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SOFT PHOTON PRODUCTION IN THE BOOST-INVARIANT COLOR-FLUX TUBE MODEL

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Soft photon production in the boost-invariant color-flux tube model

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Abstract: Starting from the classical expressions for emission of radiation we calculate soft photon production in the boost-invariant color-flux tube model. In the center-of-mass system of the initial tube we find that for large energies ($\sqrt{s} \sim 20$ GeV) the production of photons with frequencies: 20 MeV $< \omega <$ 50 MeV, and emitted perpendicularly to the collision axis is strongly enhanced; it exceeds considerably production of photons given by the Low limit. For the emission more collinear with the collision axis and for decreasing ω the effect becomes weaker and, eventually, in the limit $\omega = 0$ we recover precisely the Low formula. We also find that for smaller energies ($\sqrt{s} \sim 5$ GeV) the emission of photons is well reproduced by the Low formula. Generally speaking, the observed enhancement is related to the existence of a large, *i.e.* extended in time, region of photon emission. This, in turn, results from the time dilation accompanying the space-time evolution of tubes. Strong time dilation effects follow from the boost-invariance of our model and, for large s, considerably enhance radiation of soft photons. By the same token, this enhancement decreases with decreasing s, because dilation decreases.

1. Introduction

Production of very soft photons in hadronic collisions has been studied in several experiments [1-6]. The data [2-5] show an excess of very soft photons above the expectation from the bremsstrahlung off incoming and outgoing hadrons, *i. e.* the Low formula [7]. On the other hand, the data [1, 6] do not show any anomalous radiation.

The negative result of [1, 6] seems to contradict the effects observed in [2-5]. We must have in mind, however, that all these experiments are different. The incident energy changes from 10.5 to 450 GeV, the projectiles and/or targets are different. Finally, the photons are measured in different regions of the phase space. Consequently, the direct comparison between the data given in [1-6] may be misleading; *a priori* it is possible that all the data are consistent but show an excess of soft photons only in some specific situations.

The excess of photons found in [2-5] has triggered many theoretical studies whose aim was to explain the possible origin of the observed effect. The emission of very soft photons from any intermediate stage of the radiating system is suppressed by the, typically, short collision time [7]. Therefore, the excess of very soft photons may indicate that the space-time region of particle emission is much larger than the typical 1 fm. Indeed, the phenomenon of intermittency [8], may suggest the existence of very long-living objects which might be responsible for such phenomena [9]. Various models and/or mechanisms for the intermediate stage contribution are discussed in [10] with the general conclusion that it is very unlikely to get the enhancement unless the intermediate state extends over a sufficiently large time interval, much larger than 1 fm (see also [11-14]).

Since we are discussing the soft photons we can employ classical formulas for the emission of electromagnetic radiation. Indeed, this seems to be an appropriate approach because the soft component of the spectrum is not very sensitive to the details of processes. This kind of calculations have been done, *e. g.*, for the Lund model [15], but the results [16] indicate that one cannot expect any significant enhancement in the frequency region up to 100 MeV. For higher frequencies (100-500 MeV) the photon spectrum is within a factor 2 to 3 from the Low limit [7].

In this paper we stick to the classical description of radiation and calculate the soft photon emission in the framework of the model which has been formulated in [17, 18]. The model describes the space-time evolution of color-flux tubes including their decays due to the Schwinger tunneling mechanism [19]. Such color-flux tubes are believed to be formed in high-energy interactions and quite a few models make an use of the tubes (strings); in particular, the already mentioned Lund model [15]. However, the detailed implementation of the tube fragmentation is different in each particular model. Consequently, their results differ from each other [20] hence it is reasonable to study the same phenomenon using various approaches.

So far, there is no direct relation between some of the data (*e. g.* production of hadrons) and the results of our simulation because the hadronization process is still missing in our code. In fact, we may describe only these phenomena which relate directly to the sequential decays of the color-flux tubes: the electromagnetic radiation belongs to this class of processes. Although in our simulation the tubes do not hadronize, we can study the influence of their non-trivial (boost-invariant) space-time structure on the long wavelength region of the photon spectrum.

The way in which the boost-invariance is implemented in our simulation will be discussed in more detail in Section 2. Now, we only note one of the characteristics of this symmetry: the space-time development of the system is governed/parametrized by the invariant time τ rather than by the normal time t. This property leads naturally to time dilation effects. For example, if two tubes decay at the same invariant time τ then the decay time in the laboratory frame of the faster-moving one is longer than the decay time of the slower-moving tube. This standard relativistic effect leads to elongation of various lifetimes and one of its consequences is that some of the regions of photon emission are much larger than 1 fm. It is, therefore, very interesting to check how such large times affect soft photon production.

The paper is organized as follows. In the next Section we briefly recall our model. In Section 3 we introduce the classical description of the emission of radiation and discuss its limitations. Our results on soft photon production are presented and discussed in the context of the Low theorem in Section 4. Conclusions and the list of references close the paper.

Throughout the paper we use the units in which $\hbar c = 1$ ($\hbar c = 200$ MeV/fm, 1 GeV $= 5$ fm^{-1}).

2. A short review of the model

In the model under consideration [17, 18] we take into account only elementary color-flux tubes (*i. e.* spanned by one $q\bar{q}$ pair). Such tubes are presumably created in e^+e^- annihilations at very high energies. The first one is spanned by the receding members of the $q\bar{q}$ pair created by the virtual photon in which the e^+e^- pair annihilates. The next tubes are formed through the subsequent decays of the first one.

