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Abstract

A list of a few methods for plasma diagnostics via fluctuation (noise) analysis of random (both temporally and spatially) system parameters is reviewed. Analogy is drawn with certain noise analysis methods, used in the diagnostics of fission reactors. These methods have been applied also to fusion measurements to some extent. However, the treatment of fusion plasma fluctuations is dominated by an approach that allows for temporal randomness, but assumes periodicity in space. This approach suits well a large class of phenomena such as magnetic fluctuations (MHD effects), but is much less suited to treat localised effects such as turbulence and density fluctuations. This paper discusses the potentials of the former approach, i.e. ordinary noise analysis methods of non-periodic variables in fusion plasma diagnostics. A new recommendation is to use the crossed beam correlation analysis of soft X-ray signals for determining the local short-range correlations in the plasma and to perform a systematic exploration of the plasma spatial correlation structure with that and other methods.

I. INTRODUCTION

It has been known from the early days of fission reactor research that fluctuations in the fission chain, that is in the number of particles, carry important information on the system. In zero power reactors, i.e. critical assemblies, the transport takes place in a steady medium and accordingly, the fluctuations carry information on cross sections, the delayed neutron fraction and other static nuclear parameters. In power reactors there is another source of the fluctuations, namely the fluctuations of the medium itself in which the transport takes place. Examples of such processes are the boiling of the coolant/moderator in boiling water reactors (BWRs) and vibrations of control and fuel rods in both BWRs and pressurized water reactors (PWRs). The neutron flux fluctuations, also called neutron noise, carry information on these technological processes. Neutron noise measurements can be used to extract this information, and thus to monitor the system (for optimum control) and diagnose its status, including incipient failures (for early warning).

In a fission reactor, the neutron-neutron collisions can be neglected besides the collisions of the neutrons with the host atoms. Thus, the neutron transport can be described by linear theory, and the neutron distribution is determined uniquely by the properties of the host material. Because of this, both the mean value of the neutron distribution and the fluctuations are determined by, and carry information on, the properties of the host material only. Due to the linearity of the transport, there are no turbulence effects in the neutron field. The mean neutron distribution is determined by the mean host atom distribution, and the spatial correlation structure of the noise is simply related to that of the fluctuations of the host material.

The fluctuations of the reactor material are most often spatially localised, at least in one or two dimensions (i.e. small spherical bubbles, thin (compared to reactor diameter) control rods). The only exception when the fluctuations are periodic in both time and space is the case of BWR instability (Section 2.2). Thus, except in that case, the description of reactor noise is based on techniques of broad-band (in both frequency and wave number domain) or non-periodic random processes. In particular, an expansion of the fluctuations into spatial eigenfunctions is seldom used, rather the relevant master or Langevin equation is solved by direct Green's function techniques.

The transport of ions and electrons in a plasma shows fluctuations, too. These fluctuations are of a character different from those in fission reactors since the plasma transport is non-linear due to ion-ion and ion-electron collisions. Due to the non-linearity, not even the static (bulk) ion and electron distributions can be determined via calculations with the same ease (and precision) as the static neutron distribution in a reactor. This is even more valid for the fluctuations. There exist both turbulence effects (spatially localized) and macroscopic phenomena (MHD modes, consisting of low-order toroidal and poloidal spatial Fourier components). This means that the fluctuations take place both on a microscopic and a macroscopic level. Knowledge of these fluctuations is just as important, if not more important, as in fission reactors. For one thing, he amplitude of fluctuations is larger in relation to the mean density than in the case of fission reactors, hence they play a more important role. In fission reactors, the fluctuations are small enough such that the reactor operational state is described sufficiently well by the average neutron distribution. Thus the fluctuations are only used for diagnostics. In a fusion device, the fluctuations are much larger. Thus, besides of using these for diagnostics, they are also important even when describing the operational state of the reactor.

It would thus appear that besides of investigation of the spatially harmonic oscillations, the study of spatially localized fluctuations in plasma diagnostics is also important. Such methods have indeed been applied in charge exchange measurements and in correlation microwave reflectometry ([1], [2]). We feel however, that the potential of these methods and the possibility of applying other similar methods has not been fully realized and utilized. The reason is the dominance of spatially harmonic methods in plasma diagnostics (see e.g. [3]). While this approach is very suitable to describe MHD phenomena, it becomes apparently ineffective in describing localised perturbations and their propagation. These latter are usually recognized as "phase locking" [4] after a lengthy and unnecessary harmonic analysis, which still lacks to yield some important parameters such as correlation length (disturbance size).

The purpose of the present paper is to advocate the application of noise analysis methods with which the local plasma correlation structure can be determined. These methods bear obvious resemblance to noise analysis techniques of conventional fission reactors. Since noise analysis has long been pursued at fission reactors, it is the purpose of this paper to describe some of these methods by a reactor noise analyst for sharing them with the fusion community. The methods selected are all thought to have some relevance to fusion plasma diagnostics. The objective is to show how features of a pattern, or changes thereof, of the statistical descriptors (power spectra, phase and coherence) can be attributed to a physical process.

II. EXAMPLES OF NEUTRON NOISE APPLICATIONS IN FISSION REACTORS

2.1. Boiling water reactor noise - determination of steam velocity and detecting detector vibrations

The first case we discuss here concerns the application of in-core neutron noise for diagnostics of BWRs.The relevance of this application to plasma diagnostics arises due to some similarities between turbulent two-phase flow and turbulent plasma transport. Since the water acts as neutron moderator in BWRs, the boiling process, that is random generation and transport of bubbles, leads to fluctuations in the neutron flux. As described elsewhere ([5], [6]), these fluctuations consist of a sum of a "global" (reactor-wide) and a local component. The local component exists in the neighbourhood of the perturbation with a short (few cm) relaxation length. Due to this component, each passing bubble leads to a short pulse in the in-core detector signal. If only this component existed, then the signal of two detectors placed along an axial line at a distance $z_2 - z_1$ to each other would have signals similar to each other but with a time delay

$$\tau = \frac{z_2 - z_1}{v}, \tag{1}$$

where v is the propagation velocity. That is,

$$\delta\phi(z_2,t) = \delta\phi(z_1,t-\tau) . \tag{2}$$

Applying now the Wiener-Khinchin theorem and using (2), the cross-spectrum (CPSD) between the two detectors can be expressed by the autospectrum of one of the signals as

$$CPSD(z_1,z_2,\omega) \cong \delta\phi(z_2,\omega)\,\delta\phi^*(z_1,\omega) =$$
$$\cong \delta\phi(z_1,\omega)\,\delta\phi^*(z_1,\omega)\,e^{-i\omega\tau} = \tag{3}$$
$$= APSD(z_1,\omega)\,e^{-i\omega\tau}$$

where APSD stands for the autospectrum of one signal. According to (3), the phase of the cross-spectrum is a linear function of ω:

$$\varphi(\omega) = -\omega\tau \tag{4}$$

Since the boiling process itself, to a good approximation, is usually a white noise, $APSD(z,\omega)$ and thus $|CPSD(z_1,z_2,\omega)|$ are constants in the frequency range of interest, usually between 0 and 10-15 Hz in a typical BWR. If (2) holds, that is the two signals are identical except for a time delay, then the coherence equals unity:

$$\text{Coh} \equiv \frac{|CPSD(\omega)|}{[APSD(z_1)\,APSD(z_2)]^{1/2}} = 1 \tag{5}$$

