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Dynamical Effects of QCD Vacuum Structure

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Abstract

We review the role of the QCD vacuum structure in the determination of the properties of states and processes occurring in the confinement regime of QCD. The finite range of the vacuum correlations is discussed, and an analytical form is suggested for the correlation functions. The role of the vacuum quantum numbers in the phenomenology of high-energy scattering is reviewed. The vacuum correlation model of non-perturbative QCD is mentioned as a bridge between the fundamental theory and the description of the experiments.

1. Physical Vacuum and Confinement in QCD

The self interactions of the intermediating boson fields (gluons) in Quantum Chromodynamics are responsible for the characteristic behaviour of the running strong coupling constant. At high energies (small distances) the coupling becomes weaker, with the consequent phenomenum of asymptotic freedom showing quarks moving freely inside the hadrons. Perturbative methods can be applied in these conditions, and progress is being achieved through skill, time and effort put in the calculations.

On the other extreme, that of low energies (and distances of the order of the sizes of the hadronic systems), there appears the phenomenum of confinement of colour, with strict rules for the formation of the hadronic states that can be observed individually. The mechanism for colour confinement is still mysterious, and its manifestations, such as the hadronization in inelastic processes, are very complex. Lattice results confirm confinement, while simple hadronic spectroscopy based on confining potentials is able to give systematic descriptions of the hadronic states. However, the fundamental origin of the confinement property is not understood, either in mathematical or in physical terms.

The transition from the confinement regime to the asymptotically free regime of QCD is observed in the behaviour, as a function of the momentum (or of the distance), of the characteristic Green's functions. Indications are that the transition is sharp, which in a way helps in the use appropriate methods valid for each of the two regions. In the study of hadronic properties, the operators representing the physical quantities of interest are expanded in terms of products of coefficient functions, representing the high momentum range and evaluated with perturbative methods, and matrix elements representing the low momentum region, which must be evaluated non-perturbatively.

After some simplifications and approximations, only a few of these non-perturbative matrix elements are ultimately needed. With the approximation that in the infrared calculations the momentum can be put equal to zero, the relevant matrix elements are parametrized through quantities representing properties of the vacuum. These quantities are the condensates of quarks and gluons, defined as vacuum expectation values of products of operators representing the fundamental quark and gluon fields. Thanks to appropriate methods of calculation (Borel summation and dispersion relation techniques), only matrix elements of products of low dimensions are actually important. In practice, these are only three quantities : the quark condensate $\langle \bar{\psi}\ \psi \rangle$, the gluon condensate $\langle g^2\ F_{\mu\nu} F^{\mu\nu} \rangle$, and the mixed condensate $\langle \bar{\psi} \sigma_{\mu\nu} F^{\mu\nu} \psi \rangle$. The operator product with four quark fields may be assumed to reduce by factorization to the square of the simpler quark condensate with two fields, while the matrix element formed with three gluon fields is estimated to be very small.

These condensates, which are responsible by observable effects and influence all hadronic parameters, show the rich and complex structure of the QCD vacuum . The existence and properties of hadronic states cannot be explained through perturbative methods and usual Feynman diagrams alone. Thus the mesonic $q\bar{q}$ states are formed in the physical vacuum, which is filled with multi-gluon states, and the corresponding interactions must be taken into account in the evaluation of masses and transition rates for these states. Thus non-perturbative contributions are most important in the region of low momentum and determine the confinement regime of QCD and influence the whole of hadron physics.

The continuation of the phenomenological amplitudes, through dispersion relation techniques, to the unphysical region, for a comparison with the above mentioned theoretical expansions, leads to the sum-rule method of non-perturbative QCD[1]. This method, first used to explain the states of the charmonium system, has since its invention been applied to the evaluation of a large number of hadronic parameters[2].

Most of these applications make use of the static properties of the vacuuum, e.g. the value of the gluon condensate

$$\langle 0|g^2 F_{\mu\nu}(0)\ F^{\mu\nu}(0)|0\rangle_A \equiv \langle g^2\ FF\rangle\ ,$$

with the two gluon fields evaluated at the same point and thus representing correlations at zero range . The bracket $\rangle_A$ represents the average over the low frequency fluctuations of the vacuum. However, hadron physics is not so simple as to have all of its parameters and phenomena described through averages over vacuum fluctuations. Some phenomena depend on similar matrix elements evaluated with fields at two different points of space-time, thus reflecting a dynamical behaviour of the vacuum. The dependence of such matrix elements on the distance between the two points define characteristic correlation functions, governed by parameters called correlation time and correlation length. The existence of such finite range correlations has been confirmed by lattice calculations[3]. Clearly an important problem is to obtain the dependence of the two-point function on the coordinates of the two points.

