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托卡马克电阻性鱼骨模理论

THEORY OF TOKAMAK RESISTIVE  
FISHBONE MODES

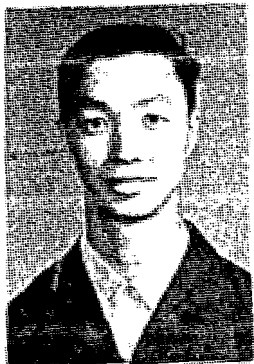


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# 托卡马克电阻性鱼骨模理论

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## 摘 要

在托卡马克等离子体中, 高能粒子可以激发一种称为鱼骨模的内扭曲模。现有鱼骨模理论是建立在理想磁流体近似基础上的, 它预言有两支不稳定分支。对于高能粒子的比压值,  $\omega \approx \bar{\omega}_{dm}$  的一支 (Chen-White 分支) 的激发阈值较高; 而  $\omega \approx \omega_{*i}$  的一支 (Coppi 分支) 的激发阈值很低, 因而对托卡马克等离子体的加热效率及约束非常不利。不过, 研究发现电阻效应对这支模有实质性的影响。文中详细分析了电阻性鱼骨模理论, 给出了新的稳定性参数区结构图。可以看到, Coppi 分支被一支增长率很低的弱不稳定模所代替, 同时, 严格意义的稳定区不复存在。作者用一种新方法计算了该模的增长率随高能离子比压的变化, 并与其它不稳定模作了对比。讨论了对未来大型托卡马克装置的影响。

# THEORY OF TOKAMAK RESISTIVE FISHBONE MODES

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## ABSTRACT

A special kind of internal kink mode, the fishbone, can be excited by the energetic particles in tokamak plasmas. Theoretical analyses of fishbone modes based on the ideal MHD framework have predicted that two branches of modes exists. One is the Chen—White branch with  $\omega \approx \bar{\omega}_{dm}$ , corresponding to a higher threshold in  $\beta_h$ ; the other is the Coppi's branch with  $\omega \approx \omega_{*i}$ , and a much lower threshold in  $\beta_h$ . The latter mode would put a rather unfavourable restriction on heating efficiency and on plasma confinement. However, It is found that the resistivity effect is essential for this mode. In this paper, a new resistive fishbone mode analysis is carried out. In the  $(\gamma_{mhd}, \beta_h)$  space, the stability diagram shows complicate structure, the Coppi's branch is replaced by a weakly unstable mode and there is no longer closed stable region. The growth rate of this mode varies with  $\beta_h$ , its peak value is still very low compared to other internal modes. The implications of these results to future tokamak experiments are discussed.

# INTRODUCTION

Interactions between MHD modes and energetic particles which can be produced through various high power auxiliary heatings and/or current drive in tokamak plasmas are very important. Of them, the interaction between trapped hot ions and the internal kink mode has attracted much attentions. One of these internal kink modes, the fishbone mode, first observed in the PDX device in NBI heating<sup>[1,2]</sup> and later<sup>[3]</sup> in other devices like PBX, D III -D and JET, can severely eject hot ions then affect heating efficiency. Based on the ideal MHD approximation, theoretic analyses<sup>[4-9]</sup> have been drawn to this issue. Resistivity was considered in some publications as perturbation<sup>[5,6]</sup>. These theories predict that there are two branches of unstable modes excited by hot trapped ions. The first has a characteristic frequency  $\omega \approx \bar{\omega}_{dm}$ , the average precession frequency of trapped hot ions in tokamak configurations, having a relatively higher threshold in  $\beta_h$ , the beta value of hot trapped ions. This is the so-called Chen-White branch<sup>[4]</sup>. The other, with  $\omega \approx \omega_{*i}$ , the plasma ion diamagnetic drift frequency, has a much lower threshold in  $\beta_h$ . This is the Coppi's branch<sup>[5,6]</sup>. Its easy excitation then put a great threat on large tokamaks and on future D, T reactors<sup>[9]</sup>. On the other hand, however, experiments on many tokamaks do not show such kink of vulnerability in exciting fishbone modes. The disparity of theories and experiments has not been discussed so far. In a previous paper<sup>[10]</sup>, we indicated that resistivity effect can substantially (not in a perturbed way) reform the stability diagram in  $(\gamma_{mhd}, \beta_h)$  space: the Coppi's branch is replaced by a very weakly unstable mode which extends deeply into originally stable region so that there is no longer restrict stable region. The growth rate of this weakly unstable mode is now calculated in this paper. Comparison with other internal modes is made. It seems that this mode should not put great restriction on future tokamak projects.

The structure of this paper is as follows: In Section II, a brief review of fishbone mode theory is outlined; in Section III, resistive fishbone mode analysis is carried out; in Section IV, some comparisons and discussions are given; finally, in Section V, concluding remarks are drawn.

## 1 A BRIEF REVIEW OF FISHBONE MODE THEORY

The main characteristics of fishbone modes observed in experiments can

be summarized as: (i) this is an oscillation of  $m/n=1/1$  MHD structure, its oscillographs look like fish skeletons; (ii) the mode can be excited during high power NBI heatings and other heating with huge amount of hot ion populations. Meanwhile, the magnetic fluctuations concomitant with the mode can eject hot ions severely, this in turn reduced the beta value of hot ions until it becomes lower than the threshold value and the mode suppressed. The competition between these processes form the fishbone cycle. The above-mentioned characters imply that interaction between the internal kink modes and the hot ions should be responsible for the mode excitation. In PDX device, frequencies of magnetic perturbations lie in the range (10~20) kHz, approximately, these are the precession frequencies of hot trapped ions produced by the perpendicularly NBI heating. Based on this, Chen et al.<sup>[4]</sup> developed a theory for the fishbones, indicating that this is a new branch of internal kink mode (different from that supported by the pressure gradient of the background plasma), excited through resonant interaction between the precession motion of hot ions and MHD modes. The Chen-White theory gives a threshold value in the pressure of hot ions as:

