RECEIVED
JAN 05 1995
OSTI

Institute for Fundamental Theory Preprint UFIFT-HEP-95-27

UNIVERSITY OF FLORIDA
Gainesville, Florida 32611

Anomalous Abelian Symmetry in the Standard Model

PIERRE RAMOND

Institute for Fundamental Theory
Department of Physics, University of Florida
Gainesville FL 32611

Based on an invited talk at SUSY-95
Ecole Polytechnique, Palaiseau, France

ABSTRACT: The observed hierarchy of quark and lepton masses can be parametrized by nonrenormalizable operators with dimensions determined by an anomalous Abelian family symmetry, a gauge extension to the minimal supersymmetric standard model. Such an Abelian symmetry is generic to compactified superstring theories, with its anomalies compensated by the Green-Schwarz mechanism. If we assume these two symmetries to be the same, we find the electroweak mixing angle to be $\sin^2\theta_w = 3/8$ at the string scale, just by setting the ratio of the product of down quark to charged lepton masses equal to one at the string scale. This assumes no GUT structure. The generality of the result suggests a superstring origin for the standard model. We generalize our analysis to massive neutrinos, and mixings in the lepton sector.

1. Introduction

There is little doubt that the standard model of strong and electroweak interactions is an effective field theory, the chiral remnant of a more fundamental theory at very short distances. The nature of this theory and the value of the energy scale at which it is operative is one of the fundamental questions in particle physics.

Since experimental scales are very much removed from the cut-off, it may be difficult to infer its structure from the data. However there are two phenomena which are not scale-dependent, the cosmological constant and anomalies. Clearly the absence of cosmological constant is the major puzzle, the hint left behind to tell us when we have understood the theory that underlies our universe. Anomalies are far more tractable, but remarkably all anomalies in the standard model automatically cancel, even the mixed hypercharge-gravitational anomaly. Hence if anomalies are to be useful, there must be another gauged symmetry in the standard model, whose scale of breaking is much higher than the electroweak scale. Does such a symmetry manifest itself in the present data? Below we argue that it is indeed present, and is needed to reproduce the mass ratios quarks and leptons. Further we show how to relate its anomaly coefficients to observables, and that these coefficients cannot vanish.

The presence of such an anomalous symmetry in the standard model requires its underlying theory to naturally provide this symmetry, as well as a mechanism that compensates its anomalies. It is most satisfying that superstring theories contain an anomalous $U(1)$ which comes equipped with its own anomaly cancellation, the Green-Schwarz[1] mechanism. This cancellation implies[2] a relation between the anomaly coefficients and the Weinberg angle. Remarkably we find that in the MSSM, the relation leads exactly to $\sin^2\theta = 3/8$ at the cut-off, in remarkable agreement with (extrapolated) data[3]. Furthermore this symmetry is automatically broken a few orders of magnitude below the cut-off[4]. This provides the small expansion parameter needed to understand the hierarchy of quark and lepton masses. Below we do not address more specific aspects of compactified superstring theories[5], but rather concentrate on its more generic features.

We proceed by showing how this Abelian symmetry can be used to understand the hierarchy of quark and lepton mases and mixing angles, in the language of an effective low energy theory. We consider only the standard model extended to $N = 1$ supersymmetry[6], which allows for its perturbative extrapolation to near Planckian scales, where the gauge couplings [7]and some Yukawa couplings[8] appear to converge, suggesting that the $N = 1$ standard model at short distances is much simpler than at experimental scales.

However, many other parameters are different at short distances, suggesting in fact a strong hierarchy among the masses of the quarks and the leptons, parametrized by the orders of magnitude estimates[9]

$$\frac{m_u}{m_t} = \mathcal{O}(\lambda^8) \; ; \qquad \frac{m_c}{m_t} = \mathcal{O}(\lambda^4) \; ; \tag{1.1}$$

$$\frac{m_d}{m_b} = \mathcal{O}(\lambda^4) \; ; \qquad \frac{m_s}{m_b} = \mathcal{O}(\lambda^2) \; , \tag{1.2}$$

where, following Wolfenstein[10], we use the Cabibbo angle λ, as expansion parameter. The charged lepton masses also satisfy similar relations

$$\frac{m_e}{m_\tau} = \mathcal{O}(\lambda^4) \; ; \qquad \frac{m_\mu}{m_\tau} = \mathcal{O}(\lambda^2) \; . \tag{1.3}$$

The mass hierarchy appears to be geometrical in each sector. The equality

$$m_b = m_\tau \; ,$$

known to be valid in the ultraviolet[8], yields the estimate

$$\frac{m_d m_s m_b}{m_e m_\mu m_\tau} = \mathcal{O}(1) \; . \tag{1.4}$$

Our basic assumption is that these hierarchies stem from family symmetries. The simplest is an Abelian symmetry, which sets the dimension of non-renormalizable Yukawa interactions, as originally suggested by Froggatt and Nielsen[11]. Some aspects of this old idea have been revisited in the recent literature [12,13,14]. The work presented below is the result of a collaboration with P. Binétruy and S. Lavignac[3,15]. There are also closely related work in the recent literature, but with different emphases[16,17,18,19].

