"IDEAL GAS" GLUON PLASMA WITH MEDIUM DEPENDENT DISPERSION RELATION
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"Ideal Gas" Gluon Plasma with Medium Dependent Dispersion Relation

An "ideal gas" model with temperature dependent particle mass is constructed for the gluon plasma equation of state. This simple model gives us an example of a system with temperature dependent effective Hamiltonian. To satisfy thermodynamical relations in these systems, standard statistical mechanics formulas have to be supplemented by special requirements which are considered in details. A self-consistent "ideal gas" formulation is used to describe Monte Carlo lattice data for the thermodynamical functions of SU(2) and SU(3) gluon plasma.
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I. INTRODUCTION

Recently an ideal gas model of the gluon plasma (GP) was considered [1,2] to describe the Monte Carlo (MC) lattice data in SU(2) and SU(3) gluodynamics for the thermodynamical functions of the deconfined phase, i.e., at temperatures $T$ above the critical value $T_c$. It was assumed that the system of interacting massless gluons (the same arguments can be applied when quarks are included [2]) can be effectively represented at $T > T_c$ as an ideal gas of "massive" non-interacting "gluons" (and "quarks"). The gluon dispersion relation for particle of energy $\omega^*$ and momenta $k$ in such a system is assumed to be

$$\omega^*(k, T) = (k^2 + m^2(T))^{1/2}, \quad (1)$$

where temperature dependent "gluon mass" $m = m(T)$ appears due to the medium effects. The ideal gas picture of gluons and quarks with "thermal masses" was further used in Refs. [3,4] for some physical applications. The "massive gluons" were always treated as non-interacting "quasi-particles" with quantum numbers of real gluons and have, therefore, only two states of polarizations (transverse polarized modes).

We stress that one needs some additional physical input, as missed in the previous studies [1-4], to realize this simple "ideal gas" model in a thermodynamically self-consistent way. To illustrate our argument we begin with ideal Bose gas formulas for particle of mass $m$ and zero chemical potential. The pressure and energy density are given by [5]

$$p_{id}(T, m) = -T \frac{d}{2\pi^2} \int_0^\infty k^2 dk \ln [1 - \exp(-\omega/T)] , \quad (2)$$

$$\epsilon_{id}(T, m) = \frac{d}{2\pi^2} \int_0^\infty k^3 dk \frac{\omega}{\exp(\omega/T) - 1} , \quad (3)$$

where $\omega = (k^2 + m^2)^{1/2}$ and $d$ is the degeneracy factor ($d = 6$ for SU(2) and $d = 16$ for SU(3) gluons). Both Eq. (2) and Eq. (3) with $\omega = \omega^*(k, T)$ (1) were assumed in Refs. [3,4] as the starting point for the GP equation of state. However, when the particle mass in Eqs. (2,3) is $T$-dependent the fundamental thermodynamical relation between the pressure $p(T)$ and energy density $\epsilon(T)$ [5], namely,

$$\epsilon(T) = T \frac{dp(T)}{dT} - p(T) \quad (4)$$
is not satisfied. One can easily check that the only solution of Eq. (4) for \( m(T) \), with \( p(T) \) and \( \epsilon(T) \) given by Eqs. (2,3), is \( m = m_0 = \text{const.} \). This implies that the ideal gas mode (2,3) is thermodynamically consistent for particles of constant mass only - any temperature dependence \( m = m(T) \) violates the thermodynamical identity (4)!

In Refs. [1,2] only the ideal gas form for the pressure function (2) with temperature dependent mass was assumed. The energy density is then calculated from the thermodynamical relation (4). In this case the energy density function of \( T \) and \( m \) does not have the ideal gas form (3). One can equally well start with the ideal gas form (3) with \( \omega = \omega^*(k,T) \) (1) for the energy density and solve differential equation (4) to find a \( p(T) \) function. The obtained function is then different from the ideal gas formula (2). One can, therefore, construct two different "ideal gas" formulations with different \( m(T) \) functions from the same MC lattice data fitting either \( p_{MC}(T) \) by Eqs. (1,2) or \( \epsilon_{MC}(T) \) by Eqs. (1,3). Both these two formulations are not, however, physically reasonable in the sense that they do not correspond to the statistical mechanics of an ideal gas with the dispersion relation (1). We'll show that, in a thermodynamically consistent formulation of an ideal gas system with \( \omega = \omega^*(k,T) \), neither \( p(T) \) nor \( \epsilon(T) \) has the simple form (2,3). They should contain additional medium contributions. It turns out that only entropy density preserves its ideal gas form.