In a non-Abelian theory such as QCD, a complication arises because a matrix element of a product $F_{\mu\nu}(x)F_{\rho\sigma}(y)$ of fields at different points is not gauge invariant. In order to form gauge invariant quantities, the two arbitrary points x and y must first be connected by parallel transport to the same arbitrary reference point w through Schwinger strings. Thus a tensor is defined

$$\mathbf{F}_{\mu\nu}(x;w) = \phi^{-1}(x,w)\ \mathbf{F}_{\mu\nu}(x)\ \phi(x,w)\ , \tag{1}$$

where $\phi(x,w)$ is a non-Abelian Schwinger string from point w to point x

$$\phi(x,w) = \mathbf{P}\exp\left[-ig\int_0^1 d\sigma\cdot(x-w)_\mu\ \mathbf{A}_\mu(w+\sigma(x-w))\right]\ . \tag{2}$$

P denotes path ordering, which is necessary in order to give to the exponential a well defined meaning.

The field strength tensor in eq.(1) has the *global* gauge transformation property

$$\mathbf{F}_{\mu\nu}(x,w) \rightarrow \mathbf{U}(w)\ \mathbf{F}_{\mu\nu}(x;w)\ \mathbf{U}^{-1}(w)\ , \tag{3}$$

so that the correlator $\langle \mathbf{F}_{\mu\nu}(x,w)\ \mathbf{F}_{\delta\sigma}(y,w)\rangle_A$ is gauge-covariant. Under the approximation that the correlator is independent of the reference point w, and thus only depends on the difference $z = (x - y)$, its most general form[4,5] is given by

$$\begin{aligned}\Big\langle g^2\ F^C_{\mu\nu}(x,w)\ F^D_{\rho\sigma}(y,w)\Big\rangle_A &= \frac{\delta^{CD}}{N_c^2-1}\,\frac{1}{12}\langle g^2\ FF\rangle \\ &\cdot \Big\{\kappa(\delta_{\mu\rho}\,\delta_{\nu\sigma} - \delta_{\mu\sigma}\,\delta_{\nu\rho})\cdot D(z^2/a^2) \\ &+ (1-\kappa)\cdot\frac{1}{2}\Big[\frac{\partial}{\partial z_\mu}(z_\rho\delta_{\nu\sigma} - z_\sigma\delta_{\nu\rho}) + \frac{\partial}{\partial z_\nu}(z_\sigma\delta_{\mu\rho} - z_\rho\delta_{\mu\sigma})\Big]D_1(z^2/a^2)\Big\}\ .\end{aligned} \tag{4}$$

This quantity reduces to the vacuum condensate $\langle g^2\ FF\rangle$ as x tends to y . The above expression introduces two arbitrary scalar correlation functions $D(z^2/a^2)$ and $D_1(z^2/a^2)$, where a is a characteristic correlation length. N_c is the number of colours, $C, D = 1,\ldots,N_c^2 - 1$, and the factors in eq.(4) are chosen in such a way that $D(0) = D_1(0) = 1$.

The parameter κ fixes the relative strengths with which the functions D and D_1 enter in the correlator. Lattice calculations[3] have shown that the functions D and D_1 fall off exponentially with the distance, and that the ratio $\kappa/(1-\kappa)$ is rather large (about 3), so that $D(z^2/a^2)$ gives the dominant contribution.

The vacuum correlation model of Dosch and Simonov[4,5] assumes that the vacuum fluctuations are mainly of stochastic type, characterized by these two correlation functions and their finite correlation lengths. They have proved that a confining linear potential between quarks is a consequence of the tensor term with the function D, thus showing in a specific model that the confinement properties of QCD may be due to the structure of the vacuum[6]. With the form of eq.(4) for the correlator, the area law[4,5] for a Wilson loop with the string tension ρ is given by

$$\rho = \frac{\kappa\pi}{144} \langle g^2\, FF\rangle\, a^2 \int_0^\infty D(-u^2)\, du^2 \,. \tag{5}$$

Thus only the tensor structure proportional to D leads to confinement, and it must be remarked that this term is specific to non-Abelian gauge theories.

