Quarterly Progress Report

Project Title: Nuclear Waste Criticality Analysis

Principal Investigator: William G. Culbreth

Covering the Period: 1 October - 31 December 1995

Date: January 19, 1996

Summary:

The work to date includes the preparation of a report related to criticality in spent fuel, a report on the Oklo reactors and their relevance to Yucca Mountain, and the creation of a computer program to model the Oklo reactors. The objective of the program includes a computational model of the only known natural analogue to an underground nuclear waste repository and the possible application of the model to predict the long-term behavior of Yucca Mountain. A final summary of all work completed will be presented after the end of the project on February 29, 1996.

Work Completed to Date:

1. Preparation of Background Information

The topic of nuclear criticality as it applies to spent nuclear fuel is often not well understood. The professional staff of the Department of Energy and its contractors may benefit by a brief overview of nuclear fission and how sustained fission can be avoided in spent fuel. To provide this overview, we prepared a 7 page summary of atomic physics, nuclear fission, criticality control, and how it applies to the proposed Yucca Mountain site. This summary, titled: "What is Nuclear Criticality and Why is it of Concern at Yucca Mountain" is included as appendix A.

2. The Oklo Reactor Phenomenon and its Application to Yucca Mountain

The natural reactors that occurred in French Gabon, Africa over 2 billion years ago give Yucca Mountain researchers a unique analogue to what might be expected to occur in an underground nuclear waste repository in the far future. The Oklo reactors may be useful in predicting radionuclide migration rates from the proposed repository and may indicate potential criticality problems in the event that water enters the repository horizon. A report titled: "Review of the Oklo Natural Reactors and their Pertinence to Yucca Mountain" details our analysis of the Oklo reactors during this quarter. A copy of the report is included as appendix B.
3. **Student Involvement**

Two students and an postdoctoral research associate have been employed on the project. They have been continued their work by conducting criticality research and through literature reviews and have assisted in computational studies related to the project. Louise Steeps completed an independent study course on nuclear criticality as a part of her work as a senior in a civil and environmental engineering. She is currently learning to create input files to KENO-V to conduct criticality simulations of the Oklo reactors.

Dr. James Ventresca, a computational fluid mechanician, has been assisting in the development of a computer program to model the behavior of the Oklo site.

Cliff Lawson, an undergraduate student in civil engineering, is assisting with literature searches and is continuing his education into nuclear waste management.

5. **Oklo Simulation Program**

Work has been progressing on a simulation program. The results of the criticality portion of the program are being compared to results from the Monte-Carlo simulation code, KENO-V, for accuracy. A draft outline of the code was included as appendix D in our previous quarterly report.

Our goal by the end of February is to have a working model of the Oklo site including the effects of the unsteady heat transfer, phase transformation in water, and heat generation by fission. Radionuclide migration, fuel burnup, and time variant water intrusion will be added later. The results of the program will include multiplication factors, neutron fluxes, and temperature distributions as functions of time.

6. **Scale Software**

The new version of the Oak Ridge criticality code SCALE 4.3 has been ordered and is expected this January. The SCALE software will be used for further analyses of criticality in MPC's.

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Appendix A
Overview

It has often been noted that 10 CFR 60 mandates that nuclear criticality must be controlled in an underground repository at Yucca Mountain and during the handling or storage of spent nuclear fuel. Not everyone employed on the Yucca Mountain Project is a "nuclear engineer" and yet those with a scientific or engineering background should understand what the concept of criticality implies. If you want a brief overview of this phenomenon, please keep reading!

The Building Blocks of Matter

All materials on earth are composed of electrons, protons, and neutrons. While neutrons and protons have roughly the same mass, it would take 1836 electrons to equal the mass of one neutron. The number of electrons surrounding an atomic nucleus, however, governs the chemical behavior of the nucleus. To maintain a neutral charge on an atom, stable atoms have an equal number of electrons and protons. Elements are defined by the number of protons in the nucleus and are listed on the periodic charts that we all remember from high school chemistry. Neutrons have no charge and they do not effect the chemical behavior of an element. For this reason, elements may be composed of isotopes containing the same number of protons, but varying numbers of neutrons.