$$\beta_{hc} = \omega_{dm}s/\pi\omega_A, \quad \beta_h = (R/\gamma_1)\beta_{hc} \quad (1)$$

where  $\omega_{dm} = cE_h / (ZeBRr_1)$ ,  $E_h$  is the energy of trapped ions,  $r_1$  is the radius of the  $q = 1$  surface;  $\omega_A = \frac{B}{\sqrt{12\pi R\rho}}$ , with  $\rho$  the mass density of the background plasma.  $s = r (dq/dr)_{r_1}$  is the magnetic shear at  $q=1$  surface. Generally, this threshold has a value of about  $10^{-2}$ . When  $\beta_h > \beta_{hc}$ , the fishbone is excited, the rapid growing magnetic fluctuations will eject energetic ions then reduce their pressure until it becomes lower than the threshold value, the mode is suppressed and heating raises this pressure again. A new cycle of fishbone then appears. The Chen-White theory is based on the ideal MHD approximation, as can be seen, non-ideal effects like the resistivity has little to do with this branch.

Later, Coppi and his coworkers found<sup>[5,6,8]</sup> that in the framework of MHD theory, there will be another branch of fishbone mode with  $\omega \approx \omega_{*i}$ ,  $\omega_{*i} = -\frac{c}{ZeBnr_1} \left( \frac{dp_i}{dr} \right)$ , calculated also at the  $q=1$  surface. This branch has a much lower threshold in  $\beta_h$  for exciting the mode. This is the so-called Coppi's branch, or sometimes the ion fishbone branch. To form suitable cycle to explain experi-

ments, it is necessary to introduce resistivity effect to stabilize the mode. They did it in a perturbed way.

According to the unified theory for internal kink mode, the dispersion relation of both the ideal and the resistive internal kink mode can be written as<sup>[7,11]</sup>

$$\delta W_c + \delta W_h - \frac{8i\Gamma[(\Lambda^{3/2} + 5)/4]}{\Lambda^{9/4}\Gamma[(\Lambda^{3/2} - 1)/4]} \frac{[\omega(\omega - \hat{\omega}_{*i})]^{1/2}}{(\omega_A/s)} = 0 \quad (2)$$

where  $\Gamma$  is the complex gamma function, in its argument there is a parameter  $\Lambda$  which is define as

$$\Lambda = \frac{-i[\omega(\omega - \omega_{*i})(\omega - \hat{\omega}_{*e})]^{1/3}}{S^{-1/3}\omega_A s} \quad (3)$$

Besides what have been defined before, other parameters are define as

$$\hat{\omega}_{*e} = \omega_{*e} + 0.71 \frac{c}{eBr_1} \left(\frac{dT_e}{dr}\right)_\eta$$

$$\omega_{*e} = \frac{c}{eBnr_1} \left(\frac{dP_e}{dr}\right)_\eta$$

$$S = \omega_A \tau_{RS}$$

$$\tau_R = (4\pi r_1^2 / \eta c^2)$$

and  $\eta$  is the plasma resistivity. In all quantities cited above we have picked up the magnetic shear  $s$  outside the relevant quantities because there are some confusions for the definition of  $\omega_A$  in Refs. [7, 9]. For other quantities,  $\delta W_c$  is the perturbed plasma energy of the interior region,  $\delta W_h$  is the variation of the hot ion energy. To simplify the analysis without loss of essential things, we introduce here the deeply trapped ion model for the hot ion contribution. i. e.

$$\delta W_h = \beta_h \left(\frac{R}{r_1}\right) \frac{\omega}{\omega_{dm}} \ln\left(1 - \frac{\omega_{dm}}{\omega}\right) \quad (4)$$

here after we define  $\hat{\beta}_h = \beta_h R/r_1$ ; for  $\delta W_c$ , depending on the shape of the safety factor profile  $q(r)$ , the usually adopted form is

$$\delta W_c = 3\pi \left(\frac{r_1}{R}\right)^2 (1 - q_0) (\beta_q^2 - 13/144) \quad (5)$$

for a parabolic  $q(r)$ :

$$q(r) = q_0 + (1 - q_0)(r/r_1)^2, \quad r < r_1$$

in equation (5), the equivalent beta value for the background plasma  $\beta_p$  is defined as

$$\beta_p = - \frac{8\pi}{B_p^2(r_1)} \int_0^1 dx x^2 dP/dx, \quad x = r/r_1 \quad (6)$$

for easy comparisons with literatures, we introduce parameters

$$\gamma_{\text{mhd}} = - \delta W_c \quad (7)$$

$$\gamma_R = S^{-1/3} \omega_A S \quad (8)$$

and  $\gamma_R$  represents the typical MHD growth rate. From the conventional understanding, when the resistivity becomes smaller, (the plasma temperature becomes higher),  $\gamma_R \rightarrow 0$ , then we have  $\Lambda \gg 1$ . In this limit, the function involving the parameter  $\Lambda$  will asymptotically takes the form

$$F(\Lambda) = \frac{(8i\Gamma[(\Lambda^{3/2} + 5)/4])}{\Lambda^{9/4}\Gamma[(\Lambda^{3/2} - 1)/4]} \rightarrow (1 - 5\Lambda^{-3}/4) \quad (9)$$

if only the first term on the right hand side is taken, Eq (2) then becomes that for the ideal internal kink modes

$$\frac{-i[\omega(\omega - \omega_{*i})]^{1/2}}{(\omega_A/s)} - \gamma_{\text{mhd}} + \beta_h \frac{\omega}{\omega_{\text{dm}}} \ln(1 - \frac{\omega_{\text{dm}}}{\omega}) = 0 \quad (10)$$