In our approach quarks and antiquarks (called later partons) are classical particles with well defined positions and momenta. They carry color charges which are the source of the color (chromoelectric) field. In between two color charges the field is confined to a tube. The intensity of the field is characterized by the string tension σ. In our calculations we use its standard value, *i. e.* $\sigma = 1$ GeV/fm. Partons move along the classical trajectories and the color tubes decay through the Schwinger mechanism [19]. The system: partons + field is self consistent: the field creates particles and causes their motion, whereas the color charges of partons determine the value of the field.

In order to describe the electromagnetic phenomena we have introduced the electric charges of partons, so far absent in the model. We take into account only the two lightest flavors. Each time when a quark-antiquark pair is created its members are given the electric charges. In unit of the electron charge they are: 2/3, -2/3 or -1/3, 1/3 respectively. Up and down quarks tunnel with the same probability.

The model is formulated in 1+1 dimensions. The space-time positions of partons are given by the vectors $x^\mu = (t, z)$. The energy and the momentum of a parton $p^\mu = (E, p_L)$ satisfy the condition $E^2 - p_L^2 = m_T^2$, where $m_T = \sqrt{m^2 + p_T^2}$ is the transverse mass. The quantity m is the rest mass of a quark. The z-axis along which the tubes are spanned and the partons are moving will be later called the collision axis.

Instead of the variables t, z and E, p_L we use the variables which have simple transformation properties under Lorentz boosts. Namely, we introduce the rapidity and the spatial rapidity (quasirapidity) of a parton

$$y = \frac{1}{2}\ln\frac{E + p_L}{E - p_L}, \quad \eta = \frac{1}{2}\ln\frac{t + z}{t - z}, \tag{1}$$

and also the invariant time

$$\tau = \sqrt{t^2 - z^2}. \tag{2}$$

Let us now discuss in more detail the assumptions of our model [17, 18] which determine the space-time evolution of our system. The coordinate system is chosen in such a way that the members of the initial pair start their motion at the point ($t = 0, z = 0$) corresponding, of course, to $\tau = 0$. The other partons are created by tunneling in chromoelectric field at some well defined space-time points characterized by the invariant time τ^{tun}. The space-time trajectory of each parton can be parametrized by τ such that $\tau_j^{init} < \tau < \tau^{final}$ ($j = 1, 2, ..., N_{par}$). Except for $j = 1, 2$ (where we have $\tau_j^{init} = 0$) $\tau_j^{init} = \tau_j^{tun}$; here the index j numerates partons and N_{par} is their number in an event.

If we plot the parton trajectories $\eta_j(\tau)$ in the coordinate system (η, τ) we obtain characteristic yo-yo structures as depicted *e. g.* in Fig. 1 of Ref. [17]. One may ask the question how one of such structures changes if we change the Lorentz reference frame. The answer is that this structure is only shifted to the left or to the right but the relative distances do not change. This is not a completely trivial result coming from the fact that we use spatial rapidity as the dynamic variable. In fact, this invariance takes place because the partons tunnel along the hyperbolae of a constant invariant time. Because of this we know, independently of the Lorentz reference frame, which tube is created earlier or sooner with respect to other tubes. *We want to emphasize that in our approach the boost-invariance is understood just by this invariance of the structure of parton trajectories, protecting the causal character of the cascade.*

We close this Section presenting how some average characteristics of our system change with time. When τ increases the number of yo-yo's, N^{yo-yo}, also increases and their masses, M^{yo-yo}, get smaller. We observe also that during the tunneling partons gain non-zero transverse momenta what causes that a yo-yo formed by them has a non-vanishing transverse momentum P_T^{yo-yo}. The time evolution of these quantities is presented in Figs. 1, 2 and 3 for various center-of-mass energies $\sqrt{s}$. The latter were chosen to be: 4, 10 and 23 GeV because such values correspond to the collisions studied in [1, 2, 5].

3. Classical description of the parton radiation

We calculate the number of photons emitted per unit solid angle per unit frequency interval from the classical formula [21]

$$\frac{d^2N}{d\omega d\Omega} = \frac{\alpha}{4\pi^2\omega}\left|\sum_{j=1}^{N_{par}}\int_{-\infty}^{+\infty} q_j \frac{d}{dt}\left[\frac{\vec{n}\times(\vec{n}\times\vec{v}_j(t))}{1-\vec{n}\cdot\vec{v}_j(t)}\right] e^{i\omega[t-\vec{n}\cdot\vec{r}_j(t)]}dt\right|^2 . \tag{3}$$

where $\vec{r}_j(t)$ is the position of the jth parton at the time t, $\vec{v}_j(t)$ is its velocity, $\vec{n}$ is the direction from the parton to the observer, α is the fine structure constant and q_j is the parton's electric charge in unit of the electron charge.

Introducing two polarization vectors $\vec{\epsilon}_1$ and $\vec{\epsilon}_2$ defined by the relations

$$\vec{\epsilon}_1 \times \vec{\epsilon}_2 = \vec{n}, \quad \vec{\epsilon}_2 \times \vec{n} = \vec{\epsilon}_1, \tag{4}$$

we find that $\vec{n}\times(\vec{n}\times\vec{v}_j) = \vec{\epsilon}_1 v_j \sin\theta$ where θ is the angle between the vectors $\vec{n}$ and $\vec{v}_j$ (see Fig. 4). In our model all partons move along the collision axis. Consequently, all $\vec{v}_j$'s are parallel, $\vec{v}_j = (0,0,v_j)$, and the angle θ is the same for all quarks and antiquarks. Similar simple relations hold for the position vectors: $\vec{r}_j = (0,0,z_j)$ and $\vec{n}\cdot\vec{r}_j = z_j\cos\theta$. From these symmetries, the fact that $\vec{\epsilon}_1$ is a unit vector, and eq. (3) follows the formula

$$\frac{d^2N}{d\omega d\Omega} = \frac{\alpha\sin^2\theta}{4\pi^2\omega}\left|\sum_{j=1}^{N_{par}}\int_{-\infty}^{+\infty} q_j \frac{d}{dt}\left[\frac{v_j(t)}{1-v_j(t)\cos\theta}\right] e^{i\omega[t-z_j(t)\cos\theta]}dt\right|^2 . \tag{5}$$

