CTH-ED-191

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TECHNICAL REPORT No. 191

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Technical Report No. 191

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Submitted to the School of Electrical Engineering,
Chalmers University of Technology, in partial fulfilment
of the requirements for the degree of
Doctor of Philosophy

Göteborg, June 1989

ISBN 91-7032-435-2

CTH, Göteborg 1989

ABSTRACT

In present-day tokamak fusion machines, instabilities and turbulence driven by temperature gradients can have a considerable impact on the confinement qualities. This thesis is mainly devoted to analyzing the nonlinear evolution of these instabilities and the associated turbulent transport.

A combined analytical and numerical study of the ion temperature gradient driven turbulence is presented. An analytical expression for the ion thermal conductivity is derived and found to be in good agreement with simulation results. The scaling properties of χ_i are investigated and compared with experimental results.

The transport due to the simultaneous presence of a trapped electron mode and an ion temperature gradient mode is analysed. It is found that the coupling of the modes can give rise to inward diffusive fluxes of both particles and energy. The tendency of the system to equilibrate density and temperature scale lengths is compared with recent experimental trends.

The nonlinear behaviour of the instabilities is also studied in the context of low dimensional dynamical systems. Here, the relation between the fully nonlinear fluid models and the low dimensional models is discussed. The influence of a high frequency RF-field on the ion temperature gradient driven mode is investigated analytically. The consequences for mode stability and transport are considered.

Descriptors : Drift wave turbulence, anomalous transport, low dimensional systems.

PAPERS

This paper serves as an introduction to and a summary of a doctoral thesis comprising the following papers:

I Transport due to toroidal η_i-mode turbulence in tokamaks. H. Nordman and J. Weiland, Nucl. Fusion 29, 251 (1989).

II Effects of parallel compressibility on ion temperature gradient driven modes. H. Nordman, R. Singh and J. Weiland, CTH-IEFT/PP-1989-06 (Submitted for publication to Plasma Phys. Contr. Fusion).

III Transport due to η_i-modes in the presence of RF fields. R. Singh and H. Nordman, CTH-IEFT/PP-1989-04 (Submitted for publication to Nucl. Fusion).

IV Transport due to collisionless toroidal temperature gradient driven modes with trapped electron effects. J. Weiland, A. Jarmén and H. Nordman, CTH-IEFT/PP-1988-22 (Submitted for publication to Nucl. Fusion).

V Simulation of toroidal drift wave turbulence driven by temperature gradients and electron trapping. H. Nordman and J. Weiland, CTH-IEFT/PP-1989-08 (Submitted for publication to Nucl. Fusion).

VI Low dimensional model of ion temperature gradient driven turbulence. H. Nordman, CTH-IEFT/PP-1989-07 (Submitted for publication to Phys. Rev. A).

VII Generalized complex Lorenz system for plasmas and the transition to turbulence. H. Nordman and J. Weiland, Phys. Rev. A 37, 4044 (1988)

1. INTRODUCTION

The presence of anomalous, i.e. nonclassical losses of particles and energy are considered to present a major obstacle to tokamak fusion. It is generally assumed that these anomalies are the result of the observed microscopic low frequency turbulence in the plasma. Small scale fluctuations in the electric and magnetic fields induce fluctuations in the velocities and positions of the particles, thus leading to transport.

The cause of the microturbulence is attributed to low frequency drift instabilities driven by density or temperature gradients. Among the various types of drift instabilities that could be excited in a fusion plasma, the temperature gradient driven drift wave is considered to be the most likely cause of the ion energy transport. This is a robust, reactive instability that is expected to exist in a large part of the operational regimes of present-day tokamaks.

In this thesis, the toroidal branch of this instability, driven by a combination of an ion temperature gradient and magnetic field curvature, is considered. The studies here deal mainly with the nonlinear evolution of the instability and the associated turbulent ion energy transport. The effect of electron trapping is considered, both in the linear and nonlinear regimes. In this context, the relation between particle and energy transport is investigated.

The local transport properties obtained here, relevant for the so-called good confinement region in the fusion plasma, do not yield sufficient information to predict global confinement characteristics. To this end, these studies must be complemented with global transport simulations, taking the relevant boundary effects into consideration.

2. APPROACHES TO TURBULENCE

The object of the present section is to present some of the analytical and numerical methods used in this thesis. The renormalized strong plasma turbulence theories originally developed for ordinary fluids are not treated here. It will, however, be of interest to compare the derived diffusion-coefficients with the classical strong turbulence result $D \sim \gamma/k^2$ obtained by Dupree [1] by renormalization techniques.

