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Abstract

Narrow-width mechanism of ${}^5_\Xi H$ is discussed by calculating conversion widths to all its possible decay channels. Since the conversion processes have small reaction Q values, the three- and four- body decays are strongly suppressed owing to small phase volumes available. Decay widths to the two-body channels are significantly reduced by the distortion of emitted-particle waves. This mechanism brings about a narrow width of ${}^5_\Xi H$. The total width is estimated to be 0.87 MeV, in which the largest contribution comes from the decay into the ${}^4_\Lambda H^*$+Λ channel.

1. Introduction

Double-strangeness hypernuclear systems are particularly interesting, since they provide unique information concerning the $\Lambda\Lambda$ and ΞN interaction. Especially, very light systems are valuable to be studied, since their simple structures enable us to extract the interaction property. Myint et al. investigated the structure of A=5 double-strangeness systems by a coupled calculation between Ξ- and $\Lambda\Lambda$- channels, and concluded the possible existence of a ${}^5_\Xi H$ nuclear state with extremely narrow width 0.2 MeV [1]. They, however, mentioned no narrow-width mechanism based on partial conversion widths. Recently, Dover et al. discussed the mechanism in the framework of plane-wave Born approximation (PWBA) by taking into account only the three-body decay channel of ${}^3H$+Λ+Λ [2]. In the case of small Q values shown in Eqs.(1)-(6), the two-body decay contributions to the width of ${}^5_\Xi H$ would be more significant than the three-body ones, since available phase volumes become so small for the three-body decays. Thus, we reinvestigate the narrow-width mechanism of ${}^5_\Xi H$ in the distorted-wave Born approximation (DWBA) by focusing our attention on the two-body decay widths. The detailed analysis of the mechanism is useful, since it concerns whether any Ξ-nuclear state can be observed by such strangeness-transfer reactions as (K^-, K^+).

2. Formulation

In the case of ${}^5_\Xi H$, the number of decay channels is very limited. This is of great advantage to reveal the narrow-width mechanism. All the decay processes are followings:

$$
\begin{aligned}
{}^5_\Xi H \;\to\; & {}^4_{\Lambda\Lambda}H + n && + 1.23 \text{ MeV} && (1)\\
\to\; & {}^4_\Lambda H + \Lambda && + 8.95 \text{ MeV} && (2)\\
\to\; & {}^4_\Lambda H^* + \Lambda && + 7.90 \text{ MeV} && (3)\\
\to\; & {}^3H + \Lambda + \Lambda && + 6.91 \text{ MeV} && (4)\\
\to\; & {}^3_\Lambda H + \Lambda + n && + 0.78 \text{ MeV} && (5)\\
\to\; & {}^2H + \Lambda + \Lambda + n && + 0.65 \text{ MeV}, && (6)
\end{aligned}
$$

where ${}^4_\Lambda H^*$ is the excited state with spin unity. The elementary conversion process of $\Xi^- p \to \Lambda\Lambda$ releases the energy 28.33 MeV, which is accidentally very close to the binding energy 28.30 MeV of the alpha particle. Then, the reaction Q values become small for these processes which break the alpha nucleus in ${}^5_\Xi H$.

As a typical example, we give the formula for the decay process (2) in the cases of plane wave and distorted wave for the emitted particle. The other processes can be treated in a similar way. The following wave functions are used for the initial and final states in the calculation.

$$\Psi_i({}^5_\Xi H)=\left(\frac{a^2}{2\pi^2}\right)^{\frac{3}{4}}\exp\left(-\frac{1}{3}a\xi_2^2-\frac{3}{8}a\xi_3^2\right)\bullet\Psi_\Xi(r_1), \qquad (7)$$

$$\Psi_f({}^4_\Lambda H+\Lambda)=\left(\frac{a'a''}{2\pi^2}\right)^{\frac{3}{4}}\exp\left(-\frac{1}{3}a'\xi_2^2-\frac{3}{8}a''\xi_3^2\right)\bullet\varphi_{k_\Lambda}(r_1), \qquad (8)$$

The notations are explained in Fig.1 and $\varphi_{k_\Lambda}(r_1)$ is $\exp(-ik_\Lambda r_1)$ for the plane-wave case. Strength parameters in the wave functions are taken to be a= 0.521, a'= 0.39 and a''= 0.20 [fm^{-2}], which correspond to α, t and d cases, respectively.