The correlation functions $D(z^2/a^2)$ and $D_1(z^2/a^2)$ must have forms that can be analytically continued from Euclidean to Minkowski space-time descriptions of field theory. Let us concentrate on $D(z^2/a^2)$, and take as an ansatz the family of functions

$$D^{(n)}(\xi^2) = -6i \int \frac{d^4k}{(2\pi)^4}\, \frac{A_n k^2}{(k^2-1)^n}\, \exp(-ik\xi\rho_n) \,, \quad n \geq 4 \,, \tag{6}$$

where $\xi = z/a$, a is the characteristic correlation length, and the constants A_n and ρ_n are to be fixed by normalization. In the Euclidean metric

$$-id^4k = d^4K \,,\ K_4 = ik_0 \,,\ k^2 = -K^2 = -(|\vec{K}|^2 + K_4^2) \,, \tag{7}$$

for space-like vectors

$$\xi(0, \vec{\xi} = \sqrt{-\xi^2}\vec{e}_3) \,,\ \xi^2 = -|\vec{\xi}|^2 \,, \tag{8}$$

the normalization conditions are written

$$D^{(n)}(0) = 1 \quad , \quad \int_0^\infty d(|\vec{\xi}|)\, D^{(n)}(-|\vec{\xi}|^2) = 1 \,. \tag{9}$$

The second of these two relations has the role of a definition for the correlation length.

For the simplest choice, that is n=4, the correlation function is

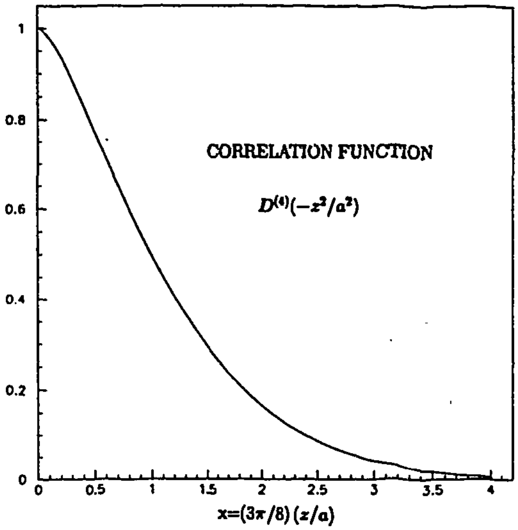
$$D^{(4)}(-|\vec{\xi}|^2) = (\rho_4|\vec{\xi}|)\left[K_1(\rho_4|\vec{\xi}|) - \frac{1}{4}(\rho_4|\vec{\xi}|)K_0(\rho_4|\vec{\xi}|)\right] \,, \tag{10}$$

where

$$\rho_4 = 3\pi/8 \,. \tag{11}$$

The function $D^{(4)}(-|\vec{\xi}|^2)$ is represented in the figure, against the variable $x = \rho_4|\vec{\xi}|$. It has a zero at $\rho_4|\vec{\xi}| = 4.43$, that, according to the values obtained for the correlation length, lies beyond the range of physical influence of the correlator. We remark that, although the analytical form chosen to define the correlation function is arbitrary, we do not expect that the consequences for phenomenology can be very different if another ansatz is adopted, since the constraints of normalization at the origin and of exponential decrease at large distances do not allow important changes in the shape of the function anyhow.

Correlation function $D^{(4)}(-|\vec{\xi}|^2)$, against $x = \rho_4|\vec{\xi}| = (3\pi/8)\,(z/a)$, where z is the physical distance, and a is the correlation length. The function is normalized to one at the origin.



2. Role of the Vacuum in High-Energy Phenomenology

A first observed manifestation of vacuum properties as a source of strong interaction dynamics occurred in the Regge phenomenology of high-energy elastic processes. A dominating and universal contribution to these processes consists in the exchange of an entity, called pomeron, carrying the quantum numbers of the vacuum.