2. Quark Masses and Mixing Angles

The most general Abelian charge assignments to the particles of the supersymmetric standard model can be written as

$$X = X_0 + X_3 + \sqrt{3}X_8 \ , \tag{2.1}$$

where X_0 is the family-independent part, X_3 is along λ_3, and X_8 is along λ_8, the two diagonal Gell-Mann matrices of the $SU(3)$ family space in each charge sector. In a basis where the entries correspond to the components in the family space of the fields $\mathbf{Q}$, $\overline{\mathbf{u}}$, $\overline{\mathbf{d}}$, L, and $\overline{e}$, we can write the different components in the form

$$X_i = (a_i, b_i, c_i, d_i, e_i) \ , \tag{2.2}$$

for $i = 0, 3, 8$. Let us note that we can always take the X-charges of the Higgs doublets to be the same by mixing an appropriate amount of hypercharge in our definition of X. This results in some redefinition of anomaly coefficients, to be discussed later.

The value of the $X-$charge of the two Higgs doublets depends on the origin of the μ term. In the minimal model, with a μ term in the superpotential, both Higgs doublets $H_{u,d}$ have zero X-charge. However, this is not a very pleasing alternative as the numerical value of the μ parameter is of the order of electroweak breaking scale. Thus many have sought to explain this suppression in terms of the structure of the supergravity Lagrangian. Two methods have emerged. In the first[20], the Kähler potential contains a term of the form $H_u H_d$, which then generates a μ parameter of the order of the gravitino mass. In another[21], the same term appears in the superpotential. In view of these alternatives, we may leave the doublets with a non-zero X-charge, s.

In the following, we assume a tree-level Yukawa coupling for *only* the third family,

$$y_t \mathbf{Q}_3 \overline{\mathbf{u}}_3 H_u + y_b \mathbf{Q}_3 \overline{\mathbf{d}}_3 H_d + y_\tau L_3 \overline{e}_3 H_d \ , \tag{2.3}$$

where the y_i's are the Yukawa couplings. This generates the relations

$$a_0 + b_0 = 2(a_8 + b_8) - s \ , \qquad a_0 + c_0 = 2(a_8 + c_8) - s \ , \qquad d_0 + e_0 = 2(d_8 + e_8) - s \ .$$

The other elements of the Yukawa matrices are zero at tree-level because of X-charge conservation.

We note again that this scheme may be too conservative as it does not address the very small bottom to top mass ratio. It could also be that only the top quark mass is tree-level[16].

Let x_{ij} be the excess X-charges at each of their entries; for the charge 2/3 Yukawa matrix they are

$$\begin{pmatrix} 3(a_8+b_8)+a_3+b_3 & 3(a_8+b_8)+a_3-b_3 & 3a_8+a_3 \\ 3(a_8+b_8)-a_3+b_3 & 3(a_8+b_8)-a_3-b_3 & 3a_8-a_3 \\ 3b_8+b_3 & 3b_8-b_3 & 0 \end{pmatrix} . \tag{2.4}$$

In the charge $-1/3$ sector, the b_i are replaced by the c_i, and in the charge -1 sector, a_i, b_i are replaced by d_i, e_i, respectively.

Introduce an electroweak singlet field θ with X-charge $-x$, to compensate the excess charge at each entry, x_{ij}, yielding an interaction of higher dimensions with no hypercharge [11,22]

$$\mathbf{Q}_i \overline{\mathbf{u}}_j H_u \left(\frac{\theta}{M_u}\right)^{n_{ij}} , \tag{2.5}$$

where the n_{ij} are positive numbers which satisfy

$$x_{ij} - x n_{ij} = 0 , \tag{2.6}$$

and M_u is some large scale. In a perturbative framework, the n_{ij} are expected to be integers. In the effective low energy theory, these interactions are the manifestation of unknown interactions which break the large chiral symmetries implied by the tree-level superpotential. The exact nature of these interactions is not the focus of this paper, but they may be related to a generalized see-saw mechanism[23].

Assume that the electroweak singlet θ develops a vacuum expectation value smaller than M_u, producing a *small parameter*, $\lambda_u \sim \theta/M_u$, as it often happens in many compactified superstring theories[4,24]. The n_{ij} then determine the order of magnitude of the entries in the Yukawa matrices[11]. Since the down quark and lepton sectors share the same electroweak quantum numbers, we expect the equivalent small parameters to be the same for the charge -1 and -1/3 matrices, that is $\lambda_d \sim \lambda_e$. We emphasize that most of the the conclusions reached with this simple assumption depend only on the *existence* of these small parameters, *not on their values.*

If we assume that the X-charge of the combination $H_u H_d$ is of the same sign as that of θ, we cannot have a μ term directly in the superpotential. However, we can have in the Kähler potential a term of the form $K = \cdots + \overline{\theta}^n H_u H_d$. This strongly favors the proposal of Giudice and Masiero[20]. Hence, in our picture, the naturalness of the μ term is just the relative sign of the X-charges.