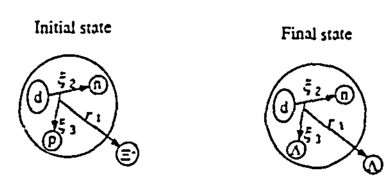

Fig.1. Schematic picture of a conversion process.

The ${}_{\Xi}^{5}$H state is bound by 1.6 MeV with respect to the Ξ^-+^{4}He threshold, whose wave function $\Psi_{\Xi}(r_1)$ is obtained by solving a Schrödinger equation with Myint et al.'s Ξ^--^{4}He potential [1]. For simplicity, we use a zero-range interaction for the $\Xi^-p \rightarrow \Lambda\Lambda$ conversion.

$$v_{\Xi^- p\rightarrow\Lambda\Lambda} = \frac{v_0}{\sqrt{2}}\,\delta\left(3/4\xi_3 - r_1\right) P^{\sigma}_{\rm singlet}, \qquad (9)$$

where $P^{\sigma}_{\rm singlet}$ is a projection operator to the spin-singlet state, which is expressed as

$$P^{\sigma}_{\rm singlet} = \Lambda^{\dagger}_{\uparrow}\Lambda^{\dagger}_{\downarrow}\cdot\frac{1}{\sqrt{2}}\left(\Xi^-_{\uparrow}p_{\downarrow} - \Xi^-_{\downarrow}p_{\uparrow}\right) \qquad (10)$$

with spin-up Λ-creation ($\Lambda^{\dagger}_{\uparrow}$), Ξ^--annihilation ($\Xi^-_{\uparrow}$), p-annihilation ($p_{\uparrow}$) and other operators. The potential strength v_0 is taken to be 285 MeV·fm^3 which is a volume integral of Shinmura's potential [1] constructed on the basis of the Nijmegen model-D potential.

The formula for the decay width $\Gamma_{{}^4_\Lambda H+\Lambda}$ is derived in such way as explained in Ref.[3];

$$\Gamma_{{}^4_\Lambda H+\Lambda} = 2C\,\frac{\mu_\Lambda\sqrt{2\mu_\Lambda}}{\hbar^3}\sqrt{Q}\,|v_0|^2\left\{\frac{8a\sqrt{a'a''}}{3\pi(a+a')}\right\}^3\left|\int_0^{\infty} r_1^2\,\bar{j}_0(k_\Lambda r_1)\exp\left(-\frac{2}{3}(a+a'')r_1^2\right)R_{\Xi}(r_1)\right|^2, \qquad (11)$$

where C is a spin-weight factor, μ_Λ is the reduced mass of $M_\Lambda M_{{}^4_\Lambda H}/(M_\Lambda + M_{{}^4_\Lambda H})$ and Q is the reaction Q value. $R_{\Xi}(r_1)$ and $\bar{j}_0(k_\Lambda r_1)$ denote radial wave functions of the Ξ^--state and of the emitted Λ-particle, respectively. In the PW case, $\bar{j}_0(k_\Lambda r_1)$ goes to a spherical Bessel function. *In the DW calculation we employ distortion potentials of Gaussian type with a range* 2.0 fm, whose depths are -55 MeV for the process (1), -36 MeV for the processes (2) and (3), and -26 MeV for the process (4). These potential depths are determined so as to reproduce the binding energy of ${}^5_{\Lambda\Lambda}$H for the two-body decay processes. For the three-body decay process (4), we only take into account the distortion between ^{3}H and each Λ by neglecting the interaction between the two Λ's.

The spin-weight factor C is obtained for the zero-range interaction as follows. Expectation values of $P^{\sigma}_{\rm singlet}$ are calculated with respect to spin states of the initial Ξ^--nucleus and of final $\Lambda\Lambda$-channel when the interacting particles Ξ^- and p are at a same point. The factor C is a sum of the squares of the expectation value over possible final states, and is

evaluated to be 3/4 for the process (1), 1/4 for the process (2), 3/4 for the process (3) and 1 for the process (4), respectively. Thus, the decay into the excited ${}^4_\Lambda$H* + Λ channel is three times as large as that into the ground ${}^4_\Lambda$H + Λ one.

3. Results and Discussions

Table 1 shows the decay width and the branching ratio (in parenthesis) of each process.

Table 1. Decay widths [MeV] and branching ratios from the ${}_{\Xi}^{5}$H state.

