3.31 Evaluation of Covariance for ^{238}U Cross Sections

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Abstract : Covariances of ^{238}U are generated using analytic functions for representation of the cross sections. The covariances of the $(n, 2n)$ and $(n, 3n)$ reactions are derived with a spline function, while the covariances of the total and the inelastic scattering cross section are estimated with a linearized nuclear model calculation.

1. Introduction

So far no covariance data has been given in JENDL-3.2. The covariance data is under preparation in order to meet requirements of the nuclear data users. The covariance data of U, Pu, C, O, Na, and structural materials have higher priority especially for design of the fast/thermal reactors and shielding. The objective is generation of the covariance data of ^{238}U for JENDL-3.2.

It is difficult to derive the covariance of the existent evaluated nuclear data, since derivation of the covariance data depends on the cross section evaluation methods, one must trace the evaluation procedure and calculate error propagation rigorously at each evaluation step. In order to evade this complication we adopt an analytic function for representation of the cross section and give the covariance for the function. Then the given covariance is rehashed into the covariance of the existent evaluated nuclear data.

2. Calculation Method

A covariance matrix for the spline-smoothed cross sections $\mathbf{M}$ is given by the least squares method, $\mathbf{M} = \mathbf{D}'\mathbf{V}^{-1}\mathbf{D}$, where $\mathbf{V}$ is a covariance matrix of experimental data, and $\mathbf{D}$ is a design matrix. The experimental data $\boldsymbol{y}$ is written as $\boldsymbol{y} = \mathbf{D}\boldsymbol{x}$, where $\boldsymbol{x}$ is a vector whose elements are the cross sections at the spline knots. The matrix $\mathbf{D}$ is arranged so as to average or to interpolate the experimental data between the spline knots.

The covariace matrix of the nuclear model calculated cross section can be generated from the covariance of the model parameters. The nuclear model calculation is linearized with the first order Taylor series expansion around a certain parameter set $\boldsymbol{x}_0$, $\boldsymbol{f}(\boldsymbol{x}) \simeq \boldsymbol{f}(\boldsymbol{x}_0) + \mathbf{C}(\boldsymbol{x} - \boldsymbol{x}_0)$, where $\mathbf{C}$ is a sensitivity matrix. The least squares method is feasible with this linearized function. The covariance of the model parameters $\mathbf{P}$ is calculated by $\mathbf{P} = (\mathbf{X}^{-1} + \mathbf{C}'\mathbf{V}^{-1}\mathbf{C})^{-1}$, where $\mathbf{X}$ is the prior covariance matrix of the parameters. The covariace matrix of the cross sections is given by $\mathbf{M} = \mathbf{CPC}'$

Usually the covariance matrix $\mathbf{M}$ is multiplied by a scale factor, χ^2. In order to rehash the covariance of the analytic function into the evaluated cross sections, we refer the χ^2 of the evaluation instead of the χ^2 of the analytic function. Note that the covariance matrices sited in the following section are not multiplied by this factor, but the χ^2's are quoted in figures.

3. Results and Discussion

Figures 1 and 2 show the covariance matrices of the $(n, 2n)$ and $(n, 3n)$ reaction cross sections. We employed the spline function assisted least squares method with interval average for $\mathbf{D}$. The spline function well reproduces the experimental $(n, 2n)$ cross sections, and that is almost equal to the JENDL-3.2 evaluation. The obtained covariance does not multiplied by χ^2 since $\chi^2 < 1$. In the case of the $(n, 3n)$, few available experimental data gave rise to large uncertainties and the ambiguous spline function.

The covariance of the total cross section was evaluated with a coupled-channels (CC) method. Optical potential parameters and deformation parameters (β_2, β_4) were taken as the model parameters. The obtained covariance matrix is shown in Fig. 3. The uncertainties are very small because there are a large number of data points.

The covariance of the inelastic scattering cross section was also evaluated with the CC method. Compound process contribution is significant below $E_n < 5$ MeV. In order to avoid the calculation being complicated, the compound cross section was calculated with a Hauser-Feshbach-Moldauer (HFM) model and the cross section was renormalized. The normalization constant was taken as the one of the model parameters. The covariance matrices of the inelastic scattering cross sections to the first excited state and the second excited state are shown in Figs. 4 and 5, respectively. There are no experimental data above 5 MeV. This method, however, is able to generate the covariance in this enegy range.

The CC calculation with 0^+-2^+-4^+ coupling scheme yields a correlation matrix of the inelastic scattering cross sections between the first and the second levels. This is shown in Fig. 6. This information is required for completion of a consistent covariance file.

