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Planar Permanent Magnet Multipoles: Measurements and Configurations*

T. Cremer, R. Tatchyn
Stanford Linear Accelerator Center, Stanford University, Stanford, CA 94309

Abstract

Biplanar arrays of N rectangular permanent magnet (PM) blocks can be used to generate high quality N-pole fields in close proximity to the array axis. In applications featuring small-diameter charged particle beams, N-poles of adequate quality can be realized at relatively low cost using small volumes of PM material. In this paper we report on recent measurements performed on planar PM multipoles, and discuss techniques for improving the field quality of such devices at distances appreciably far away from the axis. Applications to hybrid/PM insertion device designs for linac-driven Free Electron Laser (FEL) operation in the x-ray range are described.

I. INTRODUCTION

In recent years, the development of a novel class of PM multipole (N-pole) field generators has been initiated [1,2]. The basic construction principle, illustrated in Fig. 1, is to arrange N rectangular PM pieces, magnetized with the easy axis perpendicular to two opposed faces, into a biplanar array with N/2 pieces per plane. In general, while the pieces in each quadrant of the x-y plane can have arbitrary dimensions, x-placements, and magnetizations, the overall structural and field geometries possess symmetry with respect to the y-z and x-z planes, and the normal convention is to have each of the two sets of magnet surfaces closest to the x-z plane be coplanar. If we postulate an ideal N-pole generator to be a structure with N-fold rotational symmetry [3], the essential concept of planar PM multipoles is seen to be based on the reduction from N-fold to 2-fold rotational symmetry. This reduction, while allowing the design and fabrication of N-poles with fully open horizontal apertures and configurations of utmost economy and simplicity, incurs the penalty that the field away from the axis develops a much higher multipole content than an "ideal" structure with N-fold (N>2) rotational symmetry. In the following sections, selected theoretical and experimental aspects of planar PM multipole field distributions are reviewed and possible applications to FEL insertion device design are considered.

II. PLANAR PM MULTIPOLE FIELDS

The entire class of planar PM multipoles can be grouped into two families: 1) N=4n; and 2) N=4n-2; where n ε {1,2,..}.

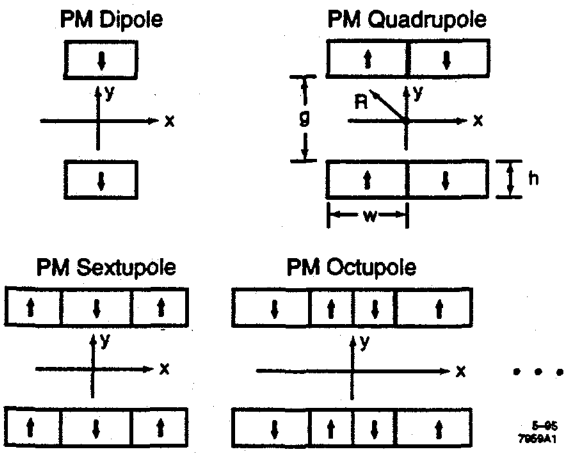

Figure 1. Planar PM multipoles composed of pieces of equal height (h), with no lateral spacing between pieces. Symmetry axis (z axis), along which all the PM pieces have equal length L, is perpendicular to the page.

Thus, the first family includes the quadrupole (quad), octupole, dodecapole, etc., and the second family includes the dipole, sextupole, decapole, etc. For each family, the corresponding magnetic scalar potential (for L>>g) in the vicinity of the symmetry axis can be approximated by a real Taylor series expansion as follows:

$$\phi_{4n} \equiv C_{11}xy + E_{13}(xy^3 - x^3y) + G_{15}(3xy^5 - 10x^3y^3 + 3x^5y) + \ldots; \quad (1)$$

and

$$\phi_{4n-2} \equiv B_{01}y + D_{21}(3x^2y - y^3) + F_{41}(5x^4y - 10x^2y^3 + y^5) + \ldots; \quad (2)$$

where the subscripted capital coefficients are functions of the PM parameters. To configure a given N-pole, the dimensions, spacings, and magnetizations of the N pieces must be designed to eliminate all the multipole coefficients lower than the desired leading N-pole coefficient. As formulas (1) and (2) explicitly indicate, the field of each such N-pole will not only exhibit its leading N-pole coefficient, but will also contain an infinitude of higher-pole terms with strengths proportional to increasing integral powers of distance (R) from the z axis.