The aim of our paper is to give a statistical mechanics formulation for systems with medium dependent dispersion relation. We present the procedure to realize system thermodynamical self-consistency and consider then self-consistent "ideal gas" model with \( m = m(T) \) to describe MC lattice data in SU(2) and SU(3) gluodynamics.

The outline of the paper is as follows. In Sec. II we formulate requirements of a thermodynamical self-consistency in a statistical mechanics for systems with medium dependent dispersion relation. A self-consistent "ideal gas" model for the GP with temperature dependent mass of particles is constructed in Sec. III. We use this model to fit MC lattice data of SU(2) and SU(3) gluodynamics in Sec. IV. Sec. V concludes our work with a summary of the results and an outlook.

II. STATISTICAL MECHANICS OF SYSTEMS WITH MEDIUM DEPENDENT DISPERSION RELATION

To construct a thermodynamically consistent "ideal gas model" with
medium dependent dispersion relation, we briefly recall how thermodynamical identities are obtained in the statistical mechanics. For a system with Hamiltonian operator $H$ and conserved charge number operator $N$, the statistical mechanics' definitions of pressure, energy density and conserved charge number density are [5]

$$
p(T, \mu) = \frac{T}{V} \ln \text{Tr} \left( e^{-\frac{H - \mu N}{T}} \right) \equiv \frac{T}{V} \ln Z(T, \mu, V) , \tag{5}$$

$$
\epsilon(T, \mu) = \frac{1}{V} \frac{1}{Z(T, \mu, V)} \text{Tr} \left( H e^{-\frac{H - \mu N}{T}} \right) , \tag{6}$$

$$
n(T, \mu) = \frac{1}{V} \frac{1}{Z(T, \mu, V)} \text{Tr} \left( N e^{-\frac{H - \mu N}{T}} \right) , \tag{7}$$

where $\mu$ is the chemical potential and the "thermodynamical limit" when $V$ goes to infinity is assumed. The thermodynamical identities are

$$
\epsilon = Ts + \mu n - p , \tag{8}
$$

$$
s = \left( \frac{\partial p}{\partial T} \right)_\mu , \quad n = \left( \frac{\partial p}{\partial \mu} \right)_T \tag{9},$$

where $s$ is the entropy density. They reduce to Eq. (4) with $s = dp/dT$ when $\mu = 0$ and mean nothing more but the identities

$$
\left( \frac{\partial \text{Tr} \left( e^{-\frac{H - \mu N}{T}} \right)}{\partial T} \right)_\mu \equiv \frac{1}{T^2} \text{Tr} \left( (H - \mu N) e^{-\frac{H - \mu N}{T}} \right) , \tag{10}
$$

$$
\left( \frac{\partial \text{Tr} \left( e^{-\frac{H - \mu N}{T}} \right)}{\partial \mu} \right)_T \equiv \frac{1}{T} \text{Tr} \left( N e^{-\frac{H - \mu N}{T}} \right) . \tag{11}$$

which are valid, of course, for any $T$ and $\mu$ independent operators $H$ and $N$. For an ideal gas Hamiltonian [5]

$$
H_{ld} = \sum_{i=1}^{d} \sum_{k} \omega(k) \ a^\dagger_{k,i} a_{k,i} , \quad (12)
$$

where the index "$i$" corresponds to the particle internal degrees of freedom, e.g., different spin and colour states for gluons, evaluations of Eqs. (5,6) are straightforward. For large $V$, i.e., in the thermodynamical limit, the
summation over \( k \) in Eqs. (5,6) can be substituted by the integration over the momentum phase space

\[
\sum_{i=1}^{d} \sum_{k} ... = d \frac{V}{(2\pi)^d} \int dk ...
\]

and Eqs. (2,3) are easily obtained. They correspond to the case \( \mu = 0 \) in Eqs. (5,6) and \( \alpha_{k,i}, \alpha_{k,i}' \) in Eq. (12) the Bose creation and annihilation operators, respectively. The calculations of Eqs. (5-7) with \( \mathcal{H} \) given by Eq. (12) for fermions and nonzero \( \mu \) values are also obvious. They lead to the well known ideal gas expressions where \( \omega(k) \) function for particle ("quasi-particle") excitation energy can take any arbitrary form.