	Actual Q values (small)		Q values fixed to 28.33 MeV (large)	
	PW	DW	PW	DW
(1)	0.23 (15.1%)	0.03 (3.2%)	0.13 (5.2%)	0.002 (0.1%)
(2)	0.26 (17.0%)	0.15 (17.7%)	0.21 (8.0%)	0.09 (3.8%)
(3)	0.76 (50.0%)	0.47 (53.5%)	0.62 (24.0%)	0.26 (11.3%)
(4)	0.27 (17.9%)	0.22 (25.6%)	1.62 (62.8%)	1.94 (84.8%)
Total	1.52	0.87	2.58	2.28

The two-body decays contribute to the total width with a significantly large ratio which amounts to 74.4 % in the DW case. The three-body decay of the process (4) is rather a minor part. The two-body decay dominance is a consequence of the small Q values which suppress substantially the available phase volumes for three- and four- body decays which are proportional to $Q^{(3n-5)/2}$ for n-body decay [3]. *Partial widths of the processes (5) and (6) are* negligible because of the too small Q values and are not included in Table 1.

For the sake of comparison, Table 1 also shows widths calculated when Q values for all the processes are assumed to be the elementary Q value, 28.33 MeV. In this large-Q case the three-body decay width of the process (4) becomes a main part of the total width which is about 85 % in the DW calculation. Then, Dover et al.'s treatment [2], in which only the process (4) was taken into account, is justified. In the case of small Q values, however, it is indispensable to evaluate contributions from the two-body decay processes which become more important than the three-body decay process. Figure 2 shows the Q value dependence of partial widths obtained by the PW calculation. Although the three-body decay monotonically decreases as Q goes to zero, the two-body decays have maxima at certain small Q values giving major contributions to the total width at the actual Q values. It is estimated in the PW calculation that by this Q value change, the three-body decay width of the process (4) is reduced from 1.62 MeV to 0.27 MeV, the two-body widths is slightly enhanced and thus the total width is reduced from 2.58 MeV to 1.52 MeV.

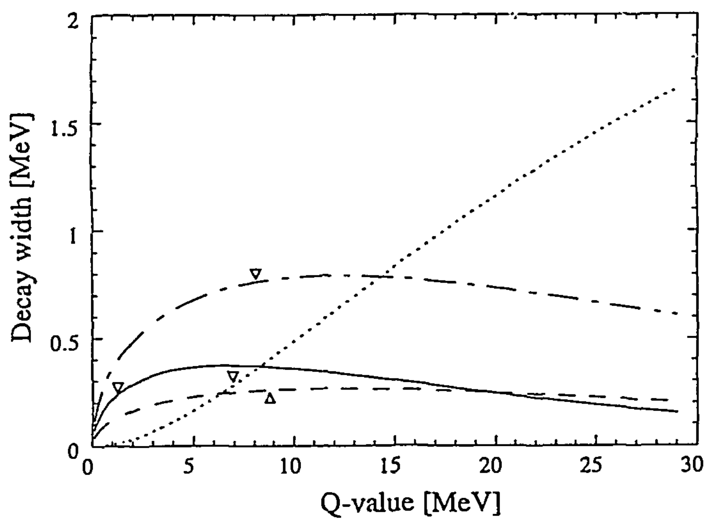

Fig.2. Q-value dependence of conversion widths in the PWBA framework. The solid, dashed, dash-dotted, and dotted lines are for the processes (1), (2), (3), and (4), respectively. The marks (▷) denote respective Q-values of the processes.

Some of the decay widths are largely affected by the distortion of wave functions for emitted particles. In particular, the two-body decay width of the process (1) is extremely suppressed by the distortion. This situation is explained in relation to the existence of a bound state. Figure 3 compares the effect of distortion potentials between ${}^{4}_{\Lambda\Lambda}$H and n. In the case of -55 MeV depth, the ${}^{4}_{\Lambda\Lambda}$H+n system has the bound state ${}^{5}_{\Lambda\Lambda}$H which substantially suppresses the decay into the ${}^{4}_{\Lambda\Lambda}$H+n channel from 0.23 MeV to 0.03 MeV by concentrating in it a major part of neutron s-wave strengths. If the distortion potential is not strong enough to have a bound state, it shifts the position of the decay peak to smaller-Q side and makes the height more prominent as seen from the comparison between the -10 MeV depth case and the PW case.

Then, the decay width of the process (1) is enhanced by the distortion as long as no bound state appears. Widths of the other two-body decays are also reduced by the distortion owing to the existence of the ${}^{5}_{\Lambda\Lambda}$H bound state, though their reduction factor of about 0.6 are not so drastic as the above case.