as were shown in Refs. [7, 9], if  $\beta_h = 0$ , there is only an unstable mode with  $\omega = \omega_{*i}/2$  when  $\gamma_{\text{mhd}}$  is larger than a threshold value

$$\gamma_{\text{mhd}} > \left(\frac{\omega_{*i} S}{2\omega_A}\right)$$

this is the ideal internal kink mode. When  $\beta_h > 0$ , a new mode, the fishbone mode can be excited. In marginal conditions, the frequency takes real value, in range  $\omega > \omega_{*i}$ , equation (10) can be splitted into a real part and an imaginary part:

$$[\omega_r(\omega_r - \omega_{*i})]^{1/2} = \pi(\omega_A/s)\beta_p\omega_r/\omega_{\text{dm}} \quad (11)$$

$$\gamma_{\text{mhd}} = \beta_p(\omega_r/\omega_{\text{dm}})\ln(\omega_{\text{dm}}/\omega_{*i} - 1) \quad (12)$$

from Eq. (11), we get

$$\omega_r = \omega_{*i}(1 - \pi^2\beta_h^2(\omega_A/\omega_{\text{dm}}S)^2)^{-1} \quad (13)$$

substitute the value of  $\omega_r$  in Eq (12) into Eq (11), a marginal relationship between  $\gamma_{\text{mhd}}$  and  $\beta_h$  is obtained. This is depicted in the figure 1, the region under the solid curve in this figure is the stable region, where a stable mode with frequency  $\omega_{*i} < \omega_r < \omega_{\text{dm}}/2$  exists. Generally, the tokamak plasma pressure is not large enough so that the value of  $\gamma_{\text{mhd}}$  is usually smaller than the corresponding maximum value depicted from this curve, so that the unstable region is in fact divided into two parts. That in the higher  $\beta_h$  side belongs to the Chen-White branch, and the region near the longitudinal coordinate is the Coppi's branch. It can be found that the threshold value of  $\beta_{hc}$  for Chen-White branch from



Eqs. (12), (13)

$$\beta_{hc} > \left(\frac{\omega_{dm}^5}{\omega_A \pi}\right) \omega (1 - 2\omega_{*i}/\omega_{dm})^{1/2} \quad (14)$$

as far as the Coppi's branch is concerned, there is no threshold in  $\beta_h$  within the ideal MHD frame work. To explain the relaxation behaviour of the fishbone, the resistivity effect is introduced in a perturbed way, i. e. , from the asymptotic expansion form, Eq. (9). Let  $\omega = \omega_r + i\gamma$ ,  $\gamma \ll \omega_r$ , considering the ion fishbone approximation  $\omega \approx \omega_{*i}$ , an approximate dispersion relation is obtained as

$$i[\omega(\omega - \omega_{*i})]^{1/2} \left(1 + i \frac{5\gamma_R^3}{4\omega(\omega - \omega_{*i})(\omega - \omega_{*e})}\right) = (i\pi\beta_h \frac{\omega_{*i}}{\omega_{dm}} - \gamma_{mhd}) \frac{\omega_A}{s} \quad (15)$$

from this it is solved

$$\beta_{hc} = \frac{5\gamma_R^3}{4\gamma_{mhd}\omega_{*i}(\omega_{*i} - \omega_{*e})} \left(\frac{\omega_{dm}^5}{\omega_A \pi}\right) \quad (16)$$

equation (17) defines a stable region in  $(\gamma_{mhd}, \beta_h)$  space as shown in figure 1 by the dashed curve. It seems that the Coppi's theory can explain the relaxation cycle of fishbone as " powerful auxilliary heating—the raising of the pressure of the hot ions—the excitation of the fishbone—the rapid loss of the hot ions and the reduction of its pressure—the supression of the fishbone". However, because its much lower threshold value, this branch of fishbone is easy to be excited and this put a great threat on the future tokamak heatings, including the alpha particle heating in burning plasmas in reactors. Supprisingly, this situation has not been reported by many tokamak experiments. Even in devices where fishbone activity is found (the DIII—D, JET, JT—60), the resultant effects seem much less severe. Compared to the PDX device, in the DIII—D, the condition for exciting fishbone is more restrict, the loss caused by fishbone type fluctuation is much smaller, in the JET device, the fishbone activity was reported, but it seems like that this activity was hidden in other internal activities. These observations are directly contradict with the Coppi's theory which predicts that the higher the plasma temperature, the narrower the stable region due to resistivity effect, and the larger the  $\gamma_{mhd}$  value, the easier the excitation of the ion fishbone mode. Sofar, these diparities have not been discussed in any detail in literatures.

In the previous paper<sup>[10]</sup>, we have checked the foundation of the fishbone theory and found that the Chen-White branch constitutes a selfconsistent description, inclusion of nonideal effect like the resistivity does not affect its be-

haviour, however, the resistivity can reform the Coppi's branch substantially. As a matter of fact, the perturbation treatment for considering resistivity effect in the region where  $\omega \approx \omega_{*i}$  is not correct. In the following, we present a resistive fishbone analysis.

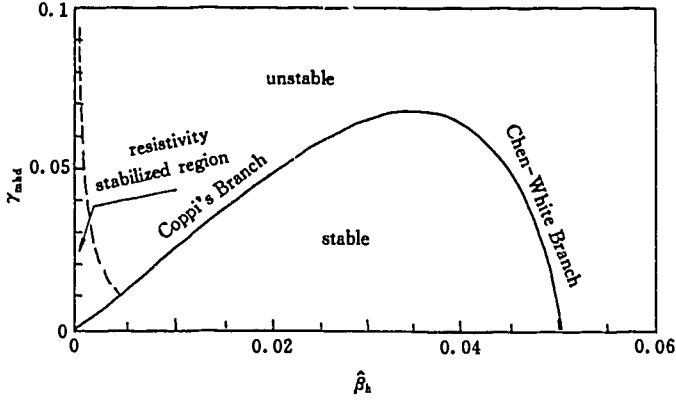


Fig. 1 The marginal stability diagram for the ideal fishbone modes.