2.1 QUASILINEAR THEORY

Contrary to the situation in fluid turbulence, plasma microturbulence is usually driven by instabilities with finite real eigenfrequency. In particular, this holds for the ion temperature gradient driven mode (η_i-mode), where $\omega_r > \gamma$ for the main part of a typical tokamak cross section. In this region, the standard weak turbulence theory may be appropriate [2]. Quasilinear theory represents a truncation of the weak turbulence expansion at first order. Hence, the theory neglects mode coupling in the evolution equations, leading to a linear relation between the fields. However, correlations between the first order field quantities are kept in the evaluation of the fluctuation-induced fluxes. To obtain a closed analytical expression for the diffusion coefficients, the nonlinearly saturated level of the fluctuation amplitude must be obtained. This is outside the scope of quasilinear theory. The most widely used estimate of the fluctuation level is the "mixing length" level $\delta n/n \sim 1/k_x L_n$. This is expected to provide a relevant model for strong turbulence situations where the saturation level has reached its maximum. In the case of the η_i-mode turbulence treated in paper I, a saturation level proportional to linear growth was found to be in better agreement with numerical simulations. This model is based on a balance between linear growth and nonlinear cascading. Hence the influence of mode coupling enters the quasilinear expressions indirectly.

Formally, the diffusion coefficients thus obtained turn into the strong turbulence result D ~ γ/k^2 in the limit $\gamma >> \omega_r$.

2.2 NUMERICAL SIMULATIONS

To solve the nonlinear evolution equations numerically represents the most direct approach to turbulence. No assumptions about fluctuating amplitudes or frequency spectrum are needed and all higher order nonlinearities are automatically incorporated. During the last years, an increasing number of numerical investigations have been made in order to understand the drift wave plasma turbulence [3-5].

In this thesis the nonlinear fluid equations are solved in a two dimensional square box with periodic boundary conditions, i.e. only homogeneous turbulence is treated. These assumptions are realistic for micro turbulence, since the nonlinear interactions in a strongly magnetized plasma are two dimensional in character. Moreover, both the background inhomogeneities and the linear drive of the turbulence vary on a larger length scale than the characteristic wavelengths of the drift waves.

A spectral code is used, allowing the computation time to scale with the number of modes as $N^2 \log N^2$. A review of the spectral methods at hand in treating nonlinear partial differential equations is given in Ref. [6].

In the low viscosity bulk plasmas encountered here, inclusion of additional, artificial damping rates can be used, thereby reducing the requirements on the size of the wave number grids and thus the computation time needed. This is a procedure often used for both ordinary fluids and plasmas.It is discussed in connection with η_i-mode turbulence in paper I and references therein.

It is found that simulation based on a 64 x 64 grid, including full dealiasing, provides a sufficient resolution; it is used in papers I, IV and V.

2.3 LOW DIMENSIONAL MODELS

Since the discovery by Lorenz [7] that a dissipative system with only three degrees of freedom can behave chaotically, low order models of turbulence have received considerable attention.

In particular, the theory of low dimensional systems has provided scenarios for the transition to turbulence that have gained experimental support both in fluids and in plasmas. This implies that, of the infinite number of degrees of freedom in a physical system, only very few will be active due to the phase space contraction associated with dissipation.

The quadratic form of the nonlinear terms in the fluid equations has made the three wave interaction model fundamental [8].

In Ref. [9] this model was considered for electron drift waves. Terry and Horton [10] studied the relation between a randomly choosen triplet interaction and a 20-mode system. They found that the qualitative features of the systems were the same.
For the temperature interchange mode, a complex version of the Lorenz system, consisting of five degrees of freedom, was found to constitute the lowest order nonlinear set [11].

In paper VII this model is generalized to include 26 modes. The properties of the complex Lorenz system are found to be rather robust with respect to the number of degrees of freedom. Paper VI treats a low order system describing toroidal η_i-modes. Because of the similarities between the temperature interchange mode and the η_i-mode, also here a complex Lorenz system is found to be relevant. In this work, the transport properties of the low dimensional system are compared with the fully nonlinear numerical solutions of the partial differential equations.

3. ION TEMPERATURE GRADIENT DRIVEN MODE

The interest in the η_i-mode ($\eta_i = d\ln T_i/d\ln n$) as a candidate for explaining the ion energy transport in tokamaks has increased during the last years. The reason for this is mainly that the experimental support favouring η_i-modes has increased recently. Also, the rather universal behaviour of tokamaks with respect to transport properties makes an instability relying on a few basic physical features attractive.

