ce}}$ for $\bar{n}_{oe} = \bar{n}_{io}$ and

$$\omega_{ci} < \omega < \omega_{ce}.$$

II. High-temperature limit : $\omega_{cd} < \omega < \omega_{ci} < \omega_{ce}$ and $\frac{k_{\perp} V_{thd}}{\omega_{cd}} > 1 > \frac{k_{\perp} V_{thi}}{\omega_{ci}} > \frac{k_{\perp} V_{the}}{\omega_{ce}}$

The dispersion relation for the Larmor radius of dust particle greater than the wavelength can be obtained from Eq.(1) as

$$\omega^2 = \frac{2}{\sqrt{2\pi}} \frac{\omega_{pd}^2 \omega_{cd}^3 \omega_{ci}^2}{k_{\perp}^3 V_{thd}^3 \omega_{pi}^2 (1 + \bar{n}_{oe} m_e / \bar{n}_{oi} m_i)}, \quad (4)$$

where $\omega_{pi} > \omega_{ci}$ has been assumed. This is a new undamped mode (dust lower-hybrid mode) in a dusty plasma. It has no analogue in ordinary electron-ion plasma. It may be mentioned here that using plasma fluid equations, Shukla and Rao¹⁴ have recently studied low-frequency modes (lower-hybrid-like) in a cold dusty plasma with relatively a higher frequency limit, $k_{\perp} V_{thi}, \omega_{ci}, \omega_{cd} \ll \omega \ll \omega_{ce}$ and $k_{\perp}^2 V_{thd}^2 / \omega_{cd}^2 = (k_{\perp}^2 V_{thi}^2 / \omega_{ci}^2) (m_d / m_i) \ll 1$. When the dust grains are charged by plasma currents (electron and ion) only, the charges on the grains become negative and $\bar{n}_{oi} > \bar{n}_{oe}$. In this situation, the motion of the electrons can be neglected as is evident in Eq.(4). Since the mass of a dust grain is many orders higher than that of ions (*e.g.*, $m_d / m_i \sim 10^{12}$), the dust Larmor radius may be much higher than the ion Larmor radius in a magnetized dusty plasma. This finite dust Larmor radius effect is the cause of this new electrostatic mode propagating perpendicular to the magnetic field in the dusty plasma.

We have shown the existence of the new electrostatic dust-lower-hybrid modes below the ion cyclotron frequency (but above the dust cyclotron frequency) in a magnetized

dusty plasma. The dust-lower-hybrid frequency exists in a cold magnetized dusty plasma because of the dynamics of the dust particles and ions and the non-equality of charge ($q_d \neq q_i$) and the number density of particles ($\bar{n}_{od} \neq \bar{n}_{oi}$). The electron motion is not important for these low frequency modes. The finite Larmor radius effect of dust motion and unequal distribution of charges and number densities of dust particles and ions are responsible for this new dust lower-hybrid mode (cf., Eq.(4)).

It may be mention here that the parametric wave interactions (*viz.*, decay instability), scattering of waves off these modes, and the effects of these ultra-low frequency modes on the dust crystalization are of great importance, and the work on these lines is in progress.

Acknowledgements :

The author is grateful to Professors N.L. Tsintsardze, Ahbijit Sen, P.K. Shukla, N.N. Rao, and B. Dasgupta for stimulating discussions during the course of the work. He also acknowledges the support of the Swedish Agency for Research Cooperation with Developing Countries (SAREC) for the Associateship visit to the International Centre for Theoretical Physics (ICTP), Trieste, Italy.

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