The standard procedure at this stage is to perform an integration by parts in order to obtain the electric current in the integrand. However, for our system which has a built-in boost invariance, we express (5) through the variables introduced in the previous Section:

$$\begin{aligned}\frac{d^2N}{d\omega d\Omega} = \frac{\alpha\sin^2\theta}{4\pi^2\omega}&\left|\sum_{j=1}^{N_{par}}\int_{\tau_j^{init}-\epsilon}^{\tau^{final}} q_j \frac{d}{d\tau}\left[\frac{\tanh y_j(\tau)}{1-\tanh y_j(\tau)\cos\theta}\right]\right.\\ &\left.\times \exp\left\{i\omega\tau\cosh\eta_j(\tau)\left[1-\tanh\eta_j(\tau)\cos\theta\right]\right\} d\tau\right|^2 .\end{aligned} \tag{6}$$

The quantity ϵ is introduced in the lower limit of the integration over the invariant time to indicate that in this integration some contribution from the very point $\tau = \tau^{init}$ should be included.

One can observe that the photon spectrum is given as the squared absolute value of an amplitude which is a coherent sum of the amplitudes representing the contribution from each parton. From both the physical and computational points of view, however, it is very convenient to look at the total amplitude somewhat differently, *i. e.* to treat it as the sum of four terms which correspond to different processes, namely

$$\frac{d^2N}{d\omega d\Omega} = \frac{\alpha \sin^2\theta}{4\pi^2\omega} \mid A \mid^2 = \frac{\alpha \sin^2\theta}{4\pi^2\omega} \left| A^{fp} + A^{tun} + A^{col} + A^{osc} \right|^2 . \tag{7}$$

In the following we shall give the physical interpretation to the amplitudes: A^{fp}, A^{tun}, A^{col}, and A^{osc} discussing each of them separately.

Radiation of the first pair

The amplitude A^{fp} describes the emission of radiation accompanying the creation of the electric current of the initial $q\bar{q}$ pair. In more realistic considerations we should take into account not the creation but rather a change of the current caused by $e^+ e^-$ annihilation into the $q\bar{q}$ pair. In this approach, however, we neglect the current of the incoming $e^+ e^-$ pair.

Using eq. (5) and taking into account the fact that for $t = 0$ the velocities of the first two partons are step functions of time we obtain

$$A^{fp} = \sum_{j=1}^{2} q_j \frac{v_j(0)}{1 - v_j(0)\cos\theta} = q_1 v_1(0) \left(\frac{1}{1 - v_1(0)\cos\theta} + \frac{1}{1 + v_1(0)\cos\theta} \right), \tag{8}$$

where

$$v_1(0) = -\sqrt{1 - \frac{4m^2}{s}}. \tag{9}$$

Here we used the fact that the electric charge and the velocity of the first parton are opposite to those of the second one. We also note that in the case considered, *i. e.* for $t = 0$, the coordinate system (η, τ) is singular, therefore we use here eq. (5) rather than (6).

Photons emitted during the tunneling

The amplitude A^{tun} describes the radiation related to the tunneling process. In this case one has to consider the changes of the electric current caused by the emergence of $q\bar{q}$ pairs from the vacuum. One of the interesting consequences of our boost-invariant formalism is that the tunneling particles, when they become real, have non-zero longitudinal velocities $v_j(\tau_j^{tun})$. Therefore, our process is analogous to the radiative beta decay, where a sudden change of the electron current leads to the emission of radiation. Using eq. (6) and remembering that the velocities are step functions at the time when the partons emerge from the vacuum we find that

$$\begin{aligned} A^{tun} &= \sum_{j=3}^{N_{par}} q_j \frac{v_j(\tau_j^{tun})}{1 - v_j(\tau_j^{tun}) \cos\theta} \exp\left\{ i\omega\tau_j^{tun} \cosh\eta_j(\tau_j^{tun}) \right. \\ &\quad \times \left. \left[1 - \tanh\eta_j(\tau_j^{tun}) \cos\theta \right] \right\}. \end{aligned} \tag{10}$$

Tunneling is a non-local process thus it does lead to fluctuations of the local and global charges of the system. If we denote by $\langle \Delta t^{tun} \rangle$ the average lifetime of the configurations of $q\bar{q}$ pairs which have a net charge different from zero, we can show that $\omega \langle \Delta t^{tun} \rangle < 1$ in our simulations, ω being the frequency of interest. Consequently, these stochastic fluctuations of charges affect little our numerical results (we thank G. Gustafson for his criticism of this aspect of our model).