A comprehensive survey of the measurements of turbulence and anomalous transport in tokamaks is found in Ref. [12], where also theory is reviewed.

3.1 LINEAR THEORY

The η_i-mode is a drift type wave driven by an ion temperature gradient. It obeys the usual drift wave ordering $v_{thi} << \omega/k_{\parallel} << v_{the}$. Thus the electrons are allowed to reach a Boltzmann distribution. It was first studied in the slab description [13] where it occurs as a modified ion acoustic wave. The toroidally induced mode studied here is of Rayleigh-Taylor nature localized to regions of unfavourable magnetic curvature. This branch was first studied by Horton et al [14] in a fluid description. The correct value of the η_i threshold for onset of instability was first obtained by Guzdar et al [15] using a kinetic treatment.

Recently, a fluid description including all curvature effects in the energy equation [16] was found to reproduce the basic kinetic results. Thus the threshold value of η_i was found to be within 20% of the kinetic results.

In paper II this work is generalized to include parallel ion motion in the electrostatic limit. Since the η_i-mode has a much longer wavelength than ordinary drift waves ($k^2\rho^2 << 1$ for the fastest growing modes), the assumption $v_{thi} < \frac{\omega}{k_\parallel}$ may become critical, thus making parallel ion motion important for low k modes.

An eigenvalue equation is derived using the ballooning mode formalism derived by Connor et al [17]. Analytical solutions were found in the strong ballooning limit where the parallel compressibility entered as a slightly stabilizing effect.

For small wave numbers, the mode frequency may become comparable to the bounce frequency of the ions. This implies that the effect of ion trapping has to be considered [18]. The toroidal branch of the η_i-mode studied here usually has a larger growth rate than the slab branch. However, as pointed out by Biglary et al, [18] the two branches complement each other making the η_i-mode a robust instability in tokamaks.

3.2 NONLINEAR EVOLUTION AND ANOMALOUS TRANSPORT

By introducing nonlinearities in the evolution equations for density, temperature etc, the process of mode coupling may lead to a transfer of energy between linearly driven modes and damped modes, leading to a nonlinearly saturated state. In the turbulent state, the fluctuation induced fluxes may be calculated by numerical simulations or can be approximated by different analytical techniques. Horton et al [19] as well as Lee and Diamond [20] have used renormalized strong turbulence theory to study the slab branch. Using invariance properties of the governing equations, Connor [21] was able to derive scaling characteristics of the diffusion coefficients.

The first numerical study of nonlinear η_i-modes was reported by Horton et al [3]. They treated the slab branch with a 3D numerical simulation including the effect of quasilinear profile flattening. Waltz [4] studied the toroidal mode in a 2D simulation including, among other things, electromagnetic effects and parallel ion motion but neglecting the stabilizing compressibility terms in the energy equation. This work was recently extended to three dimensions [5]. Nonlinear gyrokinetic simulations of the slab branch have been reported by Lee and Tang [19].

In paper I the favourable fluid model of Ref. [16] is extended to the non-linear domain. Both convective E x B and FLR nonlinearities associated with the ion polarization drift are included. The work concentrates on the scaling properties of χ_i and comparisons between analytical (quasilinear) and numerical results. An analytical expression for χ_i is derived and found to be in good agreement with the simulation results.

New scaling properties are presented which compares favourably with experimental trends. This includes the scaling with L_n/L_B which indicates a better agreement with the radial variation and the scaling with T_e/T_i favouring tokamak operation in the hot ion regime.

The η_i-modes may also interact nonlinearly with other modes in the plasma. In particular, the presence of high-frequency radio waves can have considerable effects on low frequency turbulence. This interaction is important in connection with auxiliary heating of tokamaks, where injection of RF-power represents a primary heating scheme. The presence of an RF-wave in the ion cyclotron range of frequencies (ICRF) was in Ref. [20] shown to stabilize the interchange mode in a mirror machine. Parametric interaction of an RF-wave with interchange modes has been studied theoretically by McBride et al [21] and others. Considering the close resemblance of the η_i and interchange mode branches, similar effects are expected to occur here.

In paper III the interaction of η_i-modes with a fast magnetosonic ICRF wave and electrostatic sidebands is considered. The influence of the RF-field on mode stability is discussed as well as the corresponding change in transport characteristics.

4. TRAPPED ELECTRON MODE

The trapping of charged particles in magnetic mirror configurations is known as a destabilizing mechanism for drift waves. The reason for this is that the trapped particles are unable to cancel charge separation by moving along a field line. Since the fraction of trapped electrons is approximately 50% in present-day tokamaks, these effects deserve serious consideration.