When \( \omega(k) \) in Eq. (12) is substituted by \( \omega^*(k,T,\mu) \), we still obtain ideal gas expressions from Eqs. (5-7). For example, both \( p(T) \) and \( \epsilon(T) \) as obtained from Eqs. (5,6) with \( \omega = \omega^*(k,T) \) (1) and \( \mu = 0 \) would still have the ideal gas form (2,3). However, problem immediately arises. The Hamiltonian of the system becomes temperature dependent and statistical mechanics' definition of the energy density (6) contradicts the thermodynamical relation (4). This occurs because the identity (10) is no longer valid. The recipe for solving this problem is evident: upon introducing temperature and/or chemical potential dependent effective Hamiltonian \( H_{\text{eff}} \) one should additionally require the fulfillment of identities (10,11) to guarantee the thermodynamical consistency of the model.

Same requirements take place in the theory of nuclear matter [6] where quasi-nucleon dispersion relation

\[
\omega^*(k,T,\mu) = (k^2 + M^2)^{1/2} + U , \quad (13)
\]

with effective mass \( M(T,\mu) \) and "potential energy" \( U(T,\mu) \), appears due to the presence of interaction between nucleons and the scalar and vector fields in the mean-field approximation. These "fields" contribute to the effective Hamiltonian of the system and, therefore, produce additional "field" terms in the \( p \) and \( \epsilon \) functions to restore the thermodynamical consistency of the model (see Ref. [7]).

To generalize this procedure, we consider an effective Hamiltonian \( H_{\text{eff}}(c_1, c_2, ...) \) depending on phenomenological parameters \( c_1, c_2, ... \) which are assumed to be functions of temperature \( T \) and chemical potential \( \mu \) (or chemical potentials \( \mu_i \) if system has several conserved charges). To satisfy thermo-
dynamical identities \((8,9)\) for \(H = H_{\text{eff}}\) with \(T\) and/or \(\mu\) dependent parameters \(c_1, c_2, \ldots\), one should require additionally

\[
\left( \frac{\partial p}{\partial c_1} \right)_{T, \mu, c_2, \ldots} = 0, \quad \left( \frac{\partial p}{\partial c_2} \right)_{T, \mu, c_1, \ldots} = 0, \quad \ldots
\]  \(14\)

for the pressure function \(p = p(T, \mu, c_1, c_2, \ldots)\) defined in Eq. \((5)\) with \(H = H_{\text{eff}}\). Requirements \((14)\) are equivalent to Eqs. \((10,11)\).

To apply this procedure to an ideal gas of quasi-particle excitations we observe that in general Hamiltonian \((12)\) should be rewritten as

\[
H_{\text{eff}} = \sum_i^{\text{d}} \sum_k^\infty \omega^*(k) a_{k,i}^\dagger a_{k,i} + E_0^*,
\]  \(15\)

where

\[
\omega^*(k) \equiv \omega(k, c_1, c_2, \ldots), \quad E_0^* \equiv E_0(c_1, c_2, \ldots).
\]  \(16\)

\(E_0^*\) in Eq. \((16)\) is the system energy in the absence of quasi-particle excitations. This zero point energy is of constant value \(E_0\) in the standard case of medium independent \(\omega(k)\). It is usually subtracted from the system energy spectrum. It cannot be done, however, for \(T\) and/or \(\mu\) dependent dispersion relation \(\omega^*(k)\) as system's lowest state energy \(E_0^*\) becomes also a function of \(T\) and/or \(\mu\). This function is defined by the requirement of thermodynamical self-consistency \((14)\).