To illustrate, the configuration of a planar PM sextupole (leading coefficient D_{21}) demands the establishment of $B_{01}=0$, which in general can be used to help determine the relative dimensions and/or magnetizations of the PM pieces [2]. To illustrate an example of the higher multipole content associated with a general N-pole, a measured rotating-coil spectrum [4] of a SmCo quadrupole (see Fig. 1, top right) is shown in Fig. 2. The measured peaks agree to better than 0.5% with the corresponding amplitudes calculated with a computer code

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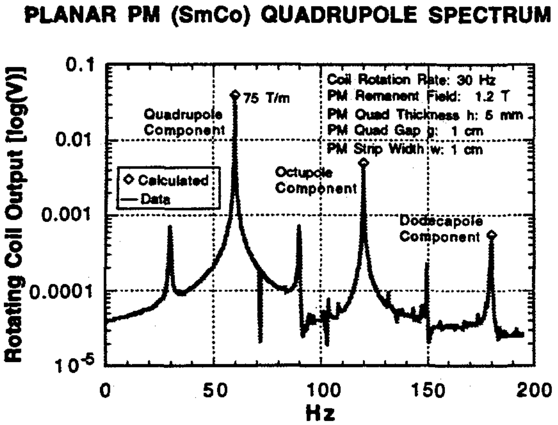

Figure 2. Rotating-coil planar PM quadrupole spectrum. Coil parameters: 50 turns; dimensions 1 cm x 2 mm. The peaks at odd multiples of 30 Hz stem from coil axis misalignments.

written to simulate both the planar multipole fields and the rotating coil apparatus.

III. APPROACHES TO PLANAR MULTIPOLE FIELD QUALITY IMPROVEMENT

Theoretically, it is well known that N-fold rotationally symmetric multipoles generating a single N-pole field component can be closely approximated. This is accomplished by shaping the (equipotential) pole surfaces to make the variable coefficient in the desired N-polar potential term equal to a constant along the pole contour. In general, this implies that the elimination of all but the N-polar term requires pole contours of infinite extent. With infinite contours, for example, the ideal quadrupole and sextupole potentials would be given, respectively, by $\phi_4 = C_{11}xy$ and $\phi_6 = D_{21}(3x^2y - y^3)$. In practice, of course, all pole contours must be finite, which results in a non-vanishing set of higher-pole field components in practical structures. Notwithstanding this, in a well-designed 4-fold rotationally symmetric quad the total field energy in the higher-pole field components can be made typically less that 10^{-6} of that in the leading component in regions proximate to the symmetry axis [5].

By contrast, the freedom of adjusting the parameters of planar PM multipoles to attain a comparable degree of field purity is seen to be severely limited. Specifically, the shapes of the PM pieces (all rectangular cross sections) cannot be modified, forcing all equipotential surfaces to be planar. Secondly, confinement to a plane prevents the rotation of the pieces as a means of approximating to a curved 2-dimensional equipotential contour. Under these constraints, the principal means of improving the field quality has to be associated with the adjustable degrees of freedom of the planar PM multipole, namely, the number, dimensions, x-placements, and magnetizations of the PM pieces. Approaching the problem from this perspective, eq's. (1) and (2) immediately suggest a systematic way of enhancing the field quality of a planar PM N-pole; namely, by the successive removal of its higher-pole field components.

The basic principle behind this approach is illustrated in Fig. 3. For an optimized planar PM quad with the field spectrum shown on the top left side, the octupole component can be nulled by centering a planar PM octupole whose leading field component, O_O, is equal and opposite to the quad's octupole component, O_Q, over the symmetry axis of the quad. We note that since the potential is given by the same canonical form for each structure (viz., by eq. (1)), making O_O=-O_Q will null the octupole component (E_{13} in our notation) for all values of R. Due to the operative principle of linear superposition, all the higher-pole components of the PM octupole will also tend to subtract from the corresponding higher-pole components of the quad.

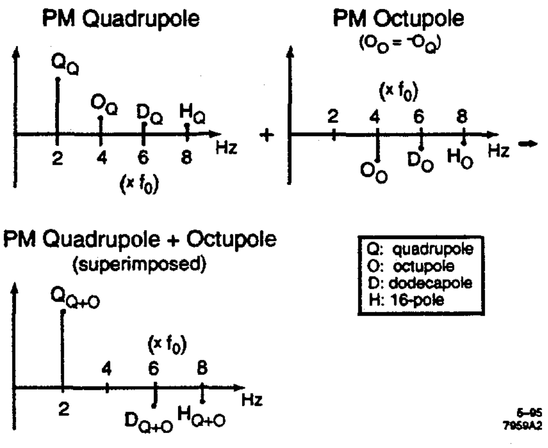

Figure 3. Linear superposition of planar PM octupole and quadrupole rotating-coil spectra as a means of nulling the octupole component in the combined structure. f_0 is the coil rotation frequency.