In reality, the two signals will not be identical due to generation, collapse and coagulation of the bubbles between z_1 and z_2. Then the coherence will be less than unity, and will be a decreasing function of $z_2 - z_1$, the axial detector separation. Due to non-constant void fraction, the velocity will not be constant either and (1) has to be modified to

$$\tau = \int_{z_1}^{z_2} \frac{dz}{v(z)} \tag{6}$$

However, (4) will still hold, and both the $|CPSD|$ and the coherence will be constants as functions of ω. Such a measurement is thus suitable for the determination of the propagation velocity, or transit time, of the perturbation.

In a reactor, however, there exists also a global or reactivity effect. This yields a reactor-wide component through the reactivity coefficient of the void, and leads to a component which is identical in the normalized signals of the two detectors. Roughly speaking, the normalized (by the static flux) detector signals consist of the sum of a global component, which is the same for all detectors, and a local component, which is propagating upwards axially and occurs with a time delay in detector signals for axially placed detectors. Since the relative weight or contribution of the local component is usually larger in BWRs than that of the global one, the joint effect of the two components will lead primarily to a slight change in the phase relationship (4) and the coherence (5). Due to the presence of the global component, instead of the linear relationship (4), the phase will not be given by a straight line as in (4), but will slightly oscillate around it. It will cross the straight line at ω values $\tau\omega = n\pi$; $n = 0, 1, 2, \ldots$. The change in the coherence is even more visible. If either only the global or only the local component existed, the coherence would be equal to unity. However, if both components exist, then, there will be an interference effect. The CPSD can approximately be given as

$$CPSD \approx a + b\cos\omega\tau \tag{7}$$

Thus the CPSD, and likewise the coherence, has maxima at

$$\omega\tau = 2n\pi; \qquad n = 0,\ 1, 2, \ldots \tag{8}$$

and minima at

$$\omega\tau = \frac{2n+1}{2}\pi; \quad n = 0,\ 1, 2, \ldots \tag{9}$$

A more detailed explanation of the above can be found in [7], where examples of phase and coherence functions, measured in Swedish boiling water reactors, are published.

For the sake of illustration of how such measurements, together with a simple phenomenological model, can be used to determine operational parameters and to detect anomalies, some figures will be reproduced here from the above mentioned publication. Fig. 1 shows a measurement corresponding to the normal state. The straight lines, representing the phase of the CPSD, are suitable to determine the transit time of bubbles between two axially placed detectors. It is also seen, from the change of the slope of the phase, that the transit time decreases with increasing axial height of the detectors, due to higher steam velocity in the upper part of the core. The coherence shows a behaviour completely consistent with (8) and (9), that is it has minima at the π-crossings of the phase.

Fig. 2 demonstrates how the behaviour of the phase and coherence can be used to detect an anomaly. As discussed in [7], the detector tube (and hence also the four detectors in it) vibrated laterally in these measurements quite violently due to the excess turbulence in the coolant flow. If the detectors are in a position where the static neutron flux has a radial gradient (which is most often the case), then the vibration of the detectors leads to a time-resolved detector signal.The energy content of the vibrations is concentrated around the fundamental frequency of the detector string, in case of large vibrations even including the first higher mode. Hence the vibrations occur as strong global components in all detector signals. Thus, over the frequency range of vibrations, the phase will tend to zero instead of being a linear function with a non-zero slope. At the same time, the sink- and peak structure of the coherence is also changed compared to the vibration-free case. More cases of detector tube vibrations and their qualification can be found in [7].

It is thus seen how cross-correlation measurements, together with a simple phenomenological model of the process, can be used to determine reactor operational parameters, monitor their changes, and detect anomalies, often in an incipient state. Examples of such applications and their theory are explained in e.g. [6], [8], [9] and [10].

2.2. BWR stability

Cases of nearly periodic, very weakly damped oscillations have been observed during the start-up of several BWRs. In certain cases the oscillations became undamped, and then had the character of a limit cycle oscillation. Both global (reactor-wide) and regional (with opposite phase in two halves of the core) oscillations were observed.

The global oscillations occur in-phase in each spatial point of the reactor, with an amplitude that is proportional to the static flux $\phi(r)$. The oscillations have a narrow-band peak (resonance) character in the neutron signal autospectrum (APSD), and that of a weakly damped (slowly decaying) oscillation in the autocorrelation function (ACF) of a detector signal (Fig. 3). It is usually assumed that these flux or reactor power oscillations correspond to a damped second order oscillator excited by a white noise,

$$\delta\ddot{\phi}(t) + 2\xi\omega_0\delta\dot{\phi}(t) + \omega_0^2\delta\phi(t) = f(t) \tag{10}$$

with $\langle f(t)f(t+\tau)\rangle = \delta(\tau)$, ξ being the damping coefficient and ω_0 the resonant frequency. From eqn (10) it follows that the autospectrum of the neutron fluctuations is given as

$$APSD_{\delta\phi}(\omega) = \frac{APSD_f}{\left(\omega^2 - \omega_0^2\right)^2 + 4\xi^2\omega^2\omega_0^2} \tag{11}$$

and the autocorrelation function as

$$ACF_{\delta\phi}(\tau) = \frac{e^{-\xi\omega_0|\tau|}}{4\omega_0^3\xi}\cos\omega_0\tau \tag{12}$$

APSD and ACF values corresponding to such oscillations are shown in Fig. 3. For weak damping the oscillations are nearly harmonic, and one can write approximately

$$\delta\phi(\underline{r}, t) = \exp\{i\omega_0 t\}\,\phi_0(\underline{r}) \tag{13}$$

where $\phi_0(\underline{r})$ is the critical (or static) flux.

Reactor stability is usually related to the decay of the envelope of the ACF, determined experimentally as the ratio $\beta = A_2/A_1$ $(=A_{N+1}/A_N)$. This quantity is called the decay ratio. $\beta = 1$ corresponds to the unstable state. From the above definition of β and eqn (12), it follows that [11]

$$\beta = e^{-\xi 2\pi} \tag{14}$$

and

$$\xi = \frac{1}{2\pi} ln \frac{1}{\beta} \tag{15}$$

Thus, the unstable case $\beta = 1$ corresponds to $\xi = 0$, that is no damping. In such a case the APSD diverges at $\omega = \omega_0$, and accordingly, the correlation time (system memory) becomes infinite, since the ACF does not tend to zero for large τ. For a non-zero damping, on the other hand, the correlation time will remain finite since the ACF tends asymptotically to zero. Generally, the condition for the existence of a finite correlation time is a broad-band, finite amplitude power spectrum, which is not fulfilled with zero damping.