Calculating now the thermodynamical functions \(p(T, \mu, c_1, c_2, \ldots)\) and \(\epsilon(T, \mu, c_1, c_2, \ldots)\) from Eqs. \((5,6)\) with effective Hamiltonian \((15)\) and quasi-particle number operator

\[ N = \sum_i^{\text{d}} \sum_k^\infty a_{k,i}^\dagger a_{k,i} \] (for simplicity, we do not write down explicitly possible anti-quasi-particle terms where \(\mu \to -\mu\)) we find

\[
p(T, \mu, c_1, c_2, \ldots) = \pm T \frac{d}{2\pi^2} \int_0^\infty k^2 dk \ln \left[ 1 \mp \exp \left( \frac{(\omega^* - \mu)}{T} \right) \right] - B^*,
\]  \(17\)

\[
\epsilon(T, \mu, c_1, c_2, \ldots) = \frac{d}{2\pi^2} \int_0^\infty \frac{k^2 dk \omega^*}{\exp[-(\omega^* - \mu)/T] \mp 1} + B^*,
\]  \(18\)

where the upper sign is for bosons and the lower for fermions. The first terms in Eqs. \((17,18)\) are the standard ideal gas expressions \(p_{id}(T, \mu, \omega^*),\ \epsilon_{id}(T, \mu, \omega^*)\) and the second ones are additional medium contributions with

\[
B^* = B^*(c_1, c_2, \ldots) \equiv \lim_{V \to \infty} \frac{E_0^*}{V}.
\]
It is \( T \) and/or \( \mu \) dependent system energy density without quasi-particle excitations. However, the statistical mechanics' expressions for quasi-particle number density \( n(T,\mu) \) (7) and entropy density \( s(T,\mu) \) (9) preserve their ideal gas form:

\[
n(T,\mu,c_1,c_2,...) = \frac{d}{2\pi^2} \int_0^\infty \frac{k^2dk}{\exp[-(\omega^* - \mu)/T] + 1} \equiv n_{id}(T,\mu,\omega^*), \quad (19)
\]

\[
s(T,\mu,c_1,c_2,...) = (\epsilon + p - \mu n)/T = (\epsilon_{id} + p_{id} - \mu n_{id})/T \equiv s_{id}(T,\mu,\omega^*). \quad (20)
\]

As we shall see this last fact is the important consequence of the above formulation for its phenomenological applications.

### III. "IDEAL GAS" GP WITH \( m = m(T) \)

Turning to our problem of the GP "ideal gas" with \( m = m(T) \) we have \( p(T,m) = p_{id}(T,m) - B^*(m) \), \( \epsilon(T,m) = \epsilon_{id}(T,m) + B^*(m) \) from Eqs. (17,18). \( p_{id}(T,m) \) and \( \epsilon_{id}(T,m) \) are given by Eqs. (2,3) with \( \omega = \omega^*(k,T) \) defined by Eq. (1). For \( m(T) = m_0 = \text{const} \), these expressions satisfy the thermodynamical identity (4) with \( B^* = B = \text{const} \). If \( m_0 = 0 \) Eqs. (19,20) coincide with the well known Bag Model equation of state for the GP, e.g., Ref. [8].

If the particle mass is temperature dependent \( m = m(T) \), the additional requirement

\[
\left( \frac{\partial p(T,m)}{\partial m} \right)_T = 0
\]

should be fulfilled. It is a special case of our general requirements (14). The condition (21) was introduced in Ref. [7] as a self-consistency equation for phenomenological extension of the mean-field theory approach of Ref. [6], where \( B^* \) has the meaning of the scalar field energy density. Explicit evaluation of \( (\partial p_{id}(T,m)/\partial m)_T \) in Eq. (21) leads to

\[
\frac{dB^*}{dm} = -m \left( \frac{d}{2\pi^2} \int_0^\infty \frac{k^2dk}{\omega^*(k,T)} \frac{1}{\exp[\omega^*(k,T)/T] - 1} \right). \quad (22)
\]

If \( B^*(m) \) is known, Eq. (22) gives us an equation for \( m(T) \) function and the model is completely defined. If, on the other hand, function \( m(T) \) is known we can calculate \( B^*(T) \) from Eq. (22) as

\[
B^*(T) = B_0 - \frac{d}{2\pi^2} \int_{T_0}^T dT' m \left( \frac{dm}{dT'} \right) \int_0^\infty \frac{k^2dk}{\omega^*(k,T')} \frac{1}{\exp[\omega^*(k,T')/T'] - 1}, \quad (23)
\]
up to an arbitrary integration constant \( D_o = B^*(T_o) \). Our equations will then take the form

\[
p(T, m) = p_{id}(T, m) - B^*(T), \quad (24)
\]

\[
\epsilon(T, m) = \epsilon_{id}(T, m) + B^*(T). \quad (25)
\]