This procedure can be, of course, repeated to eliminate the next higher (dodecapole) component of the combined quad+octupole structure (see Fig. 4). Further repetition can be used to successively remove as many of the higher multipole

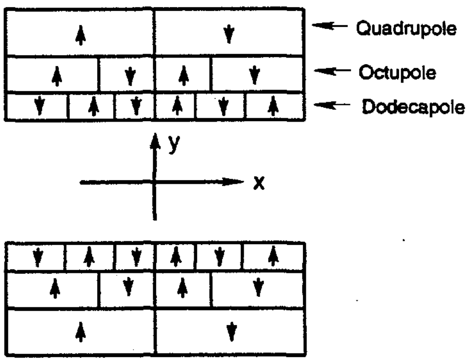

Figure 4. Planar PM quadrupole structure with nulled octupole and dodecapole field components.

components as desired. Clearly, the extirpated field components do not have to be adjacent to any other nulled component, but can be arbitrarily located. Consequently, the same method can be used to purify the field of any planar PM N-pole. Of course, practical considerations will limit the number of field components that can be nulled in this fashion, but the important result is that even with the planar PM N-pole's reduced symmetry, it is theoretically possible to make its field approximate - as closely as desired - that of an ideal (N-fold rotationally symmetric) structure. It is easy to show that if an individual PM multipole is used for each cancellation, the total number of PM pieces required for a quad with m successively nulled next-higher components would be $4+6m+2m^2$, and the corresponding number of pieces for a sextupole would be $6+8m+2m^2$.

IV. APPLICATIONS TO X-RAY FEL DESIGN

The original proposal for using planar PM quadrupoles was for applications in which the electron beam was small enough, and close enough to the axis, to make the higher-pole components negligible with respect to the leading field component [1]. Here we demonstrate calculated trajectories through a planar PM FODO lattice (Fig. 5) that support the soundness of this proposal. The computer code, developed originally for FEL simulations [6], employs a rigorously realistic model of fields generated by assemblies of PM pieces (i.e., the field experienced by the particles is everywhere divergence and curl free). The general parameters, typical of those that might be used for a 1.5 Å FEL [7], are: electron energy 12 GeV; electron emittance 4.2×10^{-11} r-m; FODO lattice period 40 cm; PM quad gap 3 mm; PM quad length 10 cm; PM piece remanent field 1.2 T; PM piece height 3.5 mm; PM piece width 1 cm; and FODO lattice betatron wavelength 8 m. For longer FEL structures the quad parameters (e.g., gap size) could be substantially relaxed. With the developments outlined above, field-improved planar PM elements can now also be considered for use in much lower-energy FELs with substantially smaller emittances (i.e., bigger beams).

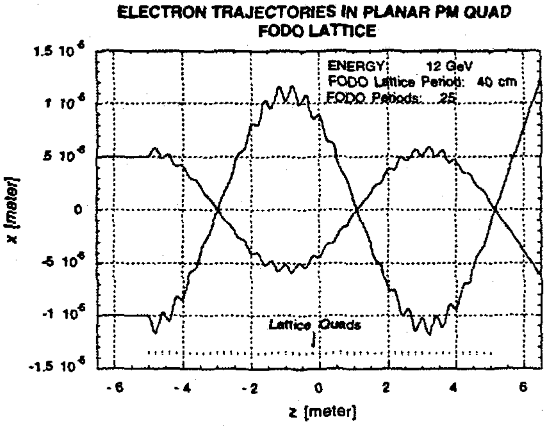

Figure 5. Electron trajectories for electrons entering FODO lattice (from the left) at 5μ and -10μ off axis in the x-z plane.

V. SUMMARY

More general discussions of the properties and various other applications of planar PM N-poles can be found in the cited prior literature . For FEL applications, rigorous modeling has been found to support the effectiveness of using planar PM N-poles. Application of the field-improvement techniques outlined above should enable improved planar PM N-poles (in particular the quad and sextupole) to find a broader range of applications than heretofore proposed. To this end, methods for further minimizing the number of PM pieces required for PM multipole field improvement are being investigated.

VI. ACKNOWLEDGMENTS

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