The stability region is under the solid curve.

The left region of the dashed curve is the resistivity stabilized region.

Parameters:  $\omega_{*i}/\omega_A = 3/40$ ,  $\omega_{dmS}/\omega_A = 1/4$ ,  $\omega_{*i}/\gamma_R = 6$ .

## 2 THEORY OF RESISTIVE ION FISHBONES

First, we point out the inconsistency of a perturbed treatment in the ion fishbone analysis. Let

$\omega = \omega_{*i} + \delta\omega_r + i\gamma$ , from Eq. (15) we obtain

$$\delta\omega_r = \pi^2 \beta_h^2 \omega_{*i} (\omega_A / \omega_{dmS})^2 \quad (17)$$

the marginal condition allows us to replace the  $\beta_h$  value in above equation by that in Eq. (17), then we have

$$\delta\omega_r = 1.56 \gamma_R^6 / (\gamma_{mhd}^2 \omega_{*i} (\omega_{*i} - \omega_{*e})^2) \quad (18)$$

with this, the value then becomes

$$\Lambda = 1.12 \gamma_R (\gamma_{mhd} (\omega_{*i} - \omega_{*e}))^{-2/3} \quad (19)$$

therefore, it is seen, in the ideal MHD approximation, when  $\eta \rightarrow 0$ ,  $\gamma_R \rightarrow 0$ , we have  $\Lambda \rightarrow 0$ . This is just the opposite limit for a correct expansion of Eq. (9) which requires  $\Lambda \gg 1$ . Then the inconsistency of Coppi's theory is obvious.

To correctly describe the behaviour of the ion fishbone, it is necessary to start from the restrict dispersion relation, Eq (2), here we write it in a form

$$F(Q) [\omega(\omega - \omega_{*i})]^{1/2} = i \left( \frac{\omega_A}{S} \right) \left[ \gamma_{mhd} - \beta_h \frac{\omega}{\omega_{dm}} \ln \left( 1 - \frac{\omega_{dm}}{\omega} \right) \right] \quad (20)$$

where  $F(Q)$  is defined by Eq (9) with  $Q = \Lambda^{3/2}$ , then

$$F(Q) = \frac{8\Gamma[(Q+5)/4]}{Q^{3/2}\Gamma[(Q-1)/4]} \quad (22)$$

$$Q = \frac{[i\omega(\omega - \omega_{*i})(\omega - \hat{\omega}_{*e})]^{1/2}}{\gamma_R^{3/2}} \quad (23)$$

the dispersion relation (21) should be solved numerically. Split the dispersion equation (21) into the real part and the imaginary part as

$$\text{Re}\{F(Q)[\omega(\omega - \omega_{*i})^{1/2}\} = \left(\frac{\omega_A}{s}\right)\pi\beta_h\omega_r/\omega_{dm} \quad (24)$$

$$\text{Im}\{F(Q)[\omega(\omega - \omega_{*i})^{1/2}\} = \left(\frac{\omega_A}{s}\right)[\gamma_{mhd} - \beta_h \frac{\omega_r}{\omega_{dm}} \ln\left(\frac{\omega_{dm}}{\omega_r} - 1\right)] \quad (25)$$

functions of  $\text{Re}\{F(Q)\}$  and  $\text{Im}\{F(Q)\}$  are plotted in figs. 2, 3 against  $Q$ . The ion fishbone mode is in a region of  $Q < 1$ , where  $F(Q)$  behaves complicatedly. It can be seen from these figures that the asymptotic expansion is approximately correct when  $Q \gg 2$ .

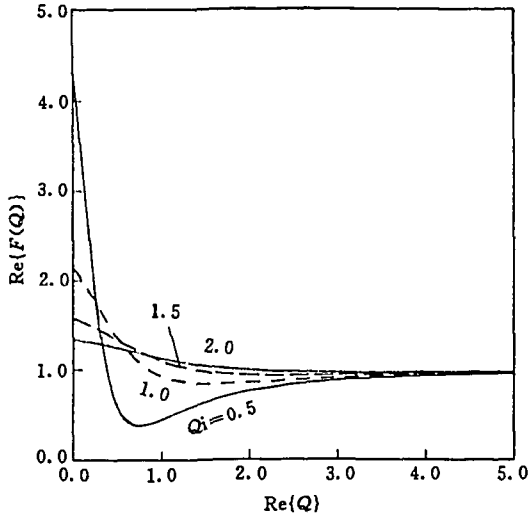


Fig. 2a,  $\text{Re}\{F(Q)\}$  versus  $Q_r$ .

Different values of  $Q_i$  are indicated near corresponding curves.

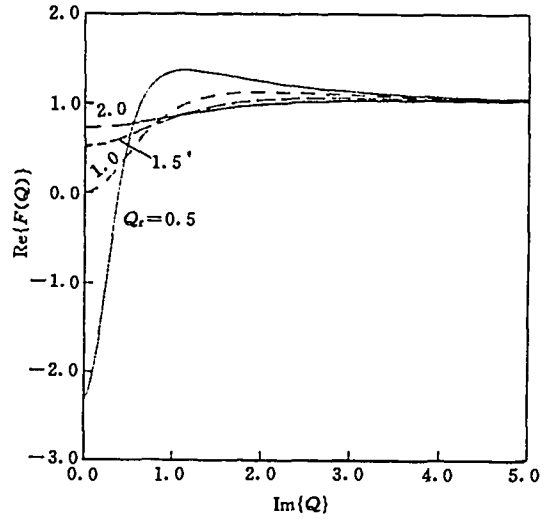


Fig. 2b,  $\text{Im}\{F(Q)\}$  versus  $Q_r$ .