The regional oscillations have a similar time or frequency behaviour, but a different space-dependence. They correspond to the higher modes of the flux (higher order spatial eigenfunctions). As described in [11], they belong to one of two categories. The first is when only one azimuthal mode exists, i.e.

$$\delta\phi(\underline{r}, t) = a_1(t)\cos\varphi\cdot\phi_1(r) = \exp\{i\omega_0 t\}\cos\varphi\cdot\phi_1(r) \tag{16}$$

where $\underline{r} = (r, \varphi)$, and $\phi_1(r)$ is the radial mode of the first azimuthal eigenfunction. This oscillation mode leads to out-of phase behaviour in two halves of the radial cross-section of the core with a static boundary (a diagonal line) between the two regions. The second possibility is that two azimuthal modes are excited with a constant phase difference, in which case one has

$$\delta\phi(\underline{r}, t) = \exp\{i(\varphi + \omega_0 t)\}\,\phi_1(r) \tag{17}$$

This corresponds to a rotating wave with angular frequency ω_0.

In both cases above, the autospectrum and the autocorrelation function of a detector signal, that is that of the noise at a given fixed $\underline{r}$, still have the form (11) and (12), respectively. It is also seen, that (13), (16) and (17) correspond formally to MHD modes with n = 0 and n = 1, respectively. We will return to this analogy later on.

2.3. Two-phase flow measurements with crossed-beam radiation correlation

This method was suggested relatively recently ([12], [13])and has not yet been widely used in diagnostics. It was tested in a series of pilot measurements at the Dept. of Reactor Physics, Chalmers, which proved its applicability [14], [15]. It is judged that this method has good potentials for application also in plasma diagnostics, this is why it is taken up here. Application of the method to JET soft X-ray data is currently under discussion.

The objective of the method is to measure local flow parameters, such as velocity, void fraction and bubble size (more precisely, correlation length of the density fluctuations) in a stationary two-phase flow with non-intrusive methods and further, to determine the flow structure or flow regime (e.g. bubbly, slug, annular etc.). This latter may be possible if the former local parameters, and the correlation length in particular, are known everywhere in the flow.

The principles of the method are described in [12] and [13], we only repeat here the basic concepts. The void and fluid parts are assigned constant density (zero for the void and unity for the fluid). It is assumed that there exists a scalar local correlation length (which may be direction dependent for anisotropic flows). The correlation length itself is easiest to define if the spatial correlations have a cut-off, which we shall assume here. For further simplicity we describe the case of isotropic flows here. The general anisotropic case is treated in [13].

The basic assumption is to write the spatial one-time correlation function of the density fluctuations $\delta\rho(\mathbf{r}, t)$,

$$\langle \delta\rho(\boldsymbol{r}_1, t)\,\delta\rho(\boldsymbol{r}_2, t)\rangle \equiv R(\boldsymbol{r}_1, \boldsymbol{r}_2) \tag{18}$$

in the form

$$R(\boldsymbol{r}_1, \boldsymbol{r}_2) = R_r(\xi) \tag{19}$$

where

$$\boldsymbol{r} \equiv \frac{\boldsymbol{r}_1 + \boldsymbol{r}_2}{2} \tag{20}$$

and

$$\xi = \frac{|r_1 - r_2|}{l(r)} \tag{21}$$

Here $l(r)$ is the local correlation length. With the scaling (21), and identifying the correlation length by the cutoff-point, one will have

$$R_r(\xi) = 0 \qquad \text{for} \quad \xi \geq 1 \tag{22}$$

In case of some simple flows, such as bubbly flow with random bubbles, $l(r)$ is equal to the local bubble diameter, or local perturbation size.

Experimental determination of $R_r(\xi)$ is made by applying two radiation beams, e.g. X-rays, to illuminate the flow transversally. The beams will be subjected to mass attenuation in the fluid, and will thus be modulated randomly in a two-phase flow. The fluctuations at the exit side are related to a line integral of the density fluctuations along the beam. One then determines the temporal cross-correlation of the detector currents:

$$R_i(\tau) \equiv \langle \delta i_1(t+\tau)\, \delta i_2(t) \rangle \tag{23}$$

where $\delta i_{1,2}(t)$ are the detector current fluctuations. Using a geometry as shown in Fig. 1, further, assuming that the density fluctuations propagate unchanged with a velocity $v(r)$ together with the flow, it can be shown that the relationship between the temporal and spatial correlation functions can be given as

$$R_i(\tau) = 2\pi l^2(r) \int_{\frac{\tau v(r)}{l(r)}}^{\infty} R_r(\xi)\, \xi d\xi \tag{24}$$

Eqn (24), together with (22) shows that the local correlation length $l(r)$ can be determined from the cutoff of $R_i(\tau)$, since the latter vanishes when $\tau = l(r)/v(r)$. This requires the knowledge of the flow velocity in the crossing point of the beams. The flow velocity can be determined with the same measurement if one of the beams is shifted in the axial direction (along the flow direction) with a distance dz. Then, the maximum of the correlation function $R_i(\tau)$ will be shifted from $\tau = 0$ to $\tau = dz/v(r)$, from which $v(r)$ can be determined. Obviously it is sufficient to perform one measurement only with the axially displaced beams to determine both $v(r)$ and $l(r)$. It is this method which is referred to as the crossed beam cross correlation method.

Thus, the local correlation length and the local velocity can be determined directly from the temporal correlation of the detector signals. In addition, in a binary flow (incompressible fluid), the average local void fraction or average local density can be determined directly from $R_i(\tau = 0)$, as described in [12]. By shifting the crossing point of the beams, or by using several parallel beams in two perpendicular sets, the above parameters can be determined in the whole radial cross-section of the flow. If the (possibly directionally dependent) correlation length is known in the whole cross-section of the flow, it is envisaged that from this, the flow structure, that is the flow regime, can be determined. This surmise, even though it is quite plausible, has not yet been confirmed experimentally. This possibility is nevertheless very important, because the concept of flow regime is central to many scientific and technological applications. There is however no existing flow regime indicator, even less a non-intrusive one.

III. APPLICATIONS OF CORRELATION ANALYSIS FOR PLASMA DIAGNOSTICS

There exist a few correlation methods that have already been used in plasma diagnostics more or less routinely. It is felt, nevertheless, that there is room for development in this area, partly by improving the performance of the existing methods, partly introducing further methods besides the already existing ones. In both cases, experience with and concepts of the methods used for fission reactor diagnostics can be utilized. We describe both general principles and one particular new method. The purpose is determination of correlation and transport properties of plasma density fluctuations. First the basic principles of such measurements will be briefly given.