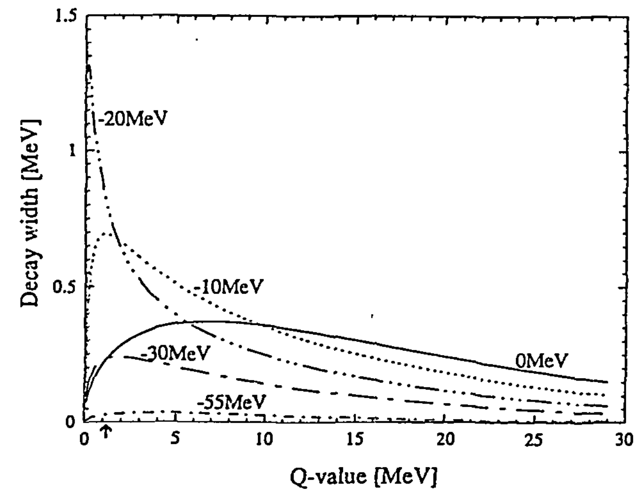

Fig.3. Distortion effects on the process (1), ${}^{5}_{\Xi}$H→${}^{4}_{\Lambda\Lambda}$H + n. Strengths of the distortion potential are shown in the Figure. The arrow denotes the Q-value position.

The total width of the ${}^{5}_{\Xi}$H state is finally obtained to be 0.87 MeV in the DWBA calculation. This value is different from Myint et al.'s result of 0.2 MeV [1]. The essential difference comes from whether or not the process (3), ${}^{5}_{\Xi}$H→${}^{4}_{\Lambda}$H* + Λ, is included in the calculations. The process is undoubtedly the main part of the total width as seen from Table 1. Myint et al. did not take into account this decay channel by treating only the ground and continuum states of ^{3}H-Λ with spin zero. We can extract a width value corresponding to their calculation from the values given in Table 1 as follows; 0.15 (${}^{4}_{\Lambda}$H+Λ) + 0.22 (^{3}H+Λ+Λ) /4 = 0.21 MeV. This value is consistent with their value 0.2 MeV. Then, the total width of 0.87 MeV can be accepted to be a reasonable value.

4. Conclusions

The total width of $^{5}_{\Xi}H$ is obtained to be 0.87 MeV. The three-body partial width is 0.22 MeV for $^{3}H+\Lambda+\Lambda$, and the two-body partial widths are 0.03 MeV for $^{4}_{\Lambda\Lambda}H+n$, 0.15 MeV for $^{4}_{\Lambda}H+\Lambda$ and 0.47 MeV for $^{4}_{\Lambda}H^{*}+\Lambda$. A substantial reduction of the three-body width was discussed by Dover et al.[2]. It, however, is not a main part of the total width. The second of the two-body widths was included in Myint et al.'s calculation [1]. The present work showed the fact that the most important is the third one, i.e., the two-body decay into the $^{4}_{\Lambda}H^{*}+\Lambda$ channel. The decay width of $^{5}_{\Xi}H$ leading to the excited hypernucleus $^{4}_{\Lambda}H^{*}$ is three times as large as that to the ground one $^{4}_{\Lambda}H$.

The narrow width of $^{5}_{\Xi}H$ is brought about by the following mechanism. All the conversion processes have very small Q values because of breaking the alpha particle in the initial hypernuclear state. The small Q values reduce phase volumes of the final channels and substantially suppress the three- and four- body decays. Thus, the two-body decay processes become more dominant than the three-body decay ones. The two-body decays are also suppressed via the distortion of emitted-particle wave functions by the presence of the bound state $^{5}_{\Lambda\Lambda}H$ which absorbs a major part of two-body strengths. The smallness of Q values and the presence of $^{5}_{\Lambda\Lambda}H$ are two ingredients which lead to a narrow width of the $^{5}_{\Xi}H$ state.

In this respect the Ξ-state of $^{7}_{\Xi}H$ is also an interesting subject to be studied. The Q values are small, and, in addition, there is no two-body decay process. Thus, the main decay process is the three-body decay of $^{7}_{\Xi}H \rightarrow {}^{5}_{\Lambda\Lambda}H+n+n$. The Ξ-state would be expected to be observed by a ^{7}Li $(K^{-}, K^{+})^{7}_{\Xi}H$ reaction. Once $^{7}_{\Xi}H$ is formed, it converts to the double Λ-hypernucleus $^{5}_{\Lambda\Lambda}H$ with a large branching ratio. Theoretical calculation is now in progress. Such an experiment is awaited in order to reveal the interaction property of double-strangeness systems.

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