Assume that all the excess charges in each Yukawa matrix are positive (The general case where some of the excess charges are negative, creates a true zero in that matrix element[12]). The charge 2/3 Yukawa matrix is

$$\mathbf{Y}_{uij} = \mathcal{O}(\lambda_u^{n_{ij}}) \ , \tag{2.7}$$

normalized to the top quark mass. It is not hard to diagonalize this matrix, setting

$$\mathbf{Y}_u = U_u D_u V_u^\dagger \ , \tag{2.8}$$

where

$$D_u = \text{diag}\ (\mathcal{O}(\lambda_u^{3(a_8+b_8)+a_3+b_3}), \mathcal{O}(\lambda_u^{3(a_8+b_8)-a_3-b_3}), \mathcal{O}(1)) \ ,$$

and the unitary matrix U_u is given by

$$U_u = \begin{pmatrix} \mathcal{O}(1) & \mathcal{O}(\lambda_u^{2a_3}) & \mathcal{O}(\lambda_u^{3a_8+a_3}) \\ \mathcal{O}(\lambda_u^{2a_3}) & \mathcal{O}(1) & \mathcal{O}(\lambda_u^{3a_8-a_3}) \\ \mathcal{O}(\lambda_u^{3a_8+a_3}) & \mathcal{O}(\lambda_u^{3a_8-a_3}) & \mathcal{O}(1) \end{pmatrix} \ , \tag{2.9}$$

These are valid for a range of parameters such that

$$3a_8 + 3b_8 > a_3 + b_3 > 0 \ .$$

We have a similar relation in the down quark sector, with the b_i replaced by c_i. It follows that the orders of magnitude of U_u and U_d are the same, but the expansion coefficients might be different. Let us set $\lambda_u = \lambda_d^y$, with $y > 0$. If $y > 1$, the orders of magnitude of the entries of the CKM matrix are

$$\mathcal{U}_{CKM} = \begin{pmatrix} \mathcal{O}(1) & \mathcal{O}(\lambda_d^{2a_3}) & \mathcal{O}(\lambda_d^{3a_8+a_3}) \\ \mathcal{O}(\lambda_d^{2a_3}) & \mathcal{O}(1) & \mathcal{O}(\lambda_d^{3a_8-a_3}) \\ \mathcal{O}(\lambda_d^{3a_8+a_3}) & \mathcal{O}(\lambda_d^{3a_8-a_3}) & \mathcal{O}(1) \end{pmatrix} . \tag{2.10}$$

If $y < 1$, the expansion parameter in the above is replaced by λ_u, that is its exponents all are multiplied by y. In either case the exponents satisfy the sum rule

$$n_{12} = n_{13} - n_{23} , \tag{2.11}$$

which implies that

$$\frac{V_{us}\,V_{cb}}{V_{ub}} = \mathcal{O}(1) , \tag{2.12}$$

in agreement with data (the right hand side is ≈ 3, and the Wolfenstein parametrization ($n_{12} = 1,\ n_{13} = 3,\ n_{23} = 2$). We note the relation between our expansion parameters with the Cabibbo angle

$$\lambda \equiv V_{us} = \lambda_{u,d}^{\;2a_3} , \tag{2.13}$$

depending on the relative magnitudes of λ_u and λ_d. The eigenvalue order of magnitude estimates are

$$\frac{m_u}{m_t} = \mathcal{O}(\lambda_u^{3(a_8+b_8)+a_3+b_3}) ; \qquad \frac{m_c}{m_t} = \mathcal{O}(\lambda_u^{3(a_8+b_8)-a_3-b_3}) ;$$
$$\frac{m_d}{m_b} = \mathcal{O}(\lambda_d^{3(a_8+c_8)+a_3+c_3}) ; \qquad \frac{m_s}{m_b} = \mathcal{O}(\lambda_d^{3(a_8+c_8)-a_3-c_3}) ,$$

The geometric hierarchy of the mass ratios in each quark sector suggests the further equalities

$$a_8 + b_8 = a_3 + b_3 ; \qquad a_8 + c_8 = a_3 + c_3 . \tag{2.14}$$

Agreement with experimental information on the quark mass ratios dictates the following

$$2(a_8 + c_8) = y(a_8 + b_8) . \tag{2.15}$$

In addition the mixing angle relation

$$V_{us} = \sqrt{\frac{m_d}{m_s}} , \tag{2.16}$$

is satisfied provided that

$$c_3 = a_3 \ , \tag{2.17}$$

if $y > 1$, and

$$c_3 = (2y-1)a_3 \ , \tag{2.18}$$

if $0 < y < 1$.We also find that

$$V_{cb} = \mathcal{O}(V_{us}^{\frac{3a_8-a_3}{2a_3}}) \ , \tag{2.19}$$

from which we may deduce that

$$3a_8 = 5a_3 \ . \tag{2.20}$$

Comparison with the data gives us six equations among seven unknown. The last unknown is y. Until we know the origins of the scales and of the expansion parameters, we cannot fix the values of λ_d and of λ_u in terms of observables. We note, for example, the interesting case $y = 2$, corresponding to $\lambda_u = \lambda_d^2$, yields $b_i = c_i$, which suggests an $SU(2)_R$ symmetry. It is quite remarkable that this simple idea is in agreement with the present data, and even predicts one successful relation among the CKM matrix elements (2.12).

For future references we note that this scheme yields the following orders of magnitude estimates

$$\begin{aligned} m_u m_c m_t &\sim v_u^3 \lambda^{3(a_0+b_0+s)} \ , \\ m_d m_s m_b &\sim v_d^3 \lambda^{3(a_0+c_0+s)} \ , \\ m_e m_\mu m_\tau &\sim v_d^3 \lambda^{3(d_0+e_0+s)} , \end{aligned} \tag{2.21}$$

These will be used to relate the product of masses to the values of the anomaly coefficients.