4. Conclusion

Covariances of the ^{238}U reaction cross sections were evaluated using analytic functions — a spline function and a linearized nuclear model calculation. The spline method went well for the $(n, 2n)$ but for the $(n, 3n)$ because of the lack of the experimental data. The linearized nuclear model method was able to generate the covariance although the experimental data were insufficient.

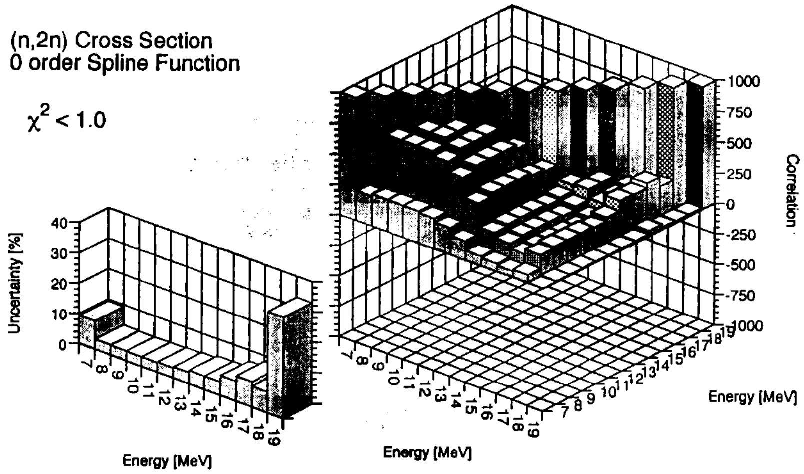

Fig.1 Covariance matrix of $^{238}U(n, 2n)$ reaction cross section