Different values of  $Q_i$  are indicated near corresponding curves.

Seven parameters are directly relevant to the characters of the fishbone mode. They are:  $(\omega_A/s)$ ,  $\omega_{*i}$ ,  $\omega_{*e}$ ,  $\gamma_R$ ,  $\gamma_{mhd}$ ,  $\beta_h$  and  $\omega_{dm}$ . We select  $\gamma_{mhd}$  and  $\beta_h$  as free parameters but fix others according to corresponding tokamak experimental conditions. The ratio of  $\omega_{*i}/\gamma_R$ , plays a special role. Usually, its value increases with the device size and lies in the range of 2—20. With the increase

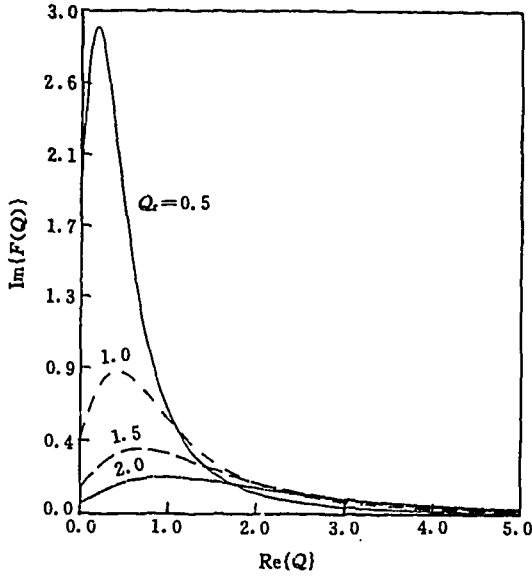


Fig. 3a,  $\text{Im} \{F(Q)\}$  versus  $Q_r$ .  
Different values of  $Q_i$  are indicated  
near corresponding curves.

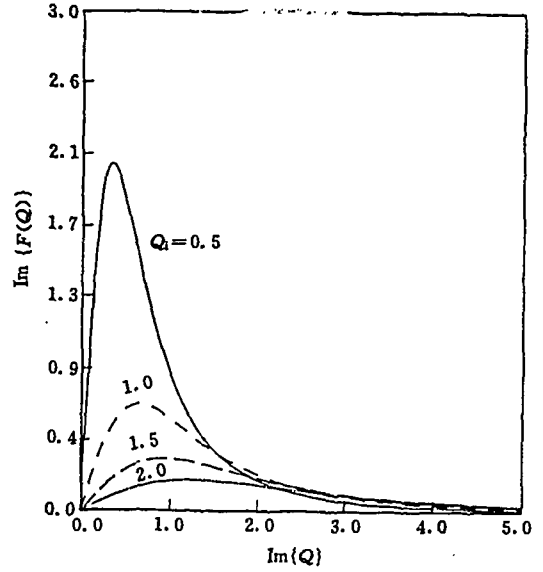


Fig. 3b,  $\text{Im} \{F(Q)\}$  versus  $Q_i$ .  
Different values of  $Q_i$  are indicated  
near corresponding curves.

in  $\omega_{*i}/\gamma_R$ , the structure of the stability diagram becomes more complicated. In marginal stable state, we can find from the relation of the real frequency and the pressure of hot ions that there are two branches of modes, one has a frequency range of  $\omega_{*i}/2 < \omega_r < \omega_{*i}$ , the other is in the range of  $\omega > \omega_{*i}$ , they coalesce when  $\omega_r \rightarrow \omega_{*i}$ . The variations of frequencies of these two modes with  $\beta_h$  are shown in figure 4, where the ideal fishbone mode is also shown by the dashed curve, determined by formula (13). It clearly shows that resistivity changes the characters of the ion fishbone mode substantially; in small  $\beta_h$  region, the Coppi's mode disappears, but the Chen-White mode which situates in the high  $\beta_h$  region does not change. Correspondingly, the marginal stability curves in the  $(\gamma_{\text{mhd}}, \beta_h)$  space are shown in Fig. 5. There are two separate curves, the right is for  $\omega_r > \omega_{*i}$ , the left one is for  $\omega_{*i}/2 < \omega_r < \omega_{*i}$ . The ideal MHD fishbone mode is shown by the dashed curve. Again, this figure shows the Chen-White branch does not change but the Coppi's branch disappears. Because the marginal stable curve is not closed, we should make sure what happens in the region where the ideal MHD theory predicts stable or unstable. This analysis is done in the following by a "constant growth rate contour analysis".

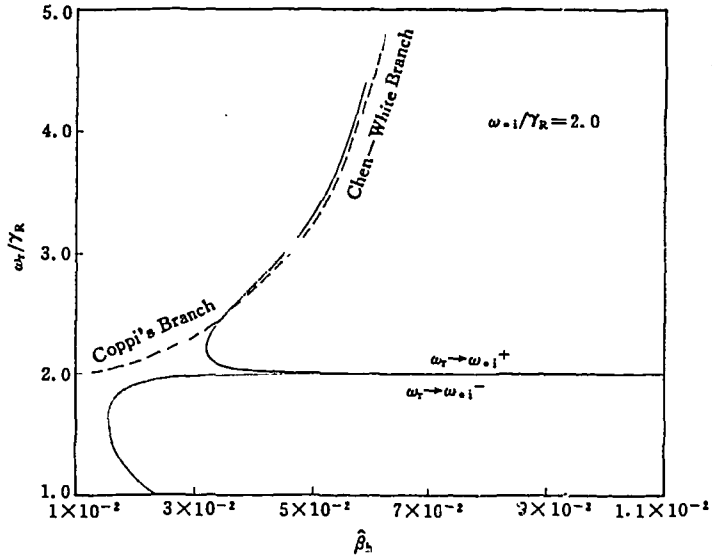


Fig. 4a The real frequency of the resistive fishbone mode versus  $\beta_h$  in marginal state.