For a constant zero value of the gluon mass Eqs. (24,25) are, as we already mentioned, reduced to the standard bag model equation of state [8]

\[ p(T, m = 0) = p_{SB} - B, \epsilon(T, m = 0) = \epsilon_{SB} + B, \]

where positive constant value of \( B \) ("vacuum pressure") is assumed and \( p_{SB}, \epsilon_{SB} \) stand for the corresponding Stefan-Boltzmann expressions: \( p_{SB} \equiv \frac{d \pi^2 T^4}{90}, \epsilon_{SB} \equiv \frac{d \pi^2 T^4}{30}. \)

This equation of state can not reproduce MC lattice data \( p_{MC}(T), \epsilon_{MC}(T) \) for SU(2) [9] and SU(3) [10] gluodynamics, since \( \epsilon = \epsilon_{SB} + B > \epsilon_{SB} \), while \( \epsilon_{MC} < \epsilon_{SB} \).

The form of Eqs. (24,25) was first used in Ref. [11] to fit MC lattice data for the SU(3) GP. It was considered as an extension of the bag model with two independent functions \( m(T) \) and \( B^*(T) \). The MC lattice data used in Ref. [11] were, however, rather poor and their fitting led to the constant values \( m = m_o \approx 2.5T_c, B^*/4 = B_0^*/4 \approx 1.25T_c. \) Therefore, the problem of temperature dependent gluon mass and self-consistency condition (22) (or (23)) which relates these two functions was not discussed.

The physical meaning of our \( \epsilon_{id}(T) \)-term \( B^* \) in Eqs. (24,25) can be rather different from that in the bag model, and we have no a priori estimate of its numerical value and even of its sign.

**IV. "IDEAL GAS" MODEL RESULTS FOR SU(2) AND SU(3) GLUODYNAMICS**

In this section, we use the "ideal gas" GP model constructed in the preceding section to describe MC lattice data for the pressure and energy density of SU(2) [9] and SU(3) [10] GP.

As we have no microscopic model for the \( B^*(m) \)-function we have to start from Eqs. (23-25) with unknown function \( m = m(T) \). A straightforward way to find it is to compare the model (23-25) with MC lattice data for the entropy density because it does not depend on \( B^*(T) \):

\[
s(T, m) \equiv \frac{p(T, m) + \epsilon(T, m)}{T} = \frac{p_{id}(T, m) + \epsilon_{id}(T, m)}{T} \equiv \epsilon_{id}(T, m). \quad (26)
\]
Namely, as we have already shown in more general case of Eq. (20), just
the entropy density function preserves its simple ideal gas form without any
additional medium contributions in a self-consistent treatment of the “ideal
gas” model.

In Fig. 1 the MC lattice data of Ref. [9] are shown for the energy density,
pressure and entropy density of SU(2) GP (at $T > T_c$) normalized to their
Stephan-Boltzmann limits $\epsilon_{SB}(T), p_{SB}(T)$ and $s_{SB}(T) \equiv s_{id}(T, m = 0) =
4\pi^2 T^3/15$. The corrections of all thermodynamical function data for the finite
size lattice effects are done according to the procedure of Ref. [12]. Requiring
our entropy density function (28) to be equal to MC lattice data $s_{MC}(T)$ of
Fig. 1, we find the $m(T)$-values at the corresponding values of temperature.
They are shown in Fig. 2, where we also show the $m(T)$-values obtained from
the prescription $p_{id}(T, m) = p_{MC}(T)$, as it has been done in Ref. [1], and
from the prescription $\epsilon_{id}(T, m) = \epsilon_{MC}(T)$. As seen in Fig. 2 and Fig. 5 below
for SU(3) case our $m(T)$-functions (full triangles) are qualitatively similar to
those obtained in Refs. [1,2] (circles), but they are, of course, rather different
quantitatively, as our “ideal gas” model is quite different from the formulation
of Refs. [1,2].