Photons from collisions

Similarly as in the case of tunneling we can extract the part of the amplitude which is related to collisions. In this case the trajectories are again not differentiable and we get

$$\begin{aligned} A^{col} &= \sum_{j=1}^{N_{par}} \sum_{\tau_j^{col}} q_j \left[\frac{v_j^{out}(\tau_j^{col})}{1 - v_j^{out}(\tau_j^{col}) \cos\theta} - \frac{v_j^{in}(\tau_j^{col})}{1 - v_j^{in}(\tau_j^{col}) \cos\theta} \right] \\ &\quad \times \exp\left\{ i\omega\tau_j^{col} \cosh\eta_j(\tau_j^{col}) [1 - \tanh\eta_j(\tau_j^{col}) \cos\theta] \right\}. \end{aligned} \tag{11}$$

The second sum in eq. (11) denotes summation over all the collision points (along the trajectory of the jth parton), whereas v_j^{in} and v_j^{out} are the velocities just before and after the collision. We want to stress here that the collisions turn out to be relatively rare in the cascade but we have to include their contribution in order to have the full and consistent description of the process.

Photons from oscillations

The typical motion of a quark or an antiquark, *i.e.* the motion in the constant chromo-electric field, gives the following contribution to the amplitude

$$A^{osc} = \sum_{j=1}^{N_{par}} \int_{osc} q_j \frac{a_j(\tau)}{(1 - v_j(\tau)\cos\theta)^2} \exp\{i\omega\tau \cosh\eta_j(\tau)[1 - \tanh\eta_j(\tau)\cos\theta]\}\, d\tau, \qquad (12)$$

where

$$a(\tau) = \frac{dv}{d\tau} = \frac{F}{m_T \cosh^2(y)\cosh(y-\eta)} \qquad (13)$$

describes the acceleration. The expression (13) can be easily derived from the classical equations of motion for a particle moving under the influence of the constant force F [18]. In our case, for elementary tubes, $F = \pm\sigma$. The integral (12) should be performed over the periods in which the velocity (rapidity) is a smooth function of τ.

The classical description considered here is well justified if the yo-yo's are highly excited systems. To be more quantitative: the yo-yo mass (the yo-yo energy measured in the yo-yo rest frame) should be much larger than the energy of its quantum mechanical ground state. The estimates of the latter usually give the values close to 1 GeV. In Fig. 2 we can see that in our simulation the yo-yo masses decrease very fast. Consequently, in order to keep our description consistent, we have to take into account rather small values of τ^{final}; in the following we assume $\tau^{final} = 0.5$ fm. At such an invariant time we have, on the average, only two yo-yo's (see Fig. 1). Nevertheless, their space-time evolution is characterized by the time extensions much larger than 0.5 fm. For $\sqrt{s} = 4$, 10 and 23 GeV we find that the average laboratory evolution time $\langle t^{final}\rangle$ is: 2.3, 5.6 and 13.0 fm. For one event we define t^{final} as the maximal time coordinate of partons at $\tau = \tau^{final}$. The quantity $\langle t^{final}\rangle$ is obtained by averaging of t^{final} over the events.

4. Photon spectra and the Low theorem

The long wavelength limit of the photon spectrum was studied by Low [7]. He showed that for $\omega \to 0$ the spectrum is determined only by the bremsstrahlung off the external currents. In our classical approach the Low theorem is a direct consequence of eqs. (5) and (6). As long as the following condition is fulfilled

$$\omega t = \omega\tau\cosh(\eta) << 1, \tag{14}$$

the exponential factor can be well approximated by unity and we obtain

$$\frac{d^2N}{d\omega d\Omega} \to \frac{d^2N_{Low}}{d\omega d\Omega} \equiv \frac{\alpha\sin^2\theta}{4\pi^2\omega}\left|\sum_{j=1}^{N_{par}} q_j \frac{v_j(\tau^{final})}{1 - v_j(\tau^{final})\cos\theta}\right|^2 . \tag{15}$$

One can see here that the soft photon spectrum is fully determined by the final (outgoing) current. There is no incoming current because we do not take into account the radiation of the $e^+ e^-$ pair.

If the condition (14) is not fulfilled, *i.e.* when the transverse momenta become larger and/or the intermediate state lives longer, we may expect deviations from the Low limit. Note also that the short evolution in the invariant time, τ, does not lead to such a strong suppression of the emission of soft photons as the short evolution in laboratory time t: the function $\cosh(\eta)$ appearing in eq. (14) accounts for the time dilation effects. In other words the invariant evolution times τ^{final} correspond to much larger laboratory evolution times t^{final}. This observation was one of our main motivations to study quantitatively the soft photon production in the boost-invariant model.

In Figs. 5a, 5b and 5c we show our results on the photon spectra for the photon frequencies: 5 MeV $< \omega <$ 50 MeV, and for the emission angle $\theta = \pi/2$. The three plots correspond to different center-of-mass energies $\sqrt{s}$ = 4, 10 and 23 GeV. In each plot we compare the spectrum to the Low limit (15). We did our calculations using the expressions given in Section 3 first for each event and, afterwards, we averaged the single event results over the sample consisting of 1000 events. The rest mass of a quark was taken to be 10 MeV. Since we are discussing at the moment the emission perpendicular to the collision axis we present the photon spectra plotting the quantity

$$\langle\frac{dN}{dk_T^2}\rangle = \langle\frac{dN}{2k_T\, dk_T}\rangle = \frac{\pi}{k_T}\langle\frac{d^2N}{d\omega d\Omega}(\omega = k_T, \theta = \pi/2)\rangle. \tag{16}$$

Here the brackets denote averaging over the events, k_T is the transverse momentum of a photon and the factor π comes from the integration of the spectrum (3) over the azimuthal angle.

First of all our results show that we can expect a significant enhancement in the soft photon production in the case when the center-of-mass energy is large ($\sqrt{s} = 23$ GeV, see Fig. 5c). For smaller energies ($\sqrt{s} = 4$ GeV, see Fig. 5a) the effect is rather negligible. This fact is in qualitative agreement with the experimental evidence [1, 2, 5] showing the effect to increase with increasing energy of collisions.