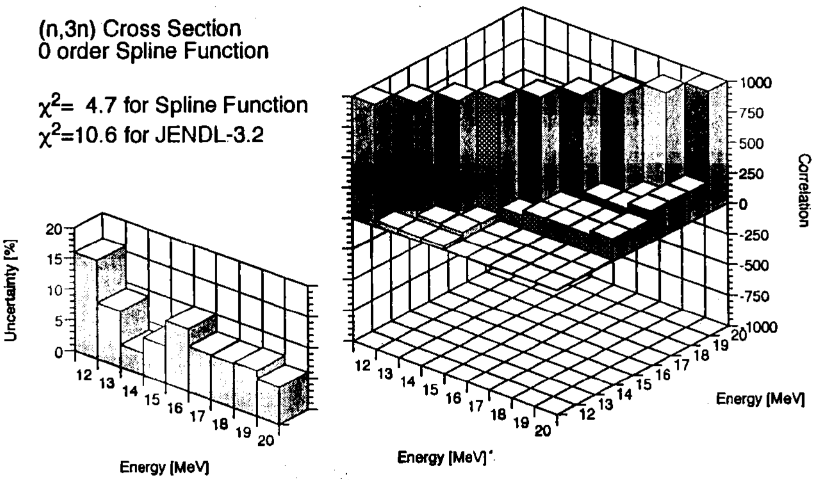

Fig.2 Covariance matrix of $^{238}U(n, 3n)$ reaction cross section

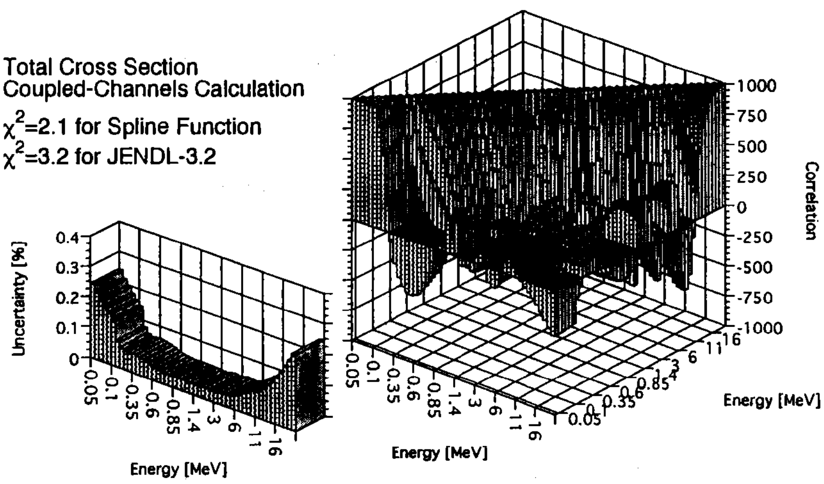

Fig.3 Covariance matrix of ^{238}U total cross section

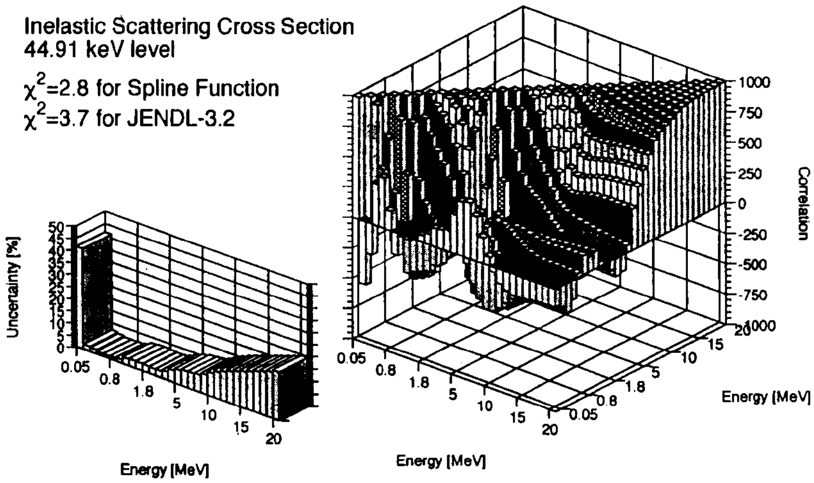

Fig.4 Covariance matrix of ^{238}U(n, n') cross section

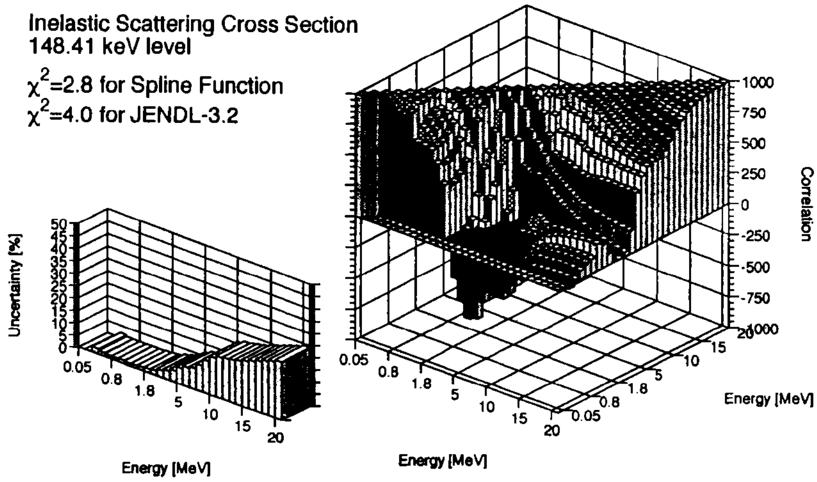

Fig.5 Covariance matrix of ^{238}U(n, n') cross section

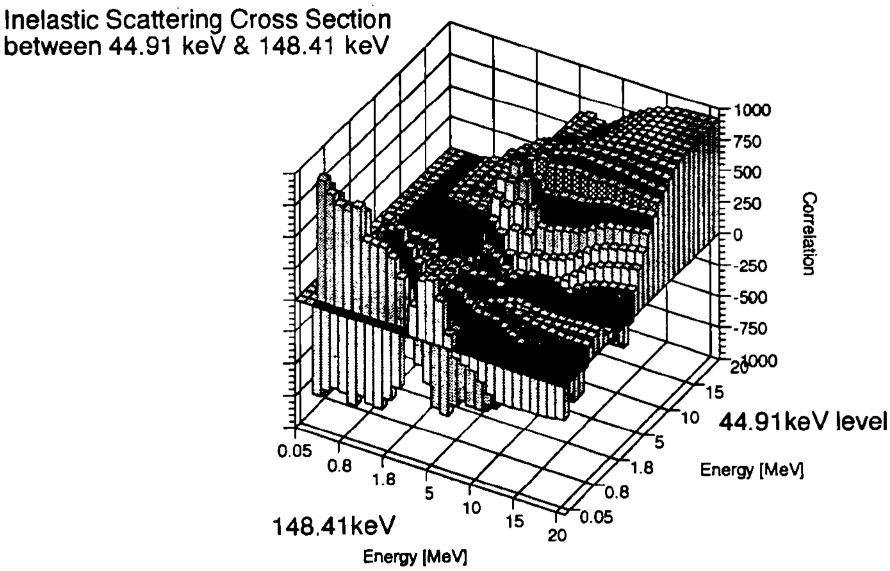

Fig.6 Correlation matrix of ^{238}U(n, n') cross sections between the first and the second levels