3.1. General considerations

Assume that one can measure a (toroidally) local quantity $S(\mathbf{r}, t)$ and its fluctuation $\delta S(\mathbf{r}, t)$. In the terminology of [1], δS is also called the disturbances by Costley et al. [1], [2]. S itself can represent e.g. the density of soft X-ray emissivity at some spatial position in the plasma, or the position of the reflecting layer in case of microwave reflectometry.

If the correlation length of the fluctuations δS is short, and the disturbances propagate toroidally with some velocity v_p, then the correlation length and propagation velocity can be determined by two detectors placed at different toroidal positions. However, the fact that in this case the plasma movement is rotational (periodic), in contrast to the propagation of two-phase flow in a straight channel, represents a new aspect.

If no periodical (MHD) component exists in $S(\mathbf{r},t)$, and $\delta S(\mathbf{r},t)$ is a broad-band fluctuation, then a finite correlation length l will exist. If $\delta S(\mathbf{r},t)$ is sufficiently broad-band, or in other words, it contains sufficiently high wave numbers, then the correlation length may be much smaller than the perimeter of the circular rotation. That is,

$$l \ll 2R\pi, \tag{25}$$

where R is the plasma major radius. In such a case, the auto power spectrum of any detector signal will be a flat broad-band spectrum, and the autocorrelation function will show no oscillations but will decay monotonically with a cut-off point at $\tau = l/v_p$. The cross-correlation function between detectors at different toroidal positions will have a maximum at $\tau_0 = dr/v_p$, where dr is the toroidal separation of the detectors. The absolute value of the cross-spectrum of the two signals will be similar to that of the autospectra, whereas its phase will depend linearly on frequency, that is

$$CPSD(\omega) = APSD(\omega)\, e^{-i\omega\frac{dr}{v}} \tag{26}$$

where $APSD(\omega)$ can be taken as constant over a broad frequency band. From the phase relationship

$$\varphi(\omega) = -\omega\frac{dr}{v} \tag{27}$$

the propagation velocity can be determined.

Such measurements were performed by Costley et al. with microwave reflectometry ([1], [2]). With correlation reflectometry, the correlation length can be measured in any direction, e.g. toroidally and radially. With two toroidal measurement points, the toroidal rotational frequency (velocity) of the plasma can be measured. It was found by Costley that the plasma properties depend strongly on the operational mode. In L-mode plasmas high correlations were found and toroidal propagation was observed.

In a tokamak torus, where the plasma rotates circularly and after one period returns to the starting point, further effects need to be accounted for, depending on the lifetime of the disturbances compared to the rotational period. Let the rotational period be T, and the lifetime (correlation time) of the disturbances t_0. Elementary consideration then yields the following. If the lifetime is larger than T but smaller than $2T$, that is the plasma has a "one-cycle memory", then some part of a signal detected at a time t will reappear again once at $t + T$. Consequently, the ACF of the signal will have, in addition to the maximum at $\tau = 0$, another smaller local maximum at $\tau = T$. In formulae, the time-resolved detector signal fluctuation $\delta i(t)$ can be written as

$$\delta i(t) = \delta S(t) + c_1 \delta S(t-T); \quad c_1 < 1 \tag{28}$$

and its ACF as

$$\langle \delta i(t+\tau)\, \delta i(t) \rangle = ACF_{\delta i}(\tau)\left[1 + c_1^2\right] + c_1\left\{ACF_{\delta i}(\tau - T) + ACF_{\delta i}(\tau + T)\right\} \tag{29}$$

In the frequency domain

$$\delta i(\omega) = \delta S(\omega)\left(1 + c_1 e^{-i\omega T}\right) \tag{30}$$

and from here, through the Wiener-Khinchin theorem,

$$APSD_{\delta i}(\omega) \propto |\delta i(\omega)|^2 = APSD_{\delta S}(\omega)\ (1 + c_1 + 2c_1 \cos\omega T) \tag{31}$$

From (31) it is seen that with a one-cycle memory, the detector APSD signal will have maxima at

$$f_0,\ 2f_0,\ 3f_0, \ldots \tag{32}$$

where f_0 is the rotation frequency $1/T$, and it will have minima at

$$\frac{f_0}{2}, \frac{3f_0}{2}, \frac{5f_0}{2}, \ldots \text{ etc.} \tag{33}$$

Continuing the same way, in case of a two-cycle memory, that is when

$$2T < t_0 < 3T, \tag{34}$$

one can write

$$\delta i(t) = \delta S(t) + c_1 \delta S(t-T) + c_2 \delta S(t-2T) \tag{35}$$

Then the corresponding ACF will have maxima at $\tau = 0$, T and $2T$. The APSD will have the form

$$APSD_{\delta i}(\omega) = APSD_{\delta S}(\omega)\ (a + b\cos\omega T + c\cos 2\omega T) \tag{36}$$

where $a \equiv 1 + c_1 + c_2$, $b \equiv 2c_1(1 + c_2)$ and $c \equiv 2c_2$. This expression has further minima at

$$\frac{f_0}{4}, \frac{3f_0}{4}, \frac{5f_0}{4}, \ldots \text{ etc.} \tag{37}$$

The temporal decay of the disturbances can thus be determined from the decay of the higher order maxima of the ACF, or from the structure of the APSD. In the case of long or infinite memory (which is actually the most common case due to the presence of MHD modes), the spectrum will be a line spectrum, being infinite at the frequencies nf_0, n=0,1,2... and zero in between. This is illustrated schematically in Fig. 5.

This latter case can be described even simpler. Slow or no decay means that the signal will be periodical, and can be expanded into a discrete Fourier series. The members of the series are the MHD modes. E.g. the toroidal modes will have the form

$$\delta S = S_0 exp\{i(n\psi + \omega_0 t)\} \tag{38}$$

with $\omega_0 = 2\pi f_0$. There is an obvious formal similarity between (38) and the rotational modes of BWR oscillations, eqn (17). For instance, one can calculate the persistence of the MHD modes by calculating the "decay ratio" from the ACF of δS.

In the frequency domain, the periodic MHD modes will lead to the appearance of spectral lines in the APSD at frequencies $f = f_0, 2f_0, 3f_0$ etc. The temporal decay of the modes will lead to a finite width of these lines, and the presence of short-scale correlations will result in a continuous background, as shown schematically in Fig. 6. The above statistical properties of the soft X-ray signals have been routinely used at JET to determine the MHD modes [16].