We measure deviations from the Low limit by constructing the following ratio

$$\mathcal{R}(\omega,\theta) = \langle\frac{d^2N}{d\omega d\Omega}(\omega,\theta)\rangle / \langle\frac{d^2N_{Low}}{d\omega d\Omega}(\omega,\theta)\rangle. \tag{17}$$

Of course, we should have $\mathcal{R}(\omega = 0,\theta) = 1$. This condition was used by us as a check of our code. The values of $\mathcal{R}(\omega,\theta)$ corresponding to the spectra depicted in Fig. 5, *i. e.* for $\omega = k_T$ and $\theta = \pi/2$, are shown in Fig. 6. We observe that $\mathcal{R}$ reaches a factor of 4 for $\sqrt{s}$ $= 10$ GeV and a factor of 12 for $\sqrt{s} = 23$ GeV.

We have also studied how the extra soft photon production depends on the direction of emission. To do it we fixed the photon energy ω to be 50 MeV and were changing the angle θ in the interval: $0.3 < \theta < \pi - 0.3$ ($-0.95 < \cos\theta < 0.95$). For the center-of-mass energy $\sqrt{s} = 23$ GeV we have found that the largest anomalous production takes place if $\theta = \pi/2$, *i. e.* in the already considered case. For the emission more collinear with the collision axis the enhancement factor $\mathcal{R}$ strongly decreases (see Fig. 7). This type of behaviour can be also observed for smaller ω's and smaller energies of collisions.

In general, we can understand this dependence by examination of the argument of the exponential function in (6). The photons present in one of the very narrow cones around the collision axis are emitted by fast partons. In this case we have either $\cos\theta \rightarrow 1$ and

$\tanh(\eta) \to 1$ (the cone around the positive z values) or $\cos\theta \to -1$ and $\tanh(\eta) \to -1$ (the cone around the negative z values). This means that the exponential function is close to 1 and the Low theorem works well even if the condition $\omega t << 1$ is not fulfilled.

The strong dependence of the effect on the direction of emission indicates that the measurements of soft photon production in different regions of the phase space can give different results. It might be that this is the reason why some data do not confirm the effect (*e. g.* [6]). This problem definitly deserves further careful studies of geometry of collisions.

So far we have been discussing the results obtained in the case when $m = 10$ MeV. To check the sensitivity of our results to the assumed value of the quark rest mass we repeated our calculations setting $m = 300$ MeV (it is like a large constituent mass). Heavy quarks tunnel not as fast as the light ones, therefore, to have similar mean characteristics of the decay process we continued our evolution till $\tau^{final} = 1$ fm. For $\sqrt{s} = 23$ GeV this invariant time corresponds to the average laboratory time $\langle t^{final} \rangle = 11$ fm. Although $\langle t^{final} \rangle$ is large we have not observed any significant enhancement of soft photon production in the considered case. This must be caused by destructive coherence phenomena.

5. Summary and Conclusions

Our numerical results show that a model of production of hadrons whose first stage is tunneling of $q\bar{q}$ pairs from color flux tubes generated in interactions of the initial particles or nuclei leads to a substantial production of soft (5 MeV $< \omega <$ 50 MeV) photons.

The ratio of the production rate of soft photons of our model to the rate obtained in the Low limit ($\omega \to 0$) depends strongly on the direction of photon emission and is very sensitive to the assumed mass of quarks. In the case of the emission perpendicular to the collision axis and for the current quark masses ($\sim$ 10 MeV) the enhancement factor can reach 10.

Thus our model is capable to accomodate the *anomalous production* (anomalous respectively to the $\omega \to 0$ limit) reported in many recent experiments. At the same time we qualitatively reproduce also the energy dependence of this phenomenon: the *anomalous production* increases with the energy injected into the flux tubes by the initial collision.

This unusually large production of soft photons is caused by the assumed boost-invariance of the process of tunneling of quarks and antiquarks – as described in detail in Section 2 and 3 (tunneling of gluons is irrelevant for production of photons). This boost-invariance generates – through the time dilations – emitters of photons whose extensions in the laboratory time reach several fermis.

There are several deficiencies in our approach which are, in general, related to the fact that we use a classical method for the calculation of the photon production. First of all, the spectrum of photons generated in the processes of creation of quark-antiquark pairs from the vacuum in the presence of the strong color field should be calculated in the framework of quantum field theory. Nevertheless, for small photon energies, hence long wavelengths, we think that the classical results are reasonable. Similar situation takes place in the radiative beta decays (*e.g.* inner bremsstrahlung): the quantum mechanical treatment leads only to the modification of the upper part of the photon spectrum. The other problem we have to take into account is that yo-yo's are quantum objects and they do not radiate while in one of the stationary states. Again we expect that the radiation of highly excited yo-yo's can be approximated by classical expressions.

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Figure Captions

Fig. 1 The (invariant)time evolution of the average number of yo-yo's for three different center-of-mass energies: $\sqrt{s} = 4$ GeV (solid line), $\sqrt{s} = 10$ GeV (dashed line) and $\sqrt{s} = 23$ GeV (pointed line).

Fig. 2 The (invariant)time evolution of the average mass of a yo-yo. The solid, dashed and pointed line correspond to $\sqrt{s} = 4$, 10 and 23 GeV.