The upper curve is for the branch of  $\omega_r > \omega_{*i}$ ; the lower is for  $\omega_{*i}/2 < \omega_r < \omega_{*i}$ . Parameters:  $\omega_{*i}/\gamma_R = 2$ ,  $\omega_A/s\omega_{dm} = 4$ ,  $\omega_{dm}/\gamma_R = 20$ . The dashed curve is for the ideal fishbone.

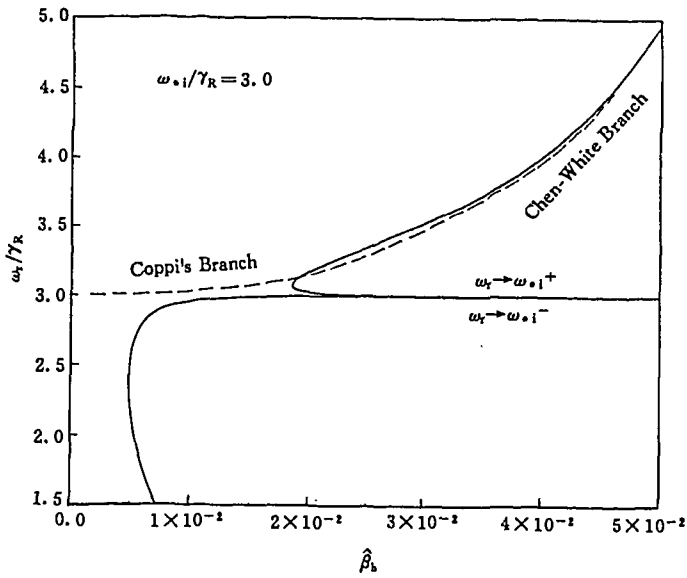


Fig. 4b The same as in figure 4a with  $\omega_{*i}/\gamma_R = 3$ .

Let  $\omega = \omega_r + i\alpha\gamma_R$ ,  $\alpha \ll 1$ . For a given  $\alpha$ , change  $\omega_r$ , in the range of  $\omega_r > \omega_{*i}/2$ , we obtain one curve in the  $(\gamma_{mhd}, \beta_h)$  plane, within the interior region of two

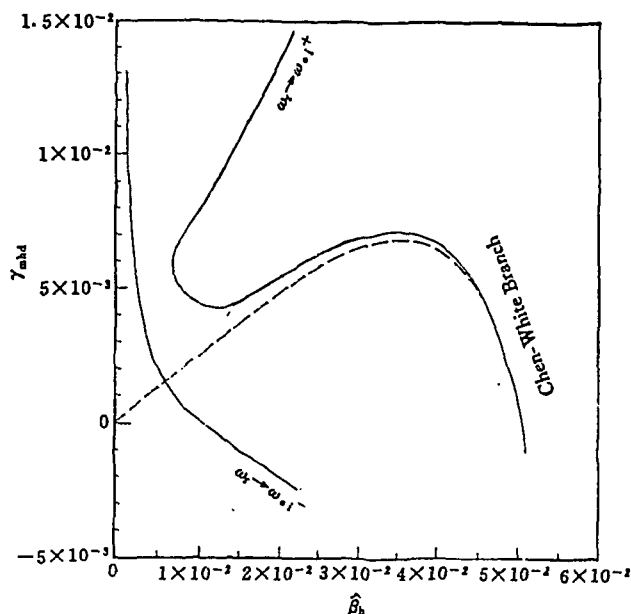


Fig. 5 The marginal stable curve of resistive fishbone mode. parameters:  $\omega_{*i}/\gamma_R=6$ ,  $\omega_{hm}/\gamma_R=20$ ,  $\omega_A/s\gamma_R=80$ .

$\alpha=\text{const.}$  curves, the mode has a growth rate of  $\alpha_1 < \gamma/\gamma_R < \alpha_2$ . (Fig. 6). It is shown that in the ideal stable region when  $\beta_h$  approaches the boundary of the Chen-White branch, the growth rate becomes extremely small so that it can be considered as stable. In the left part of the open region, however, the structures of the contours are rather complicated, some loop structures exhibit. Within these loops, there are two unstable modes with different frequencies and growth rates. The physically important mode is that with larger growth rate. In this region, however, the plasma beta should be rather large, the realistic tokamak can hardly operate in this region and we do not pay more attention to this issue.