Then, with condition (23) and MC lattice value for $p_{MC}(T_o)$ to define the
integration constant $B_o$, we reconstruct corresponding $B^*(T)$-values as shown
in Fig. 3. Once $m(T)$ and $B^*(T)$ are found we can proceed to calculate
$p(T, m)$ and $\epsilon(T, m)$ functions according Eqs. (24,25). The obtained pressure
and the energy density are compared with MC lattice data [9] in Fig. 4. Ideal
gas formulas (2,3) for $p(T)$ and $\epsilon(T)$ calculated with the same $m = m(T)$ of
Fig. 2 determined from $s_{MC}(T)$ are also shown in Fig. 4. It is seen that
lack of the thermodynamical self-consistency in this approach leads to rather
strong deviations of $p_{id}(T, m)$ and $\epsilon_{id}(T, m)$ from $p_{MC}(T)$ and $\epsilon_{MC}(T)$
data.

The complete agreement of our model results with MC lattice data is not
surprising. If MC lattice data for $p(T), \epsilon(T), s(T)$ are thermodynamically
consistent (i.e., they satisfy identity (4), or, equivalently, $s(T) = dp/dT = (p+
\epsilon)/T$), as it is, then only one of these functions, say $p(T)$, is really independent:
$\epsilon(T)$ and $s(T)$ can be calculated from $p(T)$ using thermodynamical identities.
But in our model we also have one “independent” function $m(T)$. Therefore,
if we fit completely MC lattice data for $s(T)$ and fulfill thermodynamical
identity (4) due to our Eq. (23), the model pressure and energy density
(24,25) have to coincide with MC lattice data too.

For MC lattice data of SU(3) GP [10] our fitting procedure is essentially the
same. We show only our results for $m(T)$ and $B^*(T)$ functions in Figs. (5,6).
These two functions are connected by Eq. (23) and our model equations (24,25) are again in a complete agreement with MC lattice data $p_{MC}(T)$, $\epsilon_{MC}(T)$ [10].

V. DISCUSSIONS AND CONCLUSIONS

We construct thermodynamically self-consistent "ideal gas" model with $m = m(T)$ to describe MC lattice data for SU(2) and SU(3) GP. A complete agreement with MC lattice data for $p(T)$ and $\epsilon(T)$ can be achieved as we have an arbitrary function $m = m(T)$ which is found from $s_{MC}(T)$ data. We can not, however, make definite conclusions about the physical validity of this model. To clarify its efficiency some other properties of the GP (like "heavy-quark potential" in the GP) should be calculated and compared with MC lattice data. Note that $\omega^*(k,T)$ is more a system "microscopic" characteristic than thermodynamical functions themselves.

A physical interpretation of the $m(T)$ and $B^*(T)$ behavior found from MC lattice data is also desirable. One can be surprised that $m(T)$ and absolute value of $B^*(T)$ found from MC lattice data increase at large $T$ when the GP is expected to approach the Stefan-Boltzmann limiting behavior of massless noninteracting gluons. We note that to have the essential deviations of the thermodynamical functions (24,25) from their Stefan-Boltzmann limits one needs large values of $m/T$ and $B^*/T^4$. As seen from Figs. 7-8, these quantities are large at $T$ near $T_c$, and we expect they will gradually go to zero at $T \rightarrow \infty$. A possible physical origin of the $m = m(T)$ dependence was discussed in Ref. [1].

The analysis of systems with $\omega(k) = \omega^*(k,T,\mu)$ presented here can be useful for other models of strongly interacting matter equation of state. We mention only two examples. The first one is the "cut-off" model of the GP [13] with a temperature dependent "cut-off" parameter $K(T)$ used in Ref. [9]. It corresponds to the following dispersion relation of the "ideal gas" excitations in the GP

$$\omega^*(k,T) = k \theta (k - K(T)) + (k^2 + M^2)^{1/2} \theta (K(T) - k) . \quad (27)$$

The first term in Eq. (27) represents massless high-momenta gluons and the second one is low-momenta glueballs with mass $M$. A self-consistent statistical mechanic treatment of the system (27) leads to the equations similar to Eqs. (17,18) with $\mu = 0$. Again $K(T)$-function should be extracted from
the equation $s_{id}(T, K) = s_{MC}(T)$, but not from that for the pressure as done in Ref. [9]. The second example concerns a temperature dependent pion dispersion relation, as suggested by Shuryak [14],

$$\omega^*(k, T) = \left( u^2(T) k^2 + m^2 \right)^{1/2}, \quad (28)$$

where $u(T)$ is the temperature dependent refraction index. In Ref. [14], $u(T)$-function was extracted from some "data" for the pion energy density $\epsilon_r(T)$. Again, such a procedure is not thermodynamically consistent. We intend to study both problems in details in a forthcoming publication.