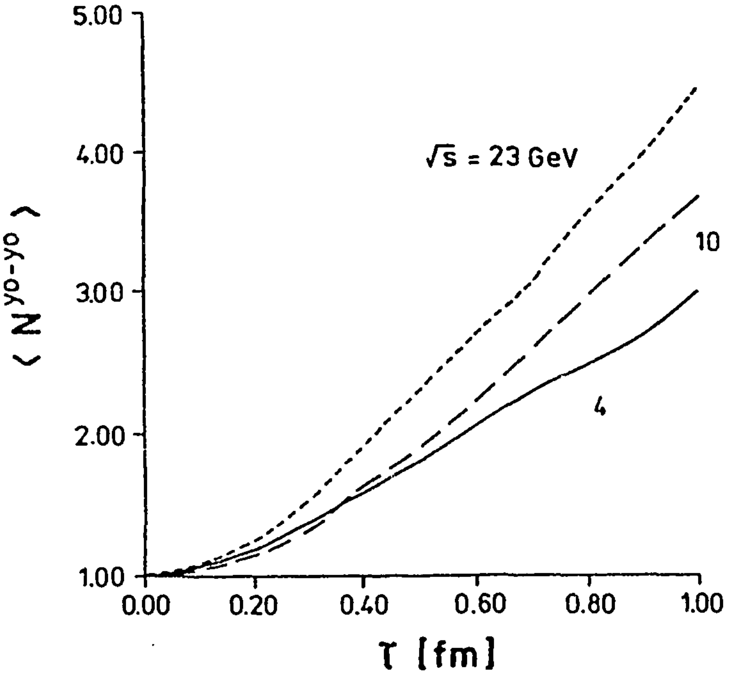
Fig. 3 The (invariant)time evolution of the average transverse momentum of a yo-yo. The three curves correspond to different initial energies as in Fig. 1.

Fig. 4 Polarization vectors ϵ_1 and ϵ_2.

Fig. 5 Spectra of photons emitted in the transverse direction ($\theta = \pi/2$) for $\sqrt{s}$= 4 GeV (a), 10 GeV (b) and 23 GeV (c). The spectra are represented by the dashed lines and the solid lines correspond, in each case, to the Low limit ($\tau^{final} = 0.5$ fm in all cases, the rest mass of the quarks $m = 10$ MeV).

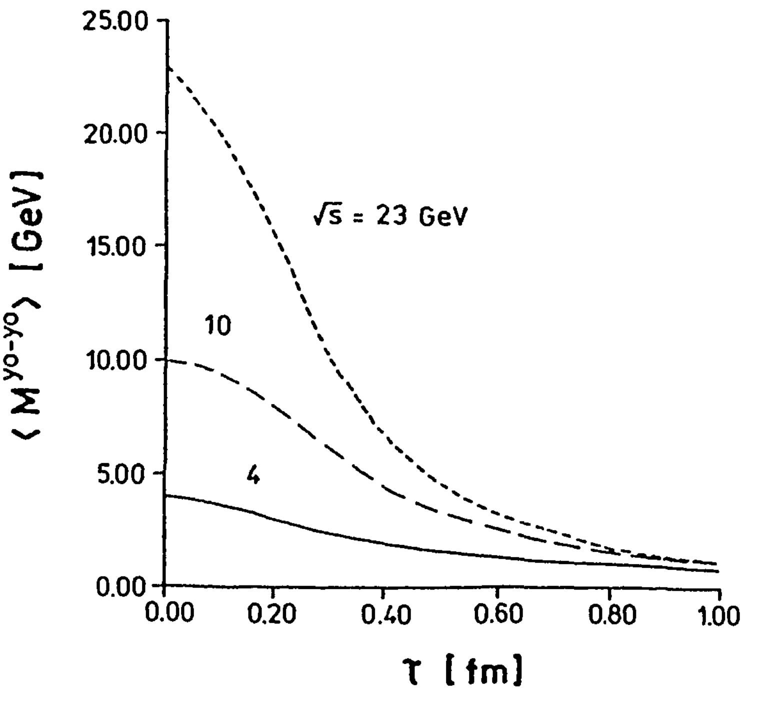
Fig. 6 Enhancement factor $\mathcal{R}$ characterizing the spectra plotted in Fig. 5. Here, again, the three curves correspond to different initial energies as in Fig. 1 (τ^{final} and m as in Fig. 5).

Fig. 7 The angular dependence of the enhancement factor for fixed: the photon energy ω= 50 MeV and the initial energy $\sqrt{s} = 23$ GeV (τ^{final} and m as in Fig. 5).



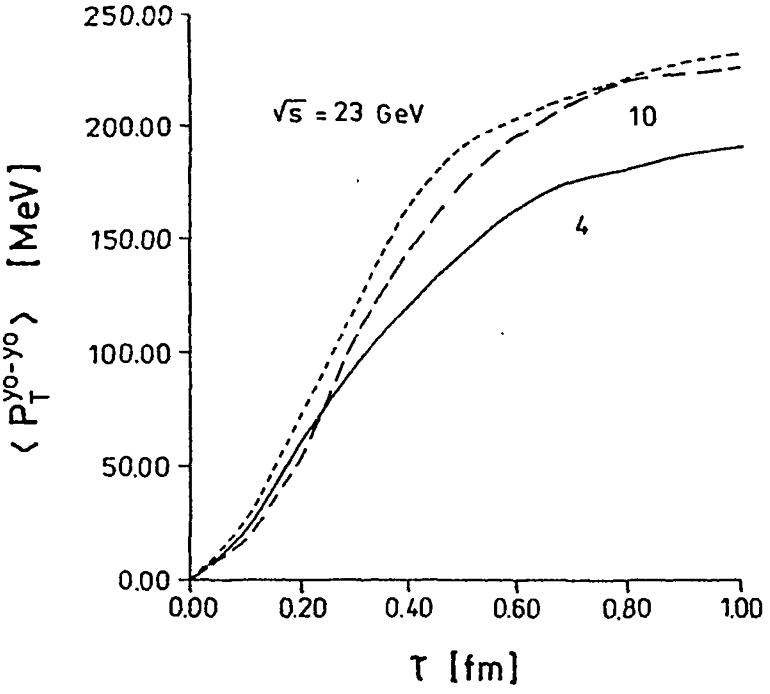
5.00
4.00
3.00
2.00
1.00
0.00
0.20
0.40
0.60
0.80
1.00
⟨ N^yo-yo ⟩
τ [fm]
√s = 23 GeV
10
4