Hydrogen is a good example of an element with several isotopes. As seen in figure 1(a), for example, we are most familiar with a hydrogen atom composed of one electron and one proton. The electron actually forms a "probability cloud" about the nucleus and doesn't follow the simple orbit as shown. The chemical behavior of hydrogen is due to the ability of the electron to form ionic or covalent bonds with the electrons of other atoms. To clearly identify this "isotope" of hydrogen, we can indicate it as H-1. This identifies the element as hydrogen and specifies that it has one nucleon. A nucleon is defined as either a neutron or a proton. Figure 1(b) shows the next isotope of hydrogen, indicated as H-2. This isotope has one electron, one proton, and one neutron. The neutron does not change the chemical behavior of the atom; it still acts like hydrogen. Another name for
this isotope is deuterium. Deuterium forms the basis for the "heavy water" used in certain Canadian reactors (and the plot for some movies about World War II). The proton and the neutron in the nucleus orbit each other in a complex "distribution cloud" similar to that of the electrons.

**Radioactive Decay**

Electrons orbiting a nucleus have discrete energy levels. If radiation strikes one of these electrons, the electron will increase its orbit to a higher energy level. The electron will "relax" to its original orbit (or quantum mechanical state) by releasing a photon. A photon is a basic quantity of electromagnetic radiation. The energy level of a photon identifies it as infrared radiation, visible light, ultraviolet radiation, or x-rays. A similar thing happens if you disturb the nucleons in the nucleus. Since the energy levels in the nucleus are quite a bit higher than in the "electron cloud," a nucleus releases gamma rays when it relaxes to its original state. Gamma rays are very intense photons of electromagnetic radiation with energy levels that can be thousands of times greater than that of x-rays.

Radioactive decay occurs in the nuclei of isotopes that are unstable. An unstable nucleus may decay by releasing gamma rays, electrons, neutrons, and even antimatter particles. Radioactive decay can also release quite a bit of energy from the nucleus. This energy appears in the form of the kinetic energy of the ejected particles.

A third isotope of hydrogen is shown in figure 1(c). This is H-3, or by its common name, tritium. The two extra neutrons make this isotope unstable.

We quantify the decay rate of an isotope in terms of its half-life. The half-life of an isotope is defined as the time that it takes for 50% of the isotope to decay into some other isotope. As shown in figure 2, tritium has a half-life of 12.6 years. It takes 12.6 years for half of a 1 kilogram sample to decay away. Eventually, tritium decays into Helium-3 and releases electrons (called β or beta particles) and a lot of kinetic energy. After 10 half-lives or 126 years, essentially all of the tritium is gone.

Radioactive isotopes are called radioisotopes. Half lives of radioisotopes may range from microseconds to billions of years. For example, the half-life of Pu-239 (plutonium) is 24,360 years. The half-life of U-238 (uranium) is 4.51 billion years. Generally, the longer the half-life, the lower

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the radioactivity of the isotope meaning that it is "safer" to be around. Certain elements in our own bodies, such as carbon and potassium, contain radioactive isotopes.

**Nuclear Fission**

As you go down the periodic chart from light nuclei like hydrogen to heavy nuclei such as uranium, the stable elements are composed of an increasing ratio of neutrons to protons. Helium, for example, is composed of two protons and two neutrons. Uranium 238 is composed of 92 protons with 146 neutrons.

Some isotopes of uranium and plutonium are extremely unstable. When these nuclei, called *fissiles* collide with a low energy neutron, they violently break apart forming two large *fission fragments*, neutrons, electrons, gamma rays, and other subatomic particles. Each particle carries away a certain amount of kinetic energy from this explosive event. The amount of kinetic energy released in the fission of one atom of U-235 is roughly equivalent to the energy released by breaking over 10,000,000 chemical bonds in a hydrocarbon/oxygen mixture.

Several isotopes make excellent fissionable fuel, including: U-233, U-235, Pu-239, and Pu-241. All we need is an initial source of neutrons and the chain reaction is off and running. As seen in figure 3(a), if a slow (referred to as a "thermal" neutron) is absorbed by a nucleus of U-235, it becomes an "excited nucleus" as seen in figure 3(b).

Within a microsecond, the excited nucleus breaks apart releasing two or more neutrons, as well as other particles. The released neutrons are important. They are very energetic and must be "slowed down" before they can go on to cause other U-235 nuclei to fission. Neutrons are slowed down by allowing them to collide with atoms that have nuclei with nearly the same mass as a neutron. In each collision, the a neutron looses a fraction of its kinetic energy by momentum transfer in much the same manner that billiard balls do during a game of pool. Light atoms, like hydrogen and helium, make ideal *moderators* to serve as subatomic counterparts to the billiard balls. Without a moderator, the highly energetic neutrons created by fission will likely leak out of the uranium fuel and never go on to cause a new fission.
(a) Before Fission: A slow neutron strikes a fissile nucleus.