3.2. Crossed beam correlation analysis of soft X-ray signals

Soft X-ray measurements are based on the radiation induced internally in the plasma, as viewed by an X-ray camera. The emissivity q can be written as

$$q_x = \sum_z n_e n_z f(z, T_e) \tag{39}$$

where n_e and n_z represent the electron and impurity ion density, and the summation goes for the impurity species. In many cases (39) can be simplified into

$$q_x = n_e^2(r, t)\, T(r, t) \tag{40}$$

In an X-ray camera, there are several X-ray detectors, each detector collecting signals along a line-of-sight, hence detecting the line integrated emissivity. There are a large number of detectors in an X-ray camera (Fig. 7), with a few sets of nearly parallel lines. For purposes of tomography, the lines from different sets cross each other. Thus there is a possibility to use the crossed beam correlation technique for determining local parameters such as correlation length of density and temperature fluctuations and propagation velocity. Again, if the correlation length is small enough, these parameters will be radially local, and with the use of several crossing beams the distribution (radial profile) of these parameters can be determined.

However, the presence of the MHD modes represents a problem. The periodic modes correspond to waves with infinite or very long correlation length. To determine the short-range correlations, the MHD modes need to be eliminated. It terms of Fig. 6 this means that the lines or peaks in spectra at nf_0 in the APSD need to be filtered out. It should be noted that in any measured signal with a non-zero mean value, the line at $n = 0$, that is at $f = 0$, is always present. This line is, however, very easy to eliminate even by hardware means, either with an offset (DC compensation) or with a high-pass filter with a sufficiently low break frequency. The advantage of doing this before digitizing the signal is that the AD conversion will have a much better resolution. If the DC part of the signal is too high, the fluctuations may not be recorded at all without eliminating the former. However, eliminating higher order periodic components from a signal is not as simple, basically due to the fact that the frequency of the components to be eliminated varies from measurement to measurement and is not known in advance. If only the MHD mode n=1 exists, high-pass filtering techniques still may be possible. Otherwise the elimination need to be performed by digital filtering techniques on the recorded signals, which is only useful if the broad-band fluctuating part of the signal is strong enough. If the elimination is successful, the crossed-beam correlation method can be applied to determine local plasma parameters from the soft X-ray signals.

Although much is known already on the turbulence and transport properties in burning plasmas via microwave reflectometry and charge exchange (CX) measurements, the evaluation of crossed-beam X-ray measurements would prove to be a useful complement. For example, toroidal transport can be measured with it better than with reflectometry if also poloidal rotation is present. In particular, a systematic study of the space- and direction-dependent correlation length would be of value. So far only toroidal and radial correlation lengths have been determined in certain points of the plasma. A more thorough investigation of the spatial correlation structure may contribute to a classification of plasma modes or regimes. Again, one can take analogies to two-phase flow, where the concept of flow regimes not only helps to classify different flows, but a prior knowledge of the flow regime makes numerical calculations on the flows much more effective and precise. There will of course be several basic differences between two-phase flows and plasmas, one of them being that in a two-phase flow, unlike in a plasma, the disturbances (bubbles or droplets) have sharp boundaries. This makes the identification of the topological structure of the flow intuitively easy even by inspection. A similar classification of plasma structure is only possible via the correlation structure in a fusion plasma, which is less straightforward.

IV. CONCLUSIONS

Some aspects of the correlation analysis methods that are being used at fission reactors as well as those either already in use or being suitable to be used at fusion reactors were described. The main emphasis is on the possibilities and significance of determining the plasma density correlation structure experimentally. Some new suggestions are given that will be tested with measured signals from both JET tokamak and the Wendelstein W7-X stellarator.

V. REFERENCES

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Fig. 1

Figure Captions

Fig. 1. Phase and coherence between two axially placed neutron detectors in a BWR measurement. Only the transport of bubbles can be seen.

Fig. 2. Phase and coherence between two axially placed neutron detectors in a BWR measurement. In this measurement, the tube holding the detectors vibrated. This can be seen through the deviations from Fig. 1.

Fig. 3. APSD and ACF values of detector noise signals in case of BWR oscillations.

Fig. 4. Layout of the crossed beam correlation measurement.

Fig. 5. APSDs of plasma density fluctuations with cyclic transport (rotation) and different temporal decay (memory time).

Fig. 6. APSD of plasma density fluctuations with a white background (short-range correlations) and a line spectrum, corresponding to the MHD modes.

Fig. 7. The old JET X-ray camera.

Fig. 2

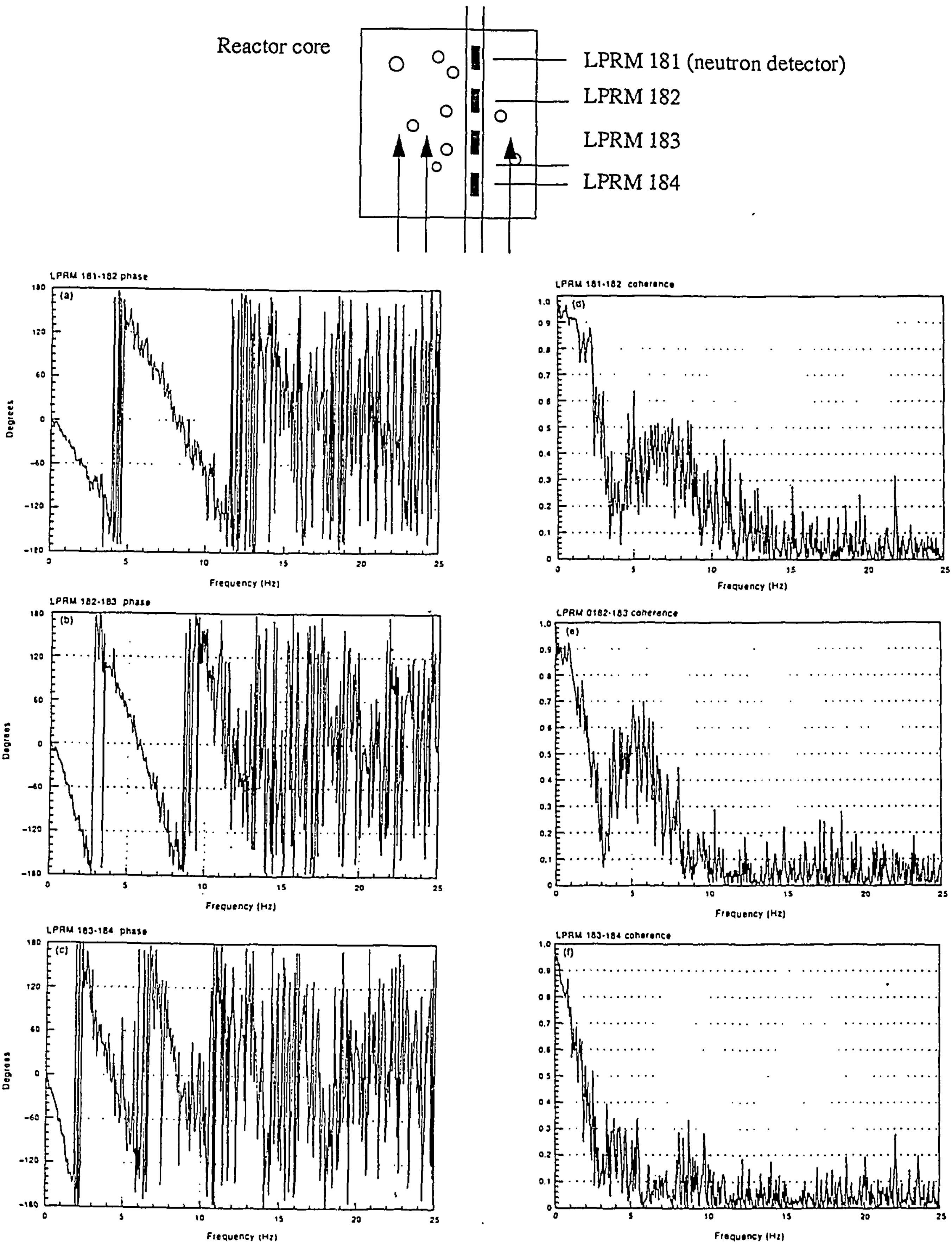
Reactor core
LPRM 181 (neutron detector)
LPRM 182
LPRM 183
LPRM 184
LPRM 181-182 phase
(a)
Degrees
Frequency (Hz)
LPRM 181-182 coherence
(d)
Frequency (Hz)
LPRM 182-183 phase
(b)
Degrees
Frequency (Hz)
LPRM 0182-183 coherence
(e)
Frequency (Hz)
LPRM 183-184 phase
(c)
Degrees
Frequency (Hz)
LPRM 183-184 coherence
(f)
Frequency (Hz)