3. Lepton Masses and Mixing Angles

An analysis akin to that in the previous section yields the charged lepton mass estimates

$$\frac{m_e}{m_\tau} = \mathcal{O}(\lambda_e^{3(d_8+e_8)+d_3+e_3}) \ ; \qquad \frac{m_\mu}{m_\tau} = \mathcal{O}(\lambda_e^{3(d_8+e_8)-d_3-e_3}) \ . \tag{3.1}$$

Geometric hierarchy of the charged lepton mass ratios implies that

$$d_8 + e_8 = d_3 + e_3 \ , \tag{3.2}$$

There are no mixing angles if the neutrinos are massless. Below, we generalize the Froggatt-Nielsen analysis to massive neutrinos, without assuming any extra symmetry[25]. We do this by adding right-handed neutrinos to the MSSM in order to generate masses for the neutrinos via the "see-saw" mechanism[26].

Let us assume that the low energy chiral remnants of the primal soup come from **27** representations of E_6. This representation carries two fields with no electroweak quantum numbers. One is an $SO(10)$ singlet, as we can see from the decomposition $\mathbf{27} = \mathbf{16} \oplus \mathbf{10} \oplus \mathbf{1}$. The other is an $SU(5)$ singlet which lives in the spinor representation of $SO(10)$ $\mathbf{16} = \overline{\mathbf{5}} \oplus \mathbf{10} \oplus \mathbf{1}$. This same field is part of an isodoublet under the right-handed $SU(2)_R$ inside $SO(10)$ $\mathbf{16} = (\mathbf{2}, \mathbf{1}, \overline{\mathbf{3}}^c \oplus \mathbf{1}^c) \oplus (\mathbf{1}, \mathbf{2}, \mathbf{3}^c \oplus \mathbf{1}^c)$. These two neutrino fields are not so "ino" as they are assumed to be very massive. With two fields, the Majorana mass matrix is

$$\begin{pmatrix} 0 & m_1 \; m_2 \\ \begin{smallmatrix} m_1 \\ m_2 \end{smallmatrix} & \mathcal{M}_0 \end{pmatrix} ,$$

where $\mathcal{M}_0$ is a 2×2 symmetric matrix, and $m_{1,2}$ are the usual $\Delta I_w = 1/2$ mass entries of electroweak order. Let the eigenvalues of $\mathcal{M}_0$ be M_1 and M_2. We can go to a basis where M_0 is diagonal, in which $m_{1,2}$ are rotated into $\hat{m}_{1,2}$, yielding the light eigenvalue

$$m_\nu = \frac{\hat{m}_1^2 M_2 - \hat{m}_2^2 M_1}{M_1 M_2} .$$

Thus if $M_1 < M_2$, it becomes just $\hat{m}_1^2/M_1$, so that it is the lighter of the singlet neutrinos that enters in the light neutrino mass. This assumes that the $\hat{m}_i$ are of the same order of magnitude, themselves much smaller than M_1.

Thus in the following we assume only one right-handed neutrino per family, and leave the more complicated analysis to others. Assume that we have three such fields, $\overline{N}_i$, each carrying X-charge. The superpotential now contains the new interaction terms

$$L_i \overline{N}_j H_u \left(\frac{\theta}{m_\nu}\right)^{p_{ij}} + m_0 \overline{N}_i \overline{N}_j \left(\frac{\theta}{m_0}\right)^{q_{ij}} , \tag{3.3}$$

multiplied by couplings of order one, and where m_0 is some mass of the order of the GUT scale or string scale. In analogy with the quark and charged lepton sectors, we assume that $p_{33} = 0$, so that there is only the tree-level coupling for the third family. Call the X-charges of the right-handed neutrinos f_0, f_3, f_8, so that at tree-level

$$d_0 + f_0 = 2(d_8 + f_8) \ . \tag{3.4}$$

All Yukawa couplings satisfy conservation of X, relating q_{ij} and p_{ij} to the X-charges of the fields. For three families, the 6×6 Majorana mass matrix is of the form

$$\begin{pmatrix} 0 & \mathcal{M} \\ \mathcal{M}^T & \mathcal{M}_0 \end{pmatrix} \ .$$

In the above $\mathcal{M}$ is the $\Delta I_w = 1/2$ mass matrix with entries not larger than the electroweak breaking scale, and $\mathcal{M}_0$ is the unrestricted $\Delta I_w = 0$ mass matrix. Assuming that the order of magnitude of the $\Delta I_w = 0$ masses is much larger than the electroweak scale, we obtain the generalized "see-saw" mechanism.

The calculation of the light neutrinos masses and mixing angles proceeds in two steps. Let U_0 be the matrix which diagonalizes $\mathcal{M}_0$, that is

$$\mathcal{M}_0 = U_0 D_0 U_0^T \ , \tag{3.5}$$

where D_0 is diagonal. Then in terms of D_ν, the 3×3 eigenvalue matrix for the light neutrinos, and U_ν be their mixing matrix, we have

$$\mathbf{Y}_e' \equiv U_\nu D_\nu U_\nu^T = -\mathcal{M}' \frac{1}{D_0} \mathcal{M}'^T \ . \tag{3.6}$$