The general features of the hadronic elastic processes (pp, $\bar{p}p$, πp, Kp, ...) at high energies are rather simple to describe[7]. For all processes, there is a strong forward peak, with the elastic differential cross-section $d\sigma^{el}/dt$ (t is the squared momentum transfer four-vector) decreasing exponentially with t. The total cross-sections first decrease with the energy, until a minimum is reached at an energy around 10 GeV, and then increase again, slowly. The values of the total cross-section $\sigma^T(s)$ and of the slope parameter of the elastic differential cross-section

$$B = \frac{d}{dt}\left(\ln \frac{d\sigma^{el}}{dt}\right)\Big|_{t=0} \tag{12}$$

are the basic characteristic quantities of these hadronic elastic processes at very low momentum transfers. These quantities are well described through the Regge exchange phenomenology, developed since the years 1960's. Actually, the Regge pole parametrization[8] yields an excellent phenomenological representation of the bulk of the data on high-energy scattering at small momentum transfers, $t \lesssim 1\mathrm{GeV}^2$. The total cross-section can be written in this approach as

$$\sigma^T = 2 \sum_i \beta_i^2(0)\left(\frac{s}{s_0}\right)^{\alpha_i(0)-1} , \tag{13}$$

and the differential elastic cross-section as

$$\frac{d\sigma^{el}}{dt} = \frac{1}{4\pi}\left[\sum_i \beta_i^2(t)\left(\frac{s}{s_0}\right)^{\alpha_i(t)-1}\right]^2, \tag{14}$$

each term of the sums corresponding to a Regge trajectory. At high energies, the process is dominated by the term with the largest value of $\alpha_i(0)$, the so-called pomeron trajectory, $\alpha_1(t) \equiv \alpha_p(t)$, with the quantum numbers of the vacuum ($J = 0$, $I = 0$, $C = +1$). It has been shown by Donnachie and Landshoff[9] that an excellent description of the scattering data at high energies and small momentum transfers can be obtained with the exchange of one pomeron, with a linearly increasing Regge trajectory $\alpha_p(t) = 1.0808 + 0.25t$. The parameter β_p determines the strength of the pomeron coupling to the hadrons. For higher momentum transfers, terms corresponding to two and more pomeron exchanges must be added to the amplitude. The value $\alpha_p(0) = 1.0808$, being larger than 1, would lead to a violation of the Froissart–Martin bound[10] for extremely high values of s, and there it must be modified by the presence of Regge cuts, which occur naturally in a Reggeon field theory.

With the pomeron trajectory alone contributing to the Regge expansions in eqs.(13),(14), we obtain for the energy dependence of the total cross-section

$$\sigma^T(s) = \sigma^T(s_0)\ (s/s_0)^{0.0808} , \tag{15}$$

and of the slope parameter

$$B(s) - B(s_0) = 2\alpha'(t)\ \log(s/s_0)\ , \tag{16}$$

where the t dependence of the residue $\beta(t)$ has been neglected. A direct relation between the values of total cross-sections and slope parameters is

$$\sigma^T(s) = \sigma^T(s_0)\ \exp\left[\frac{0.0808}{2\alpha'(t)}[B(s) - B(s_0)]\right]\ . \tag{17}$$

All these relations (15)-(17) are well fulfiled by the data at high energies.

Quantum chromodynamics, as the fundamental theory of the strong interactions, should be able to explain this successful and simple Regge phenomenology. Since the pomeron has the quantum numbers of the vacuum, it is natural to associate its exchange with the exchange of gluons. The understanding of the detailed nature of the gluon-field processes behind this phenomenology is of course an important problem.

Since at small momentum transfers the strong coupling constant becomes large, one has to rely either on refined resummation schemes in perturbation theory or on non-perturbative models. Landshoff and Nachtmann[11] have constructed a model in which the pomeron is described by the exchange of two gluons with modified propagators, containing a new length scale Λ, which implies a modification of the long-range QCD forces. Nachtmann later refined this model[12], describing a system of two quarks interacting through an external vector field (gluon field), which is supposed to vary slowly in time, compared with the frequency associated to high-energy quark motion. Since the quarks in the problem have very high energies, and only very small angle scattering is considered, the WKB (eikonal) approximation for the scattering in an external field can be used, and the quarks can be put in light-like paths.

Hadron–hadron scattering has been treated in the same framework[13], with the purpose of explaining the elastic scattering data in a non-perturbative QCD framework. This treatment requires the functional integration over the external gluon fields (denoted by the bracket $\rangle_A$), which cannot be performed exactly. Use is then made of the same vacuum correlation model[4,5] introduced in Euclidean field theory for investigations of hadron spectroscopy, where it provides an explanation of confinement (the linearly rising potential)[6] as a dynamical consequence of the vacuum structure. The basic assumption of the model is that the complicated integration over the low-frequency (non-perturbative) contributions to the gluon fields can be approximated by a cluster expansion, ideally by a Gaussian process, which is determined by the correlators of two fields.