Statistical mechanics with any $T$ and $\mu$ independent Hamiltonian operator satisfies automatically the thermodynamical identities. An ideal gas model with temperature dependent particle mass gives us a simplest example of systems with $H = H_{eff}(T, \mu)$. For such models the standard statistical mechanics formulas have to be supplemented by special requirements to guarantee that fundamental thermodynamical identities will be fulfilled. We have shown that these additional requirements can be presented in a simple mathematical form of Eq. (14). Particular examples of this kind have been known in the mean-field theory of nuclear matter [6] and its phenomenological extension [7].

"Ideal gas" models with $T$ and/or $\mu$ dependent dispersion relation $\omega^*(k, T, \mu)$ should be constructed according to Eqs. (15,16). The lowest state energy $E_0^*$ without quasi-particle excitations becomes a function of $T$ and/or $\mu$ in this case and has to be included in the "ideal gas" formulation for the thermodynamical consistency. The connection between $\omega^*$ and $E_0^*$ is given by Eq. (14).

In conclusion we repeat two main points of our study the systems with $\omega(k) = \omega^*(k, T, \mu)$:

1. The quasi-particle dispersion relation, e.g., Eqs. (1,13,27,28), should be used for or extracted from the entropy density function.
2. The pressure and and energy density for such systems consist in additional medium contributions which are to be found from the requirements of the thermodynamical self-consistency (14).
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References


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Figure Captions

Fig. 1. MC lattice data of SU(2)-gluodynamics [9] for the energy density (boxes), pressure (circles) and entropy density (full triangles) at $T > T_c$ normalized to its Stephan-Boltzmann limits.

Fig. 2. Gluon "thermal mass" $m = m(T)$ obtained from the fit to MC SU(2) lattice data [9] for the entropy density (full triangles), energy density (boxes) and pressures (circles) in Fig. 1 by the corresponding expressions for the ideal gas entropy density (24), ideal gas energy (3) and ideal gas pressure (2). The "pressure-fit" (circles) was done in Ref. [1].

Fig. 3. The $B^*(T)$ function found from Eq. (19) with $m(T)$ in Fig. 2 (full triangles) obtained from the MC SU(2) lattice data for the entropy density.

Fig. 4. MC lattice data of SU(2)-gluodynamics [9] for the pressure (circles) and energy density (boxes) at $T > T_c$ normalized to their Stephan-Boltzmann limits. Full lines are obtained from Eqs. (20,21) with $m(T)$ in Fig. 2 found from the entropy density (full triangles) and $B^*(T)$-function given in Fig. (3). Dashed lines are obtained from Eqs. (2,3) with the same $m(T)$ function but without $B^*(T)$ contributions.

Fig. 5. Gluon "thermal mass" $m = m(T)$ obtained from the fit to MC lattice SU(3) data [10] for the entropy density (full triangles), energy density (boxes) and pressures (circles) by the corresponding expressions for the ideal gas entropy density (24), ideal gas energy (3) and ideal gas pressure (2). The "pressure-fit" (circles) for these SU(3) data was done in Ref. [2].

Fig. 6. The $B^*(T)$ function found from Eq. (19) with $m(T)$ in Fig. 5 (full triangles) obtained from the MC SU(3) lattice data [10] for the entropy density.

Fig. 7. Gluon "thermal mass" $m = m(T)$ obtained from the fit to MC SU(2) lattice data [9] for the entropy density (full triangles of Fig. 2) divided by temperature.

Fig. 8. The $B^*(T)$ function of Fig. 2 divided by $T^4$. 
Fig. 1

\[ \frac{P}{P_{SB}} - \frac{\Omega}{\Omega_{SB}} - \frac{\delta}{\delta_{SB}} \]

\[ T/T_0 \]

SU(2)

Fig. 2

\[ \frac{m}{T_0} \]

\[ T/T_0 \]

SU(2)
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