Fig. 1



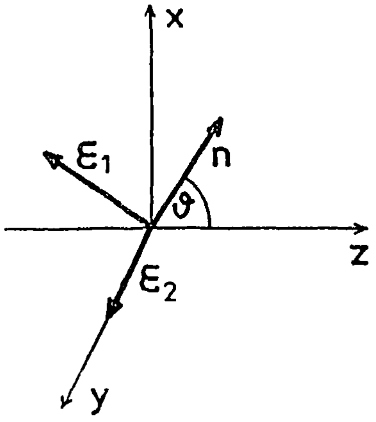
25.00
20.00
15.00
10.00
5.00
0.00
0.00
0.20
0.40
0.60
0.80
1.00
⟨ M^yo-yo ⟩ [GeV]
τ [fm]
√s = 23 GeV
10
4

Fig. 2



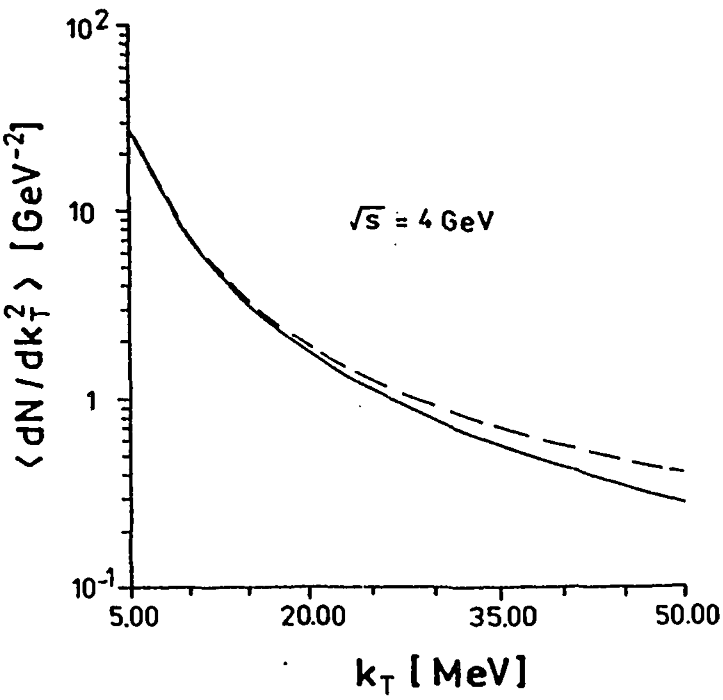
250.00
200.00
150.00
100.00
50.00
0.00
0.00
0.20
0.40
0.60
0.80
1.00
$\langle P_T^{yo-yo} \rangle$ [MeV]
τ [fm]
$\sqrt{s}$ = 23 GeV
10
4