(b) An "excited" nucleus results that quickly undergoes fission.

(c) Fission produces fission fragments, subatomic particles, and a lot of energy!

Good moderators aren’t hard to find. Water works quite well since it has a lot of hydrogen. Hydrogen has a light nucleus composed of a single proton with roughly the same mass as the incoming neutron. Another indicator of a good moderator is that it does not readily absorb slow neutrons. The probability that an atomic nucleus will absorb a neutron is expressed in terms of a cross section which is analogous to a target area with the rather odd units of barns.

If at least one neutron slows down enough to cause fission and strikes another fissionable nucleus, the fission process repeats and forms a chain reaction. If neutrons "leak out" of the nuclear fuel or are absorbed by some non-fissionable atom, they cannot contribute to the continuation of the chain reaction and are considered lost.

Remember the two large fission fragments released during the fission of a single atom of uranium? As seen in figure 3(c), each fragment shares a little less than half of the mass of the original "fissile" nucleus. There is quite a range of different nuclei created. They all tend to have an excess of neutrons in their nuclei and they get rid of these neutrons by radioactively decaying. These fission fragments form the radioisotopes in spent fuel that make it so radioactive. It turns out that fresh uranium oxide fuel really isn’t that radioactive and doesn’t get that way until it has been exposed to the high neutron flux found in a nuclear reactor core.

Figure 3 -- The Fission of a Uranium-235 Nucleus
Nuclear Criticality

In a nuclear reactor operating at a steady power level, the fission or chain reaction process is continuous and no external source of neutrons is necessary to keep the reactor going. In commercial reactors, the vast amounts of kinetic energy released during fission is used to heat water into steam to drive steam turbines and to make electricity. In a commercial reactor, we really want to sustain a chain reaction. In the Yucca Mountain spent fuel, we want to avoid it.

To sustain a chain reaction, the reactor fuel must become critical. For safety, Yucca Mountain wastes must remain subcritical. In the case of a nuclear weapon, the mixture of fuel is supercritical.

We can quantify the level of criticality by using the quantity \( k_{\text{eff}} \). This is called the effective neutron multiplication factor. This may seem a complicated name, but it only represents the ratio of neutrons in one generation to the number in the previous generation. Figure 4 graphically shows the relationship between criticality and \( k_{\text{eff}} \). If the chain reaction is increasing in its reaction rate then the reactor is supercritical and \( k_{\text{eff}} \) is greater than 1. This is a normal occurrence when the power level of a reactor is increased or the reactor is started up for the first time. Criticality in a commercial nuclear reactor is controlled by moving control rods made up of strong neutron absorbers, such as cadmium and hafnium. Boron "shims" are also used in fresh reactor fuel.

Why is Criticality an Important Issue at Yucca Mountain?

Now that we have an idea what "criticality" means, why is it of such importance in the design of the Yucca Mountain Repository? Federal law 10 CFR 60.131(a) dictates that the "multiplication" factor cannot exceed 0.95 during transportation, storage, or disposal unless at least "two unlikely events occur, simultaneously". Remember that spent fuel is in the form of fuel assemblies that were
designed with the optimal geometry to lead to criticality \( k_{\text{eff}} \geq 1 \) when flooded with water.

**What Happens if a Chain Reaction Does Occur in Spent Nuclear Fuel?**

The law is based on the concern that a criticality event could occur that would result in the release of large amounts of heat and radiation. If such an event occurs during long term disposal, the spent fuel may become a brand new source of dangerously radioactive fission products. This would complicate the goal of regulating releases of radionuclides from the repository into the environment. In certain situations, a criticality event may lead to a "steam explosion" where the energy released by fission could cause water to flash into steam violently carrying radioactive fission products away from the fission source.

There are some natural examples of criticality events that have occurred within subterranean strata of uranium-rich ore. These natural reactors existed at the Oklo uranium mining site in French Gabon, Africa over 2 billion years ago. These reactor sites were discovered in 1972 by French scientists. Some of the knowledge gained from the site have included the conclusion that radionuclides did not migrate very far from the location of the reactors. When operating at critical conditions, relatively low amounts of heat were released and radioactive fission products were created.