Fig. 1

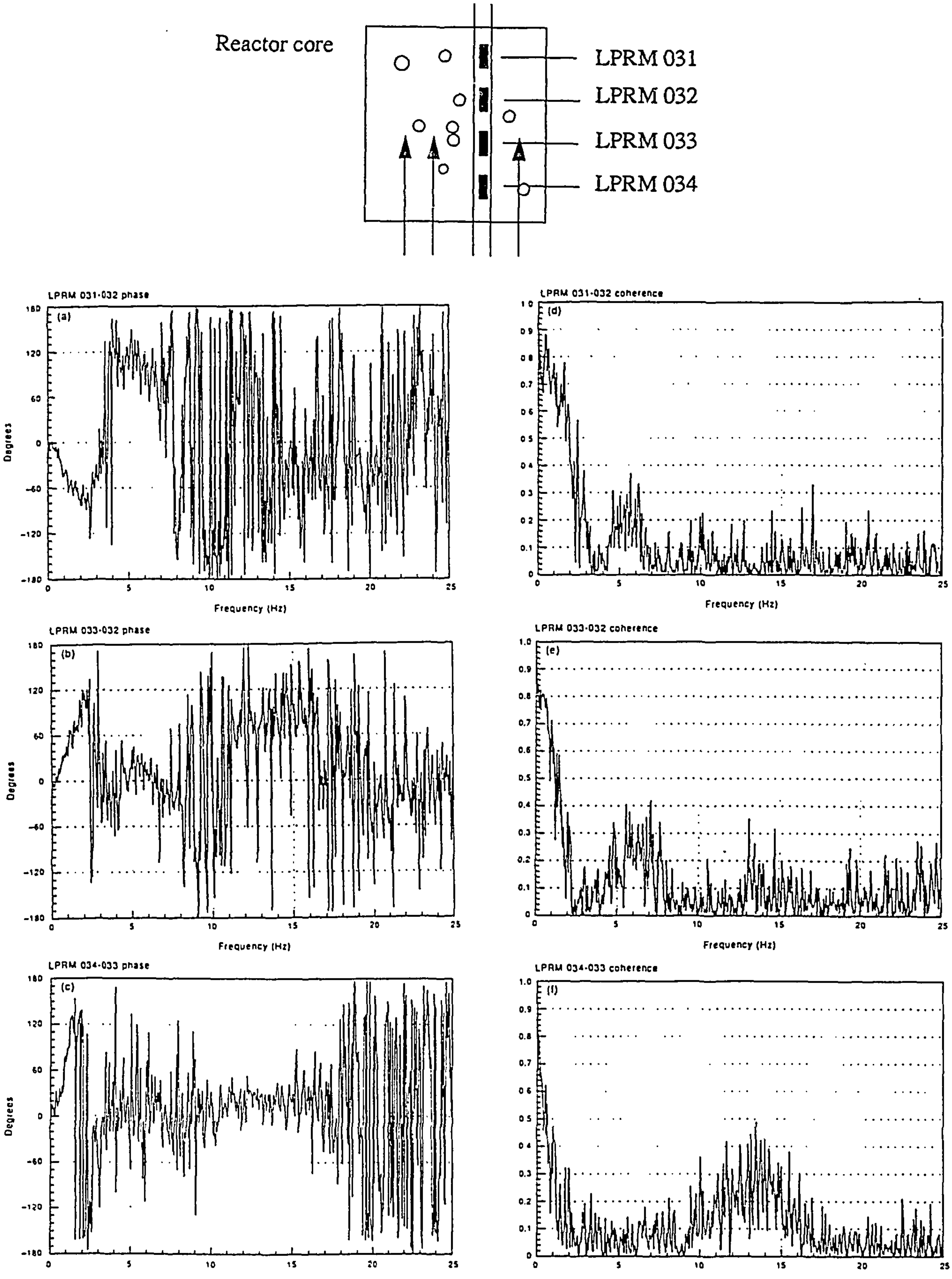
Reactor core
LPRM 031
LPRM 032
LPRM 033
LPRM 034
LPRM 031-032 phase
(a)
LPRM 031-032 coherence
(d)
LPRM 033-032 phase
(b)
LPRM 033-032 coherence
(e)
LPRM 034-033 phase
(c)
LPRM 034-033 coherence
(f)
Degrees
Frequency (Hz)