Fig. 3



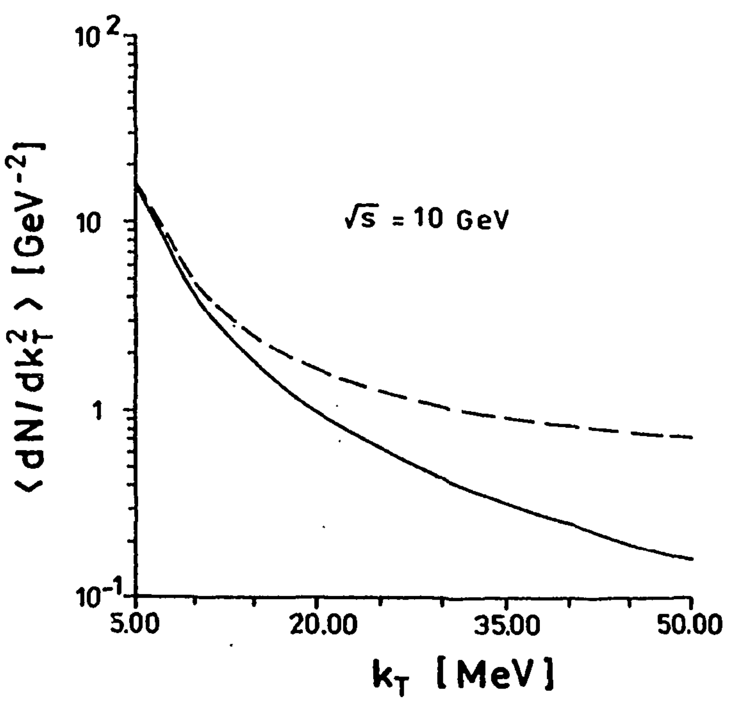
x
z
y
n
ε_1
ε_2
ϑ

Fig. 4



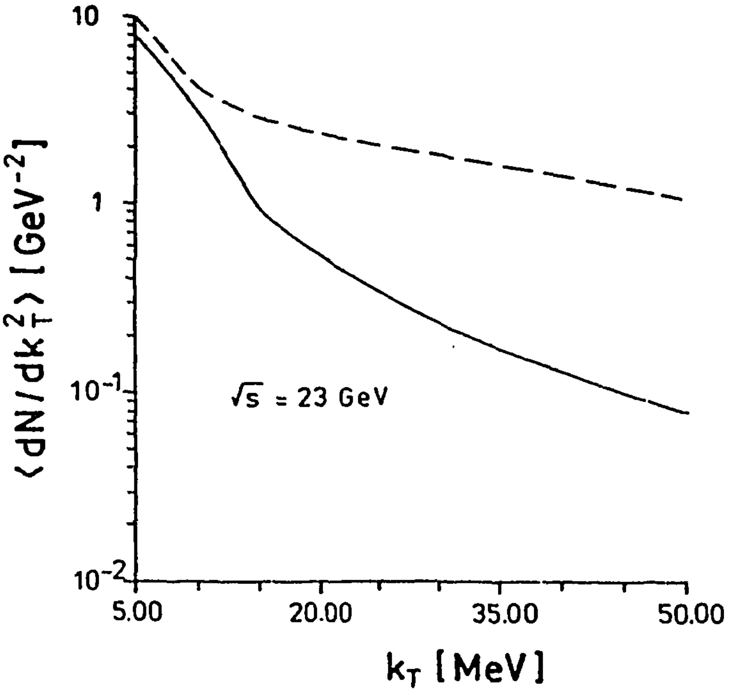
$\langle dN/dk_T^2 \rangle$ [GeV^{-2}]
10^2
10
1
10^{-1}
5.00
20.00
35.00
50.00
$\sqrt{s}$ = 4 GeV
k_T [MeV]

Fig. 5a



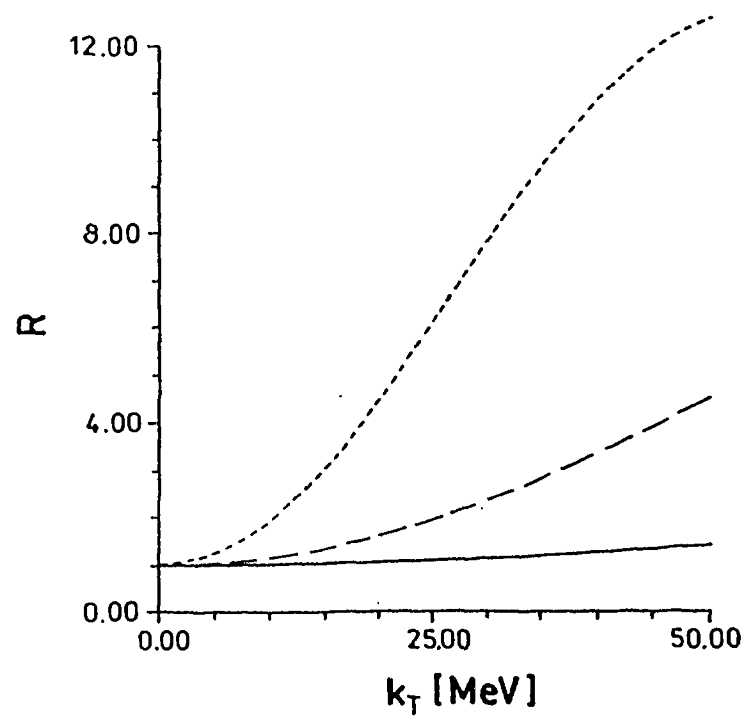
$\langle dN/dk_T^2 \rangle$ [GeV^{-2}]
10^2
10
1
10^{-1}
5.00
20.00
35.00
50.00
$\sqrt{s}$ = 10 GeV
k_T [MeV]

Fig. 5b



10
1
10^{-1}
10^{-2}
$\langle dN/dk_T^2 \rangle$ [GeV^{-2}]
$\sqrt{s}$ = 23 GeV
5.00
20.00
35.00
50.00
k_T [MeV]

Fig. 5c



12.00
8.00
4.00
0.00
R
0.00
25.00
50.00
k_T [MeV]

Fig. 6

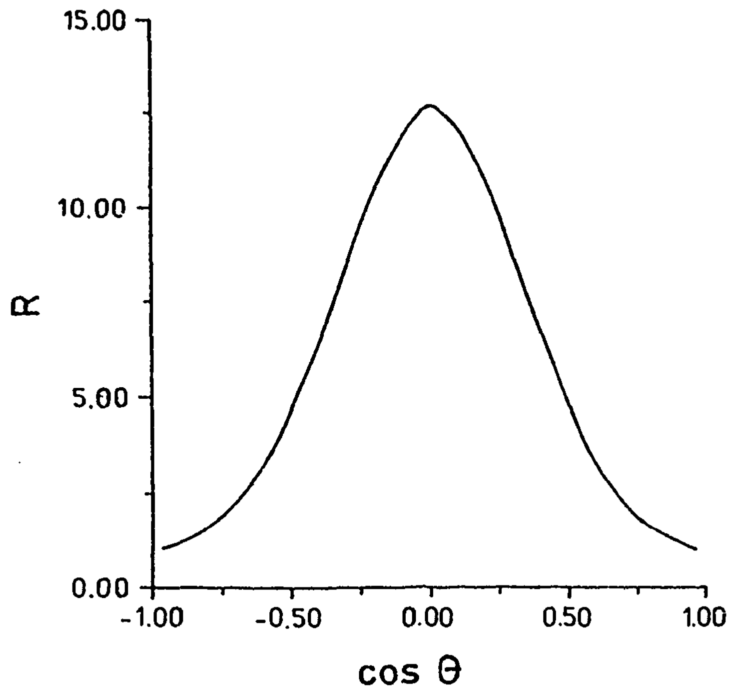


Fig. 7