In the "see-saw" limit, the matrices U_0 and U_ν are unitary, so that

$$\mathcal{M}' = \mathcal{M} U_0^* \ . \tag{3.7}$$

The orders of magnitude of the heavy neutrino mass matrix are

$$\mathcal{M}_0 = m_0 \mathcal{O} \begin{pmatrix} \lambda_0^{2(f_0+f_3+f_8)} & \lambda_0^{2(f_0+f_8)} & \lambda_0^{2f_0+f_3-f_8} \\ \lambda_0^{2(f_0+f_8)} & \lambda_0^{2(f_0-f_3+f_8)} & \lambda_0^{2f_0-f_3-f_8} \\ \lambda_0^{2f_0+f_3-f_8} & \lambda_0^{2f_0-f_3-f_8} & \lambda_0^{2(f_0-2f_8)} \end{pmatrix} \ .$$

Its diagonalization yields the three eigenvalues

$$M_1 = m_0 \mathcal{O}(\lambda_0^{2(f_0+f_3+f_8)}) \ < \ M_2 = m_0 \mathcal{O}(\lambda_0^{2(f_0-f_3+f_8)}) \ < \ M_3 = m_0 \mathcal{O}(\lambda_0^{2(f_0-2f_8)}) \ .$$

We have assumed for simplicity that the charges satisfy the inequalities

$$f_0 > 2f_8 , \qquad 3f_8 > f_3 > 0 , \tag{3.8}$$

corresponding to $M_1 < M_2 < M_3$. The diagonalizing matrix is

$$U_0 = m_0\mathcal{O}\begin{pmatrix} 1 & \lambda_0^{2f_3} & \lambda_0^{3f_8+f_3} \\ \lambda_0^{2f_3} & 1 & \lambda_0^{3f_8-f_3} \\ \lambda_0^{3f_8+f_3} & \lambda_0^{3f_8-f_3} & 1 \end{pmatrix} .$$

The electroweak breaking mass yields the matrix

$$\mathcal{M} = m\mathcal{O}\begin{pmatrix} \lambda_\nu^{3(d_8+f_8)+d_3+f_3} & \lambda_\nu^{3(d_8+f_8)+d_3-f_3} & \lambda_\nu^{3d_8+d_3} \\ \lambda_\nu^{3(d_8+f_8)-d_3+f_3} & \lambda_\nu^{3(d_8+f_8)-d_3-f_3} & \lambda_\nu^{3d_8-d_3} \\ \lambda_\nu^{3f_8+f_3} & \lambda_\nu^{3f_8-f_3} & 1 \end{pmatrix} , \tag{3.9}$$

where λ_ν is the expansion parameter, and m is a mass of electroweak breaking size. If we let $\lambda_0 = \lambda_\nu^z$, with $z > 0$. When $z \geq 1$, we find that

$$\mathbf{Y}_e' = \frac{m^2}{M_1}\mathcal{O}(\lambda_\nu^{6f_8+2f_3})\mathcal{O}\begin{pmatrix} \lambda_\nu^{6d_8+2d_3} & \lambda_\nu^{6d_8} & \lambda_\nu^{3d_8+d_3} \\ \lambda_\nu^{6d_8} & \lambda_\nu^{6d_8-2d_3} & \lambda_\nu^{3d_8-d_3} \\ \lambda_\nu^{3d_8+d_3} & \lambda_\nu^{3d_8-d_3} & 1 \end{pmatrix} , \tag{3.10}$$

is the matrix whose eigenvalues yield the light neutrino masses, and their mixing angles. It is diagonalized by the unitary matrix

$$U_\nu = \mathcal{O}\begin{pmatrix} 1 & \lambda_\nu^{2d_3} & \lambda_\nu^{3d_8+d_3} \\ \lambda_\nu^{2d_3} & 1 & \lambda_\nu^{3d_8-d_3} \\ \lambda_\nu^{3d_8+d_3} & \lambda_\nu^{3d_8-d_3} & 1 \end{pmatrix} . \tag{3.11}$$

The light neutrino masses are then

$$\begin{aligned} m_{\nu_1} &= \frac{m^2}{M_1}\mathcal{O}(\lambda_\nu^{2(3f_8+3d_8+f_3+d_3)}), \\ m_{\nu_2} &= \frac{m^2}{M_1}\mathcal{O}(\lambda_\nu^{2(3f_8+3d_8+f_3-d_3)}) , \\ m_{\nu_3} &= \frac{m^2}{M_1}\mathcal{O}(\lambda_\nu^{2(3f_8+f_3)}) . \end{aligned} \tag{3.12}$$

In order to obtain the mixing matrix which appears in the charged lepton current, we must fold this matrix with that which diagonalizes the charged lepton masses. If we let $\lambda_\nu = \lambda_e^w$, with $w > 1$, the result is

$$\mathcal{U}_\nu = \mathcal{O}\begin{pmatrix} 1 & \lambda_e^{2d_3} & \lambda_e^{3d_8+d_3} \\ \lambda_e^{2d_3} & 1 & \lambda_e^{3d_8-d_3} \\ \lambda_e^{3d_8+d_3} & \lambda_e^{3d_8-d_3} & 1 \end{pmatrix} .$$

When $0 < w < 1$, the matrix has the same form with λ_e replaced by λ_ν. It is similar to the CKM matrix. The mixing in the charged lepton current was first proposed by Maki, Nakagawa and Sakata in 1962, so we call it the MNS matrix[27]. We note that its elements satisfy

$$V_{e\nu_\mu} V_{\mu\nu_\tau} \sim V_{e\nu_\tau} . \tag{3.13}$$

It may be that $\lambda_e = \lambda_d$ and $\lambda_u = \lambda_\nu$, since they have the same quantum numbers, implying $w = y$. We also have the relations

$$\frac{m_{\nu_1}}{m_{\nu_2}} \approx (V_{e\nu_\mu})^w ; \qquad \frac{m_{\nu_2}}{m_{\nu_3}} \approx (V_{\mu\nu_\tau})^w , \tag{3.14}$$

valid only when $w > 1$. When $0 < w < 1$ the exponents in these relations is one. In this analysis, the lepton mixing matrix has the same structure as the CKM matrix. We have assumed a simple set of inequalities among the charges, to provide an example of our method. When $0 < z < 1$, the forms of the neutrino masses are the same except that $f_{3,8}$ appear multiplied by z. The mixing matrix is unchanged.