Fig. 2

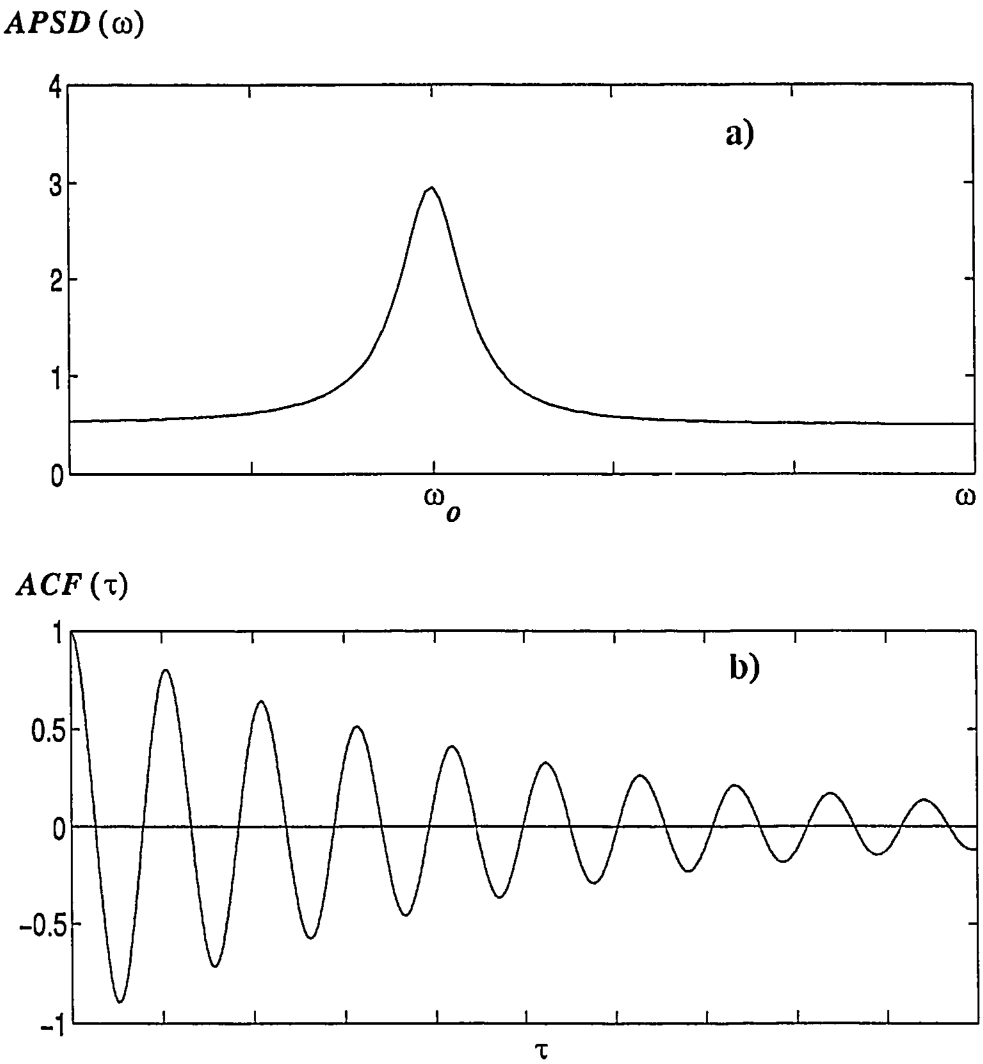

Fig. 3

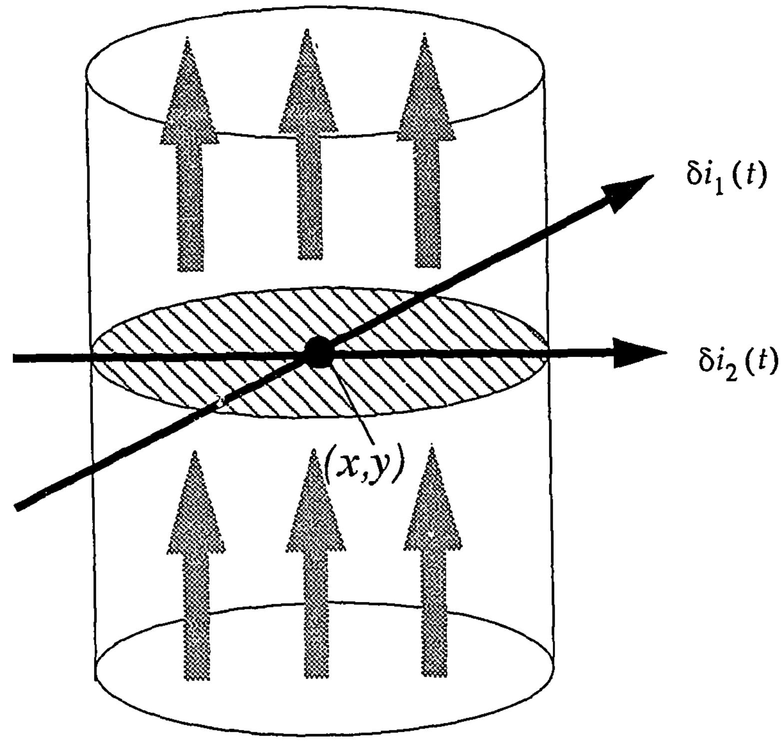

Fig. 4

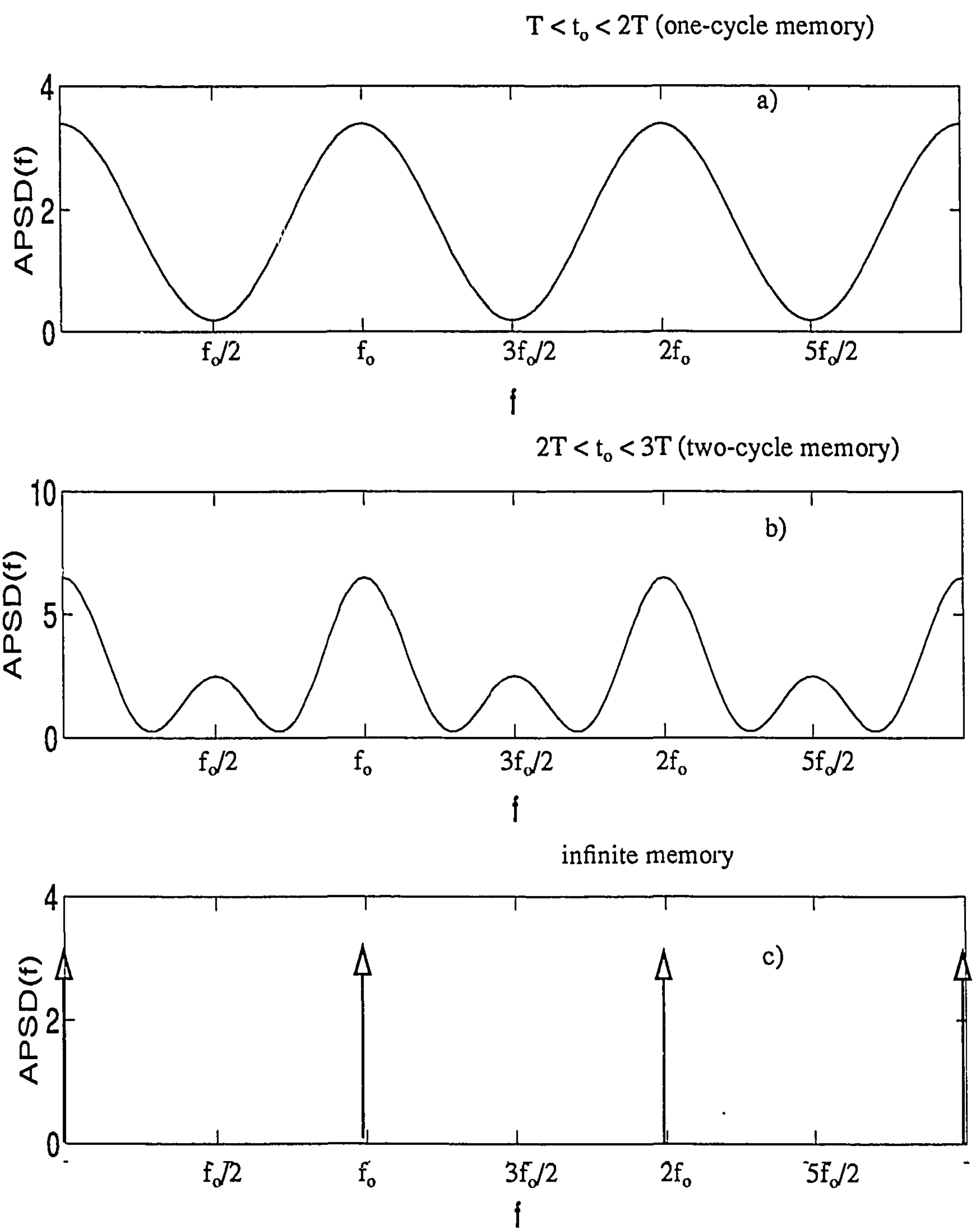
T < t_o < 2T (one-cycle memory)
a)
APSD(f)
4
2
0
f_o/2
f_o
3f_o/2
2f_o
5f_o/2
f
2T < t_o < 3T (two-cycle memory)
b)
APSD(f)
10
5
0
f_o/2
f_o
3f_o/2
2f_o
5f_o/2
f
infinite memory
c)
APSD(f)
4
2
0
f_o/2
f_o
3f_o/2
2f_o
5f_o/2
f