**How Can Criticality be Avoided?**

There are ways to decrease the chance of a "criticality event". First, we can separate the individual spent fuel assemblies so that neutrons tend to "leak" into the environment before they could cause fission in the fuel. This is difficult due to the large number of assemblies that must be disposed of. Another control method is to do whatever is possible to keep water away from the spent fuel since it is such a great moderator of neutrons (remember that a "moderator" slows down neutrons to enhance fission). One reason that Yucca Mountain was chosen for site characterization is that the water table is well below the repository.

A third technique for "criticality control" is to add neutron poisons. These "poisons" are strong neutron absorbers that decrease the number of neutrons available to go on and cause fission. Neutron poisons range from common elements like boron to toxic metals such as cadmium and expensive rare earths as samarium, hafnium, and europium. Unfortunately, although boron is inexpensive, it is very water soluble and may diffuse from the MPC during long-term storage.

Another thing in our favor that helps us avoid a criticality event is the fact that much of the fissile uranium in fresh fuel is burned up during the actual operation of the nuclear reactor. The decrease in fissile concentration makes spent fuel less of a criticality concern than fresh fuel. Unfortunately, the remaining fissile takes a long time to radioactively decay. The change in fissile uranium within spent fuel over a period of one million years barely changes. Fissile plutonium, however, decays more rapidly.

To comply with federal law and to ensure safety, the design of the repository and the treatment of spent fuel during all phases of repository operation will treat nuclear criticality with a great deal of care. Although commercial nuclear reactor fuel was designed to optimize the chances of criticality, there are safe and effective
techniques available to the repository scientists and engineers to avoid such an occurrence.

For further information, please check out the publications indicated at the references and thank you for spending the time to read through this review. Criticality promises to be an important issue during the site characterization and licensing of our nation’s first high-level nuclear waste repository and we should all be familiar with its significance.

Acknowledgements

Funding for this and other criticality related work was provided under U.S. Department of Energy Grant #7116DE012.

For Further Reading:


Appendix B
Review of the Oklo Natural Reactors and their Pertinence to Yucca Mountain

Loise E. Steeps
William G. Culbreth

December 7, 1995
UNLVWPP:61595.4

Photograph of Natural Convective Flow with Dye Injection about a Waste Package Model
University of Nevada Las Vegas  
Howard R. Hughes College of Engineering  

Review of the Oklo Natural Reactors  
and Their Pertinence to Yucca Mountain  

by  
Louise E. Steeps  
William G. Culbreth  

December 7, 1995
# Table of Contents

1. Introduction ................................................. 1  
2. Review of the Oklo Sites ................................. 1  
   2.1 Concentration of the deposits ...................... 3  
   2.2 Shape of the deposits ............................... 3  
   2.3 Moderator within the deposits ..................... 4  
   2.4 Maintaining a constant neutron flux ............... 4  
   2.5 Availability of neutron poisons .................... 5  
3. Analogy between Yucca Mountain and Oklo .............. 5  
4. Mathematical Model of Oklo .............................. 7  
   4.1 Definition of the reactor ......................... 7  
   4.2 Diffusion model .................................... 8  
   4.3 Energy model ....................................... 8  
   4.4 Criticality model .................................. 9  
   4.5 Radionuclide inventory ............................ 13  
5. Conclusion ............................................... 14  
References .................................................. 15
1. Introduction

In 1982, Congress enacted the Nuclear Waste Policy Act. According to the Act, the Department of Energy (DOE) was to submit nine possible locations for the storage of commercial nuclear spent fuel and highly radioactive waste in a permanent repository. Initially, nine possible sites were selected. Eventually six were eliminated, leaving three possible sites. As a result of the Nuclear Waste Policy Act amendment in December, 1987, site characterization was limited to Yucca Mountain, Nevada. In order for Yucca Mountain to become the site of the permanent repository, the DOE must meet the requirements set forth by the Environmental Protection Agency and CFR Part 60. The site characterization of Yucca Mountain will take up to 10 years. The information received from studies during this time will lead to the acceptance or rejection of Yucca Mountain. The results, at this point, indicate that there are no disqualifying conditions for a permanent repository in Nevada.