The early works on trapped electron modes are summarized in a review article by Kadomtsev and Pogutse, [22]. The curvature driven trapped electron mode (ubiquitous mode) was analyzed by Coppi and Rewoldt, [23]. This mode can be seen as a modified toroidal η_i-mode including the effects of electron trapping.

In paper IV, the trapped electrons are treated as a separate fluid characterized by a density and temperature.

Using the fluid model of Ref. [16] for the ions and the trapped electron fluid, a new collisionless electron mode is found with a modified η_i-mode.

The trapped electron mode has a dispersion relation similar to the η_e-mode but with a larger wavelength. The inclusion of electron trapping also leads to electron energy and particle transport. In particular, both diffusive particle effects and heat pinch effects tending to equilibrate the density and temperature scale lengths are found. These properties are also observed in tokamaks, e.g. in connection with pellet fuelling where the associated peaking of the density profile has been followed by a peaking of the temperature profile. Paper V treats the same model in more detail and contains a comparison between quasilinear and numerical results. The scaling characteristics of the transport coefficients are transformed into radial scalings which are found to compare well with experimental results.

ACKNOWLEDGEMENTS

I am grateful to Professor Jan Weiland for his generous help and advice. I am also grateful to Professor Hans Wilhelmsson, the head of the Institute for Electromagnetic Field Theory and Plasma Physics, for his support and encouragement.

Many thanks to Dr. Hans-Georg Gustavsson for help with the computer facilities, and to Bente Larsson for typing the manuscript.

To all the other members of the Institute I am grateful for many enlightening discussions.

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SUMMARY OF PAPERS

PAPER I: Transport due to toroidal η_i-mode turbulence in tokamaks.

The nonlinear evolution of the ion temperature gradient driven mode is investigated using numerical simulations. Steady state values of the ion thermal conductivity and of the fluctuation amplitude levels are computed. An analytical expression for χ_i is derived and found to be in good agreement with the simulation results. The diffusion coefficient is also found to give an improved agreement with the experimentally obtained χ_i , e.g. concerning the radial scaling properties.

PAPER II: Effects of parallel compressibility on ion temperature gradient driven modes.

The electrostatic η_i-mode studied in paper I is investigated in the linear limit including the effect of parallel ion motion. An eigenvalue equation is derived and solved in the strong ballooning limit. It is found that parallel ion compressibility is weakly stabilizing.

PAPER III: Transport due to η_i-modes in the presence of RF fields.

The coupling of high frequency RF-waves in the ion cyclotron range of frequencies (ICRF) with the η_i-mode is studied. A fast magnetosonic pump wave and electrostatic sidebands are considered. A second order dispersion relation is obtained describing the influence of the RF-wave on mode stability. The condition for destabilization is derived. A quasilinear expression for the ion thermal conductivity in the presence of RF is derived and compared with the results of paper I.

PAPER IV: Transport due to collisionless toroidal temperature gradient driven modes with trapped electron effects.

The destabilizing effects of electron trapping in combination with temperature gradients are investigated using a fluid theory. A new trapped electron mode is found together with a modified η_i-mode. A quasilinear treatment shows the presence of particle and heat pinch effects with a tendency to equilibrate the scale length of density and temperature. A nonlinear numerical simulation confirms the existence of pinch effects.

PAPER V: Simulation of toroidal drift wave turbulence driven by temperature gradients and electron trapping.

A more systematic treatment of the system in paper IV is presented. The nonlinear evolution of the instability is followed by numerical simulations. Scaling properties of the diffusion-coefficients are obtained and compared with analytical results. The agreement is found to be good for $\gamma < \omega_r$. However, the presence of electron trapping is destabilizing, thus limiting the region of applicability of the quasilinear theory.

PAPER VI: Low dimensional model of ion temperature gradient driven turbulence.

A complex version of the Lorenz system (CLS) with five degrees of freedom is found to constitute the lowest order system for nonlinear η_i-modes. It is found that the CLS and the associated ion energy transport show several features in qualitative agreement with the fully nonlinear results of paper I.

PAPER VII: Generalized complex Lorenz system for plasmas and the transition to turbulence.

In this paper, temperature interchange modes in edge plasmas are modelled by low dimensional dynamical systems. In particular, the relation between the lowest order set, a CLS, and a 26-dimensional generalisation is investigated. It is found that many features of the CLS, e.g. the limitcycle frequency and the bifurcation sequence, remain unchanged in the larger system.