The exact value of the weakly unstable mode which lies in the original Coppi's unstable and ideally stable region can be calculated by a special method. For fixed  $\gamma_{\text{mhd}}$  value, the mode becomes unstable when  $\beta_h$  is larger than a threshold value, then the growth rate increases with the increase in  $\beta_h$  first, after reaching the peaked value, it decreases rapidly but has a long tail (Fig. 7). Correspondingly, the frequency of the weakly unstable mode approaches the ion diamagnetic frequency from the lower side and then surpasses it a lit-

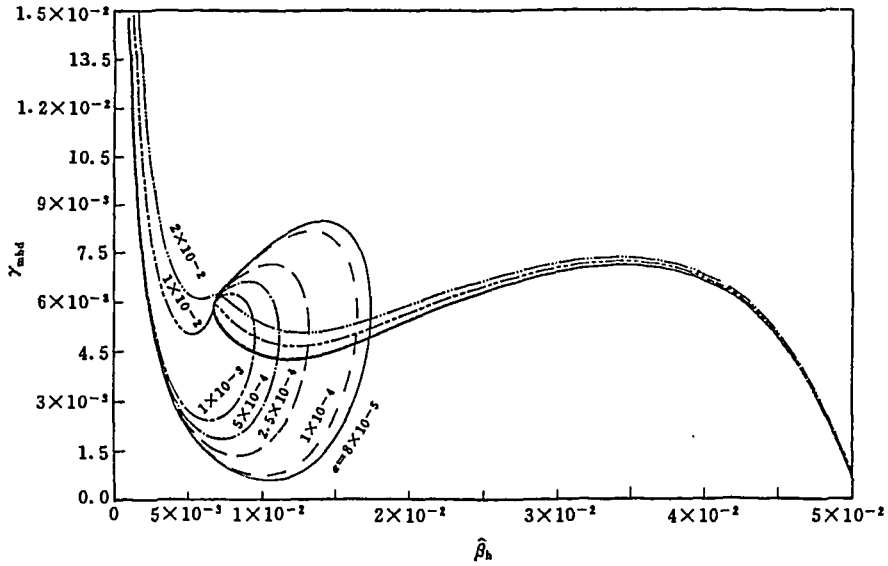


Fig. 6a The constant growth rate contours in the  $(\gamma_{\text{mhd}}, \beta_h)$  plane  $\omega = \omega_r + i\alpha\gamma_R$ , different values of  $\alpha$  are indicated near corresponding curves. Other parameters are the same as in fig. 5.

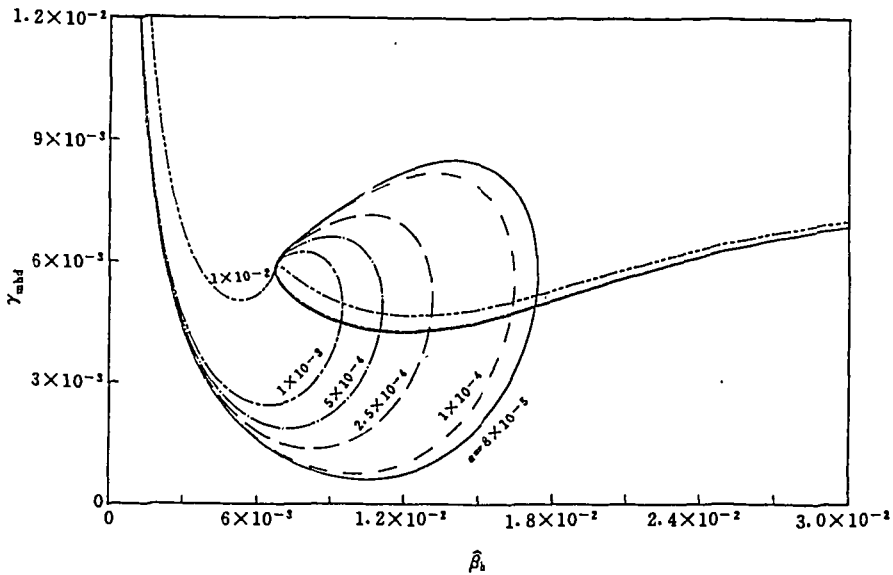


Fig. 6b The extended diagram of figure 6a in the low  $\beta_h$  region

tle. This is the reason why we call it the ion fishbone mode.

For larger  $\gamma_{\text{mhd}}$ , with the increase in  $\beta_h$ , the loop region will be intersect-

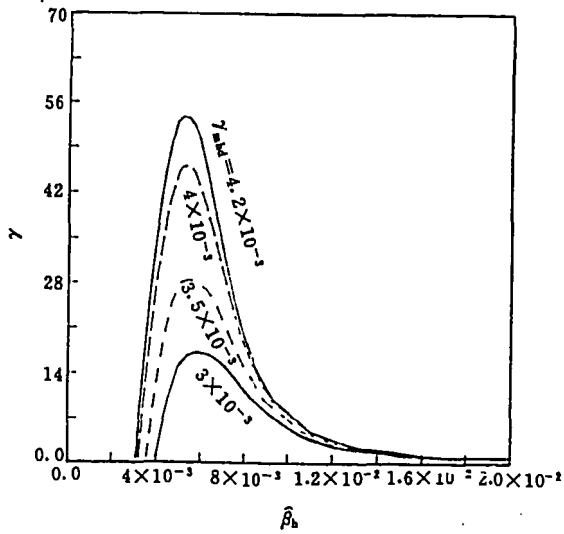


Fig. 7a The growth rate of the resistive ion fishbone mode versus  $\beta_h$

Different values of  $\gamma_{mhd}$  are indicated near the curves. Other parameters are the same as in Figs. 5, 6.