Unlike quark masses and mixing, we have little solid experimental information on the values of these parameters. The most compelling evidence for neutrino masses and mixings come from the MSW interpretation of the deficit observed in various solar neutrino fluxes. In this picture, the electron neutrino mixes with another neutrino (assumed here to be the muon neutrino) with a mixing angle θ_{12} such that

$$m^2_{\nu_1} - m^2_{\nu_2} \approx 7 \times 10^{-6}\ \mathrm{eV}^2 ; \qquad \sin^2 2\theta_{12} \approx 5 \times 10^{-3} . \tag{3.15}$$

The other piece of evidence comes from the deficit of muon neutrinos in the collision of cosmic rays with the atmosphere. If taken at face value, these suggest that the muon neutrinos oscillate into another species of neutrinos, say τ neutrinos, with a mixing angle θ_{23}, and masses such that

$$m^2_{\nu_2} - m^2_{\nu_3} \approx 2 \times 10^{-2}\ \mathrm{eV}^2 ; \qquad \sin^2 2\theta_{23} \geq .5 . \tag{3.16}$$

Fitting the parameters coming from the solar neutrino data is rather easy, suggesting that

$$V_{e\nu_\mu} \sim \lambda_e^{2d_3} \sim \lambda^2 \ ,$$

together with $m_{\nu_2} \approx 1$ meV. However it is not so easy to understand the atmospheric neutrino data. These imply

$$V_{\mu\nu_\tau} \sim \lambda_e^{3d_8-d_3} = \mathcal{O}(1) \ .$$

The relations (3.14) then suggest that w has to be large. For example the value $\theta_{23} \sim \frac{\pi}{9}$ for which $\sin^2 2\theta_{23} = .34$, yields $m_{\nu_2}/m_{\nu_3} \sim .01$, for $w = 4$. This gives $m_{\nu_3} \approx .1$ eV, which marginally reproduces the "data", and fixes the lightest neutrino mass to $m_{\nu_1} \approx 10^{-13}$ eV! The heaviest neutrino weighs one tenth of an eV, not enough to be of use for structure formation. Perhaps there are more light neutrals, coming from the extra neutral leptons in each E_6 or from $endT$ in string compactification.

Generically, though, it is difficult to understand mixing angles of order one, as suggested by the atmospheric neutrino data. The existence of only small mixing angles in the quark sectors suggests either that the interpretation of the atmospheric neutrino data is premature, or that there is fine tuning in the neutrino matrices[28].

4. Anomalies

The X family symmetry is in general anomalous. The three chiral families and the Higgs contributions to the mixed gauge anomalies are

$$C_3 = 3(2a_0 + b_0 + c_0) \ , \tag{4.1}$$

$$C_2 = 3(3a_0 + d_0) + 2s \ , \tag{4.2}$$

$$C_1 = a_0 + 8b_0 + 2c_0 + 3d_0 + 6e_0 + 2s \ . \tag{4.3}$$

The subscript denotes the gauge group of the standard model, *i.e.* $1 \sim U(1)$, $2 \sim SU(2)$, and $3 \sim SU(3)$. The X-charge also has a mixed gravitational anomaly, which is simply the trace of the X-charge,

$$C_g = 3(6a_0 + 3b_0 + 3c_0 + 2d_0 + e_0 + f_0) + 4s - x + C_g' \ , \tag{4.4}$$

where C_g' is the contribution from the particles that do not appear in the model we are discussing. We note that the mixed YXX anomaly is affected by mixing X with Y, and the last anomaly coefficient, that of the X-charge itself, C_X, is the sum of the cubes of the X-charge. Extra particles with chiral X-charge other than those in the minimal model, will contribute to both C_g' and C_X.

These anomaly coefficients can be directly related to combinations of quarks and lepton masses[3], because the X-charge of the determinant in each charge sector is *independent* of the texture coefficients that distinguish between the two lightest families.