Fig. 5

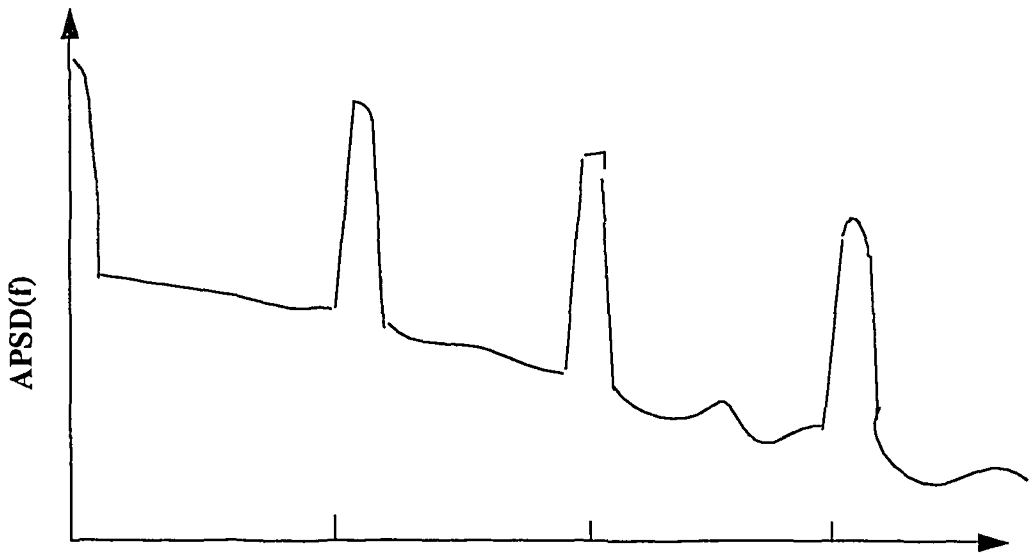
APSD(f)
f

Fig. 6

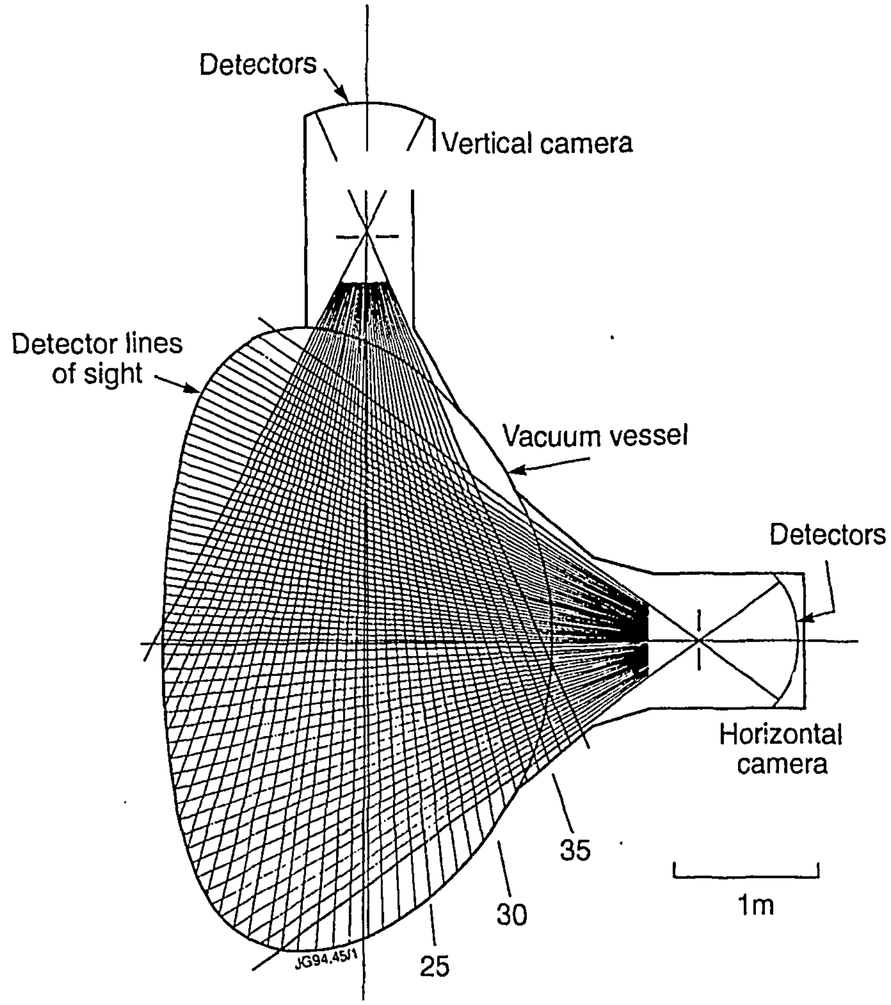
Detectors
Vertical camera
Detector lines of sight
Vacuum vessel
Detectors
Horizontal camera
35
30
25
1m
JG94.45/1

Fig. 7