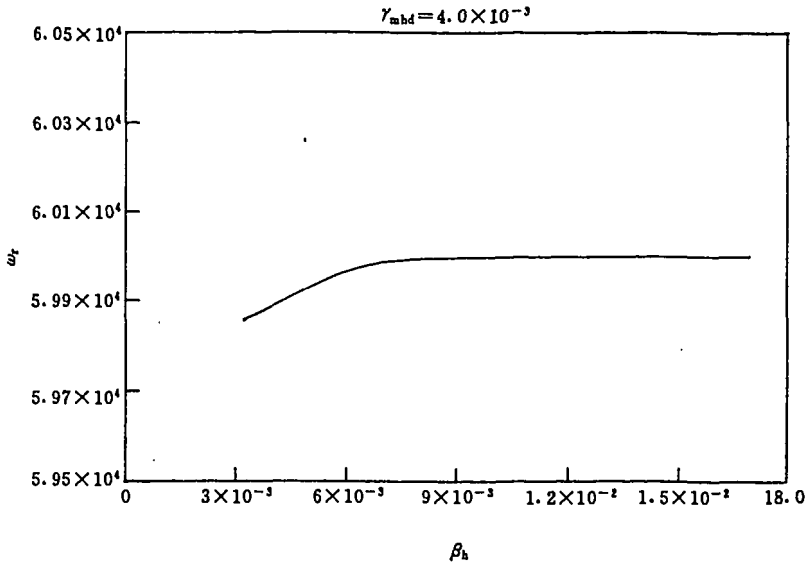


Fig. 7b The frequency of the resistive fishbone mode

ed. We can find two modes with different growth rate and frequency. One example is shown in Fig. 8. For the even larger  $\gamma_{mhd}$  value, one of the mode approaches the ideal fishbone mode with rather high growth rate, but as mentioned above, the corresponding parameters are thought to be not realistic.



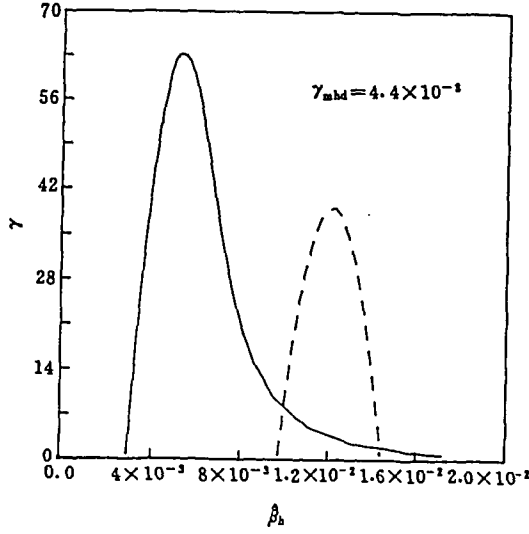


Fig. 8 Growth rates of two different modes which exist in the loop region with  $\gamma_{mhd} = 4.4 \times 10^{-3}$ . Other parameters are the same as in Fig. 7a.

### 3 COMPARISONS AND DISCUSSIONS

Compare our results to others, the following issues are essential; (i) resistivity substantially changes the structure of stability diagram. For low  $\beta_h$  case, the ion fishbone mode becomes unstable when  $\beta_h$  is greater than a threshold value  $\beta_{hc}^{(1)}$ , it can not be stabilized again for larger  $\beta_h$  (unlike the ideal case where there is a stable boundary). However, its growth rate is too small to cause physically important effects. We feel that this weakly unstable mode can be considered as nearly stable mode. (ii) Compared to the Coppi's results for the growth rate, it seems that the resistive ion fishbone has much lower growth rate. As an example, we use parameters in Fig. 7a: for  $\gamma_{mhd} = 4 \times 10^{-3}$ , the peak value  $\gamma/\gamma_R = 5 \times 10^{-3}$ , and  $\beta_h = 6 \times 10^{-3}$ , from formula (16) for Coppi's result, on the other hand, we have

$$\begin{aligned} \gamma/\gamma_R &= 2\pi\beta_h\gamma_{mhd}(\omega_A/S\omega_{dm}) - (5/2)(\gamma_R/\omega_{*i})(\gamma_R/(\omega_{*i} - \omega_{*e})) \\ &= 4.8 \times 10^{-2} - 2.8 \times 10^{-2} = 2 \times 10^{-2} \end{aligned}$$

The later is 4 times than the former. (iii) Compared to the resistive internal kink mode of which the growth rate can be estimated by

$$\frac{\gamma}{\gamma_R} = \frac{5\gamma_R^2}{2\omega_{*i}\omega_{*e}} = 0.046$$

that is almost one order of magnitude higher than the resistive ion fishbone's. Concerning experimentally observed fishbone activity, it is necessary to check other parameter, the equivalent beta of background plasma. According to Bus-sac et al. ,<sup>[12]</sup> the requirement of  $\gamma_{\text{mhd}} > 0$  means  $\beta_p > 0.3$ . Assume  $P = P_0 (1 - gx^2)$ , then  $\beta_p = g\beta_p(0) / 2$ , where  $\beta_p(0)$  is the poloidal beta of the plasma at the  $q = 1$  surface with the pressure at the magnetic axis. For  $g = 0.4$ , this means that when  $\beta_p(0) > 1.5$ , the value of  $\gamma_{\text{mhd}}$  can be positive. For negative  $\gamma_{\text{mhd}}$ , only the Chen-White fishbone can be excited. Even when  $\gamma_{\text{mhd}}$  value is positive as estimated by Coppi and his coworkers in explaining the PDX' s results, our analysis indicates that the experimentally observed mode can hardly be explained as the ion fishbone mode because its growth rate is too small.

As far as future tokamaks are concerned, the energy of energetic particles is rather high, this makes a high threshold for exciting the Chen-White fishbone, so that we feel the most important internal kink mode is the ideal internal kink mode which could be unstable due to the high plasma beta.

#### 4 CONCLUSIONS

The fishbone modes excited by energetic trapped ions in tokamak plasmas can be described by ideal MHD theory in the high frequency case where a higher threshold in beta value of the energetic trapped ions is required; for the low frequency branch, the ion fishbone mode, resistivity determines the main property of the mode, ideal treatment is not suitable any longer. Analysis carried out in this paper shows that the ion fishbone mode is a weakly unstable mode. We feel that another nonideal effects, such as the finite Larmor radius effect, the viscosity, will also be substantial. For future tokamaks with very high energetic populations (like the alphas produced by D, T reactions), the most important internal kink mode is the ideal internal kink mode.

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