Since the down and lepton matrices have the same quantum numbers, and couple to the same Higgs, we may assume they have the same expansion parameter, $\lambda_d = \lambda_e$. In that case we can relate the products of the down quark masses to that of the leptons (assuming $y_b = y_\tau$)

$$\frac{m_d m_s m_b}{m_e m_\mu m_\tau} \sim \mathcal{O}(\lambda_d^{3(a_0+c_0-d_0-e_0)}) \; . \tag{4.5}$$

From the tree-level Yukawa couplings to the third family expressed through (2.4), we can write combinations of anomaly coefficients in terms of the family-dependent charges

$$C_1 + C_2 - \frac{8}{3}C_3 = 6(a_0 + c_0 - d_0 - e_0) + 4s \; . \tag{4.6}$$

In the minimal model with a primordial μ term, $s = 0$, and the above becomes a relation between the anomaly coefficients and the ratio of products of quark and lepton masses (4.5),

$$\frac{m_d m_s m_b}{m_e m_\mu m_\tau} \sim \mathcal{O}(\lambda_d^{-(C_1+C_2-8/3C_3)/2}) \; . \tag{4.7}$$

Compatibility with the extrapolated data requires the exponent to vanish

$$C_1 + C_2 - \frac{8}{3}C_3 = 0 \; . \tag{4.8}$$

5. Green-Schwarz Cancellation of X Anomaly

Let us first assume that X is anomaly-free, that is

$$C_1 = C_2 = C_3 = 0 \; , \qquad C_g = 0 \; . \tag{5.1}$$

The last equation is not constraining as there are likely more fields in the theory with chiral X-charge. If we let $\lambda_u = (\lambda_d)^y$, the equation (2.21) imply the sum rule

$$M_u M_d \sim (v_u v_d)^3 (\frac{M_d}{M_e})^{3y/2} , \tag{5.2}$$

where we have set

$$M_u = m_u m_c m_t \ ; \qquad M_d = m_d m_s m_b \ ; \qquad M_e = m_e m_\mu m_\tau \ .$$

This becomes

$$m_t m_b \lambda^6 \sim v_u v_d \begin{cases} 1 \\ \lambda^{y/2} \end{cases} ,$$

which is either inconsistent or requires $y \sim 12$!

When $s = 0$, these relations are consistent with (4.7), but the vanishing of C_3 contradicts our hypothesis that all excess charges have the same sign. Indeed, using the tree-level Yukawa relations (2.4), (4.1), we see that

$$0 = C_3 = 6(a_8 + b_8) + 6(a_8 + c_8) ,$$

which is not consistent with our assumption that all excess charges are positive. Hence we conclude that the anomaly coefficients cannot all vanish ; we must find a way to compensate these anomalies.

The Green-Schwarz mechanism provides the required mechanism.

String theories naturally contain all of the ingredients we need to reproduce the Yukawa textures. They have an antisymmetric tensor Kalb-Ramond field which in four dimensions is the Nambu-Goldstone boson of an anomalous $U(1)$ that couples like an axion through a dimension five term to the divergence of the anomalous current. Its anomalies are cancelled by the Green-Schwarz mechanism[1]. Under a chiral transformation, this term is capable of soaking up certain anomalies, by shifting the axion field, provided that they appear in commensurate ratios

$$\frac{C_g}{k_g} = \frac{C_i}{k_i} = \frac{C_X}{k_X} , \tag{5.3}$$

where the k_i are the Kac-Moody levels. They need to be integers only for the non-Abelian factors. For the gravitational anomaly, k_g is determined by an absolute normalization.

In superstring theories, this $U(1)$ is broken spontaneously slightly below the string scale. The scale is set by the charge content of the theory[24]. It follows that singlets with masses protected by X can still be very massive, and not appear in the effective low-energy theory. The breaking comes about because of the mixed gravitational anomaly. Any Abelian charge with a non-zero C_g contains part of an anomalous symmetry. Interestingly, this anomaly is zero for hypercharge. Thus we can concentrate on X. As a result of summing the tadpole contribution to the D-term with the massless states, one finds a contribution

$$D_X = x\theta^\dagger \theta - C_g \frac{g_{string}}{192\pi^2} \frac{\alpha'}{2} , \tag{5.4}$$

where g_{string} is the string coupling, and α' is the slope parameter. This results in the breaking of the anomalous $U(1)$ slightly below the string scale

$$\frac{< \theta >^2}{M^2_{string}} = \frac{C_g}{x} \frac{g_{string}}{192\pi^2} . \tag{5.5}$$

This chiral U(1) X-charge can fix the value of the Weinberg angle, without the use of a grand unified group, as remarked by Ibàñez[2]. More recently, Ibàñez and Ross[13] applied it to the determination of symmetric textures when the field θ is vector-like.

In superstring theories, the non-Abelian gauge groups have the same Kac-Moody levels. For Green-Schwarz cancellation, it means that

$$C_2 = C_3 \qquad \text{or} \qquad d_0 = b_0 + c_0 - a_0 . \tag{5.6}$$

After this very generic requirement, we see that equation (4.7) reduces to

$$\frac{m_d m_s m_b}{m_e m_\mu m_\tau} \sim \mathcal{O}(\lambda_d^{-(C_1 - 5/3C_2)/2}) , \tag{5.7}$$

valid whenever θ is chiral. Since the right-hand side is of order one, it means that the exponent vanishes, so that in models with an *ab initio* μ term, we *deduce* that

$$C_1 = \frac{5}{3} C_2 \ . \tag{5.8}$$

However the gauge coupling constants at string unification scale with the anomaly coefficients, so that

$$\frac{C_1}{C_2} = \frac{g_1^2}{g_2^2} \ , \tag{5.9}$$

which fixes the Weinberg angle to the value

$$\sin^2 \theta_w = \frac{3}{8} \ ,$$

at the string scale, the canonical GUT value, but without the excess baggage of these theories! This is a strong hint that the $N = 1$ model does indeed come from superstrings!