The hadronic structure enters the calculation in the simplest way, through Gaussian wave-functions, with a radial parameter S, describing the sizes of the particles. Mesons are treated as simple $q\bar{q}$ systems. Baryons have been treated either through a configuration of three quarks symmetrically distributed in space or through a diquark model (in this case the baryons enter the calculation in a form totally similar to that of the mesons). At the end, hadron physics enters in the results for the observables in high-energy scattering only through the hadronic size parameters S. The relevant QCD parameters in the calculation are the gluon condensate $\langle g^2\ FF\rangle$ and the correlation length a.

The evaluation of the hadron-hadron scattering amplitudes proceeds through the evaluation of eikonal functions, and of averages over the hadronic wave-functions. After the necessary trace evaluations (the hadrons are colour-singlet objects) and numerical integrations, the profile functions that give a representation for the amplitudes in impact parameter space are otained. Then the expressions for the observables of total cross-section and logarithmic slope are constructed[13], combining QCD quantities and hadron extension parameters in the forms

$$\sigma^T = \alpha(\frac{S_1 S_2}{a^2})^{\beta/2}\ (\kappa\langle g^2 FF\rangle)^2 a^{10}\ , \tag{18}$$

and

$$-B = 1.858a^2 + \frac{\gamma}{2}(S_1^2 + S_2^2)\ , \tag{19}$$

where S_1 and S_2 denote the hadron sizes. The parameter values have been found to be

$$\alpha = 0.0062 \quad \beta = 3.090 \quad \gamma = 0.366\ . \tag{20}$$

Comparing the above expressions with those of Regge phenomenology, eqs.(15) and (16), we observe that now the parameters S representing the hadronic extensions have taken the place of the energy parameter s. If we consider hadron-hadron scattering for hadrons of equal sizes (as in pp and $p\bar{p}$ scattering), a direct relation between the observables σ^T and B, analogous to eq.(17), can be obtained by eliminating $S = S_1 = S_2$, and it has the form

$$\sigma^T_{pom} = \alpha\gamma^{-\beta/2}(\kappa < g^2FF >)^2 a^{(10-2\beta/2)}(|B| - 1.858a^2)^{\beta/2} . \tag{21}$$

It is very interesting and important that both eqs.(17) and (21) represent well the present data, which go up to the energy of 1800 GeV.

Using the experimental data for σ^T and B at a given energy, eqs.(18) and (19) provide a relation between values of the gluon condensate and the correlation length. Independent relations between these two quantities can be extracted from the lattice calculation[3], and also from the results of the vacuum correlation model derivation for the linear quark-quark potential[5,6]. These three independent relations fit together perfectly well, providing a unique determination of the values of $\langle g^2 \, FF \rangle$ and a, which are two fundamental properties of the physical vacuum of quantum chromodynamics.

3. Conclusions

The physical vacuum of QCD plays an important role in hadron dynamics. Non-perturbative methods are required to treat the low frequency contributions to the field fluctuations, which are essential to the confinement regime. Progress, leading to the description of many hadronic parameters, has been achieved in the last 15 years, mainly due to the sum-rule method. A few quantities, the condensates, are in the core of the calculation of many hadronic states and processes.

Besides the parametrized average values (values of condensates), details of the vacuum fluctuations are important in several important problems, such as in the structure of high spin states, and in the $0^{\pm}$ states of mesons and glueballs. Just because of the difficulties they present, these problems will be crucial for the progress towards the complete theory for Hadron Physics.

The existence of finite range of the vacuum correlations has been demonstrated through lattice calculations. The vacuum correlation model, which assumes a stochastic vacuum, where the low-frequency (i.e. non-perturbative) contributions to the gluon field fluctuations can be approximated by a Gaussian process, provides a mechanism that leads to confinement. The same model can be used in soft high-energy scattering, leading to total cross-sections that increase with the size of the hadrons, in good quantitative agreement with experiment. The QCD parameters determining this process, the correlation length and the gluon condensate, are found to agree with values coming from different sources of information. A parameter-free relation is produced between total cross-sections and slope parameters, which is shown to be valid up to the highest energies for which data exist.

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