We note that the mixed gravitational anomaly is exactly along the anomaly-free combination of baryon minus lepton numbers, $B - L$. In fact the most general X-charge can contain an arbitrary mixture along $B - L$, but this is already taken into account by our general parametrization. In superstring models, the Green-Schwarz mechanism extends to the mixed gravitational anomaly so that

$$\frac{C_3}{C_g} = \frac{k_3}{k_g} \ ,$$

where the Kac-Moody level is either one or two[29].

In conclusion, starting from very simple generic assumptions we are able to reproduce the data and even determine the Weinberg angle in terms of the ratio of quark and lepton masses. Our analysis, applied to neutrino masses, shows that it is ackward to accomodate both solar neutrino and atmospheric neutrino data.

This work was supported in part by the United States Department of Energy under grant DE-FG05-86-ER40272

References

1) M. Green and J. Schwarz, *Phys. Lett.* **B149** (1984) 117.

2) L. Ibáñez, *Phys. Lett.* **B303** (1993) 55.

3) P. Binétruy and P. Ramond, *Phys. Lett.* **B350** (1995) 49.

4) M. Dine, N. Seiberg, and E. Witten, *Nucl. Phys.* **B289** (1987) 585; J. Atick, L. Dixon, and A. Sen, *Nucl. Phys.* **B292** (1087) 109; M. Dine, I. Ichinoise, and N. Seiberg, *Nucl. Phys.* **B293** (1987) 253.

5) See for example, A. Faraggi, *Realistic Superstring Models,* IASSNS-HEP-94-31, 1994, and references therein.

6) For reviews, see H. P. Nilles, *Phys. Rep.* **110** (1984) 1 and H. E. Haber and G. L. Kane, *Phys. Rep.* **117** (1985) 75.

7) U. Amaldi, W. de Boer, and H. Furstenau, *Phys. Lett.* **B260** (1991) 447; J. Ellis, S. Kelley and D. Nanopoulos, *Phys. Lett.* **260B** (1991) 131; P. Langacker and M. Luo, *Phys. Rev.* **D44** (1991) 817.

8) H. Arason, D. J. Castaño, B. Keszthelyi, S. Mikaelian, E. J. Piard, P. Ramond, and B. D. Wright, *Phys. Rev. Lett.* **67** (1991) 2933; A. Giveon, L. J. Hall, and U. Sarid, *Phys. Lett.* **271B** (1991) 138.

9) P. Ramond, R.G. Roberts and G. G. Ross, *Nucl. Phys.* **B406** (1993) 19.

10) L. Wolfenstein, *Phys. Rev. Lett.* **51** (1983) 1945.

11) C. Froggatt and H. B. Nielsen *Nucl. Phys.* **B147** (1979) 277.

12) M. Leurer, Y. Nir, and N. Seiberg, *Nucl. Phys.* **B398** (1993) 319.

13) L. Ibáñez and G. G. Ross, *Phys. Lett.* **B332** (1994) 100.

14) E. Papageorgiou, "Yukawa Textures from an extra $U(1)$ Symmetry", Orsay Preprint, LPTHE Orsay 40/94.

15) P. Binétruy, S. Lavignac, and P. Ramond, in preparation.

16) V. Jain, and R. Shrock, *Phys. Lett.* **B352** (1995) 83.

17) V. Jain, and R. Shrock, Stony Brook Preprint ITP-SB-95-22, June 1994.

18) E. Dudas, S. Pokorski, and C. Savoy, Saclay preprint, MArch 1995

19) Y. Nir, *Phys. Lett.* **B354** (1995) 107.

20) G. Giudice and A. Masiero, *Phys. Lett.* **B206** (1988) 480.

21) J. Casas and C. Muñoz, *Phys. Lett.* **B306** (1993) 288.

22) S. Dimopoulos, L. Hall, S. Raby, and G. Starkman, *Phys. Rev.* **D49** (1994) 3660.

23) A. Davidson, L. Michel, M. Sage, and K. Wali, *Phys. Rev.* **D49** (1994) 1378.

24) A. Font, L.E. Ibáñez, H. P. Nilles, and F. Quevedo, *Nucl. Phys.* **B307** (1988) 109; *Phys. Lett.* **B210** (1988) 101; J. A. Casas, E. K. Katehou, and C. Muñoz, *Nucl.*

Phys. **B317** (1989) 171; J. A. Casas, and C. Muñoz, *Phys. Lett.* **B209** (1988) 214; *Phys. Lett.* **B214** (1988) 63; A. Font, L.E. Ibáñez, F. Quevedo, and A. Sierra, *Nucl. Phys.* **B331** (1990) 421.

25) H. Dreiner, G. Leontaris, S. Lola, G. Ross, and C. Scheich, *Nucl. Phys.* **B436** (1995) 461.

26) M. Gell-Mann, P. Ramond, and R. Slansky in Sanibel Talk, CALT-68-709, Feb 1979, and in *Supergravity* (North Holland, Amsterdam 1979). T. Yanagida, in *Proceedings of the Workshop on Unified Theory and Baryon Number of the Universe*, KEK, Japan, 1979.

27) Z. Maki, M. Nakagawa, and S. Sakata, *Prog Theo Physics* **28** (1962) 247.

28) S.P. Rosen and W. Kwong, Univ of Texas at Arlington preprint, UTAPHY-HEP-13 (1995).

29) S. Chaudhuri, G. Hockney, and J. Lykken, *Phys. Rev. Lett.* **75** (1995) 2264.