2. Review of the Oklo Sites

In May, 1972, H. Bouzigues found a discrepancy in the concentration of uranium 235 at the mines in Oklo, Gabon Republic of West Africa. The concentration of fissile uranium-235 was 0.7202% compared to 0.7171% found at the Oklo site. This
discrepancy was small but significant because it revealed the possibility of a natural fission reactor. After further scientific research, it became apparent that Oklo was a natural reactor site which existed over two billion years ago.

A wide range of radionuclides were created while the reactors were in operation. The fission products of a chain reaction were observed within the uranium ore. Some of the products which remained immobilized in the ore were ruthenium, palladium, silver, neodymium, and lanthanum. Neodymium is predominantly the result of a fission process. Water soluble fission products such as cadmium, strontium and cesium were not observed. Most of the site characterization studies focused upon the migration of the radionuclides from the sites. In regards to the Yucca Mountain waste repository, these studies have significant implications. However, the Oklo phenomenon reveals other potential obstacles which could affect the proposed methods of waste management. These factors include: the precipitating factors relating to criticality at Oklo, and the occurrence of criticality in enrichments.

Several conditions must have been in existence to sustain the nuclear fission process at Oklo. These included:

1) the concentration of the deposit had to be sufficient to cause a chain reaction
2) the shape of the deposit had dimensions which enabled fission,
3) an effective moderator must have been present within the ore,
4) a constant neutron flux had to be maintained,
5) a minimum quantity of neutron poisons had to be present in the reactor core.

At the time the natural reactors at Oklo were active, the amount of uranium (UO₂) in the ore ranged from 20% to 60% in each zone.[1] The U-235 concentration, at formation of the earth, was approximately 17%. The estimated concentration of U-235 at the initial operation of the natural reactor was 3.5%.[1] Criticality can be attained in a natural reactor with enrichment as low as one percent.[2] According to Hagemann and Roth [3], about four to five metric tons of U-235 underwent fission in the reactor zones.

The shape of the reactor zone, Figure 1, is another factor which would affect whether the zone could become critical. An average reactor zone was 10 meters long, 10 meters wide, and 10 to 50 centimeters thick.[1] If the ore deposit was too thin, less than half

![Fig. 1 Cross Sectional View of the Oklo Natural Reactor Site [ref. 6]](image)
a meter, the neutrons would have passed through the surface of the deposit. The core of each reactor had the shape of a lens. The core consisted of high grade uranium ore. The lenses were found between two layers. The base layer consisted of sandstone and conglomerate. The top layer was comprised of sandstones and pelites.[1,3]

In order for the fission process to occur and continue, a moderator was necessary to decrease the velocity of the neutrons. A natural occurring moderator is water. It is assumed that the ground at Oklo was saturated. The optimum ratio of water was approximately 6% by weight[2].

A constant neutron flux had to be maintained. The flux was sustained through the fission of uranium or plutonium. Nuclear fission occurs when a neutron is absorbed by the nuclei of uranium isotopes (U-235) or plutonium. The nucleus becomes unstable and splits apart while releasing a large amount of kinetic energy. Fission fragments, neutrons, and high energy photons are emitted during the fission process. A high energy neutron cannot be absorbed by U-235, thus a moderator is necessary. A neutron which has not been slowed may be absorbed by U-238, but fission does not occur.

A fission reaction could not have been sustained if there was a sufficient quantity of neutron poisons. Neutron poisons absorb the slowed neutrons without eliciting a fission process. Several rare earths; such as, boron, hafnium, cadmium, and lithium are categorized as neutron poisons. It is believed that these elements
limited the power output of the reactor, but were not available in large quantities in the Olko deposits to deter the chain reaction.

According to Hagemann and Roth [3], the life of the individual reactor zones ranged from 100,000 to 800,000 years. Kuroda states that the reactors operated intermittently over this time period.[4,5] The reactors behaved in a manner similar to geysers or hot springs. The time during which the reactors were inactive ranged from 150 to 180 minutes. [5] The temperature of the reactor during operation was between 183 degrees and 452 degrees Celsius.[4] The pressure during operation was between 0.5 and 1.5 kbars at a depth of 1500 to 4500 meters.[6]

3. Analogy between Yucca Mountain and Oklo

Yucca Mountain Nevada is undergoing site characterization studies to determine if it will become a permanent repository for nuclear spent fuel. Approximately 70,000 metric tons of spent fuel will be deposited at the nuclear waste repository. The average U-235 concentration will be approximately 1.5%. Thus, criticality conditions could be established.

The spent nuclear fuel rods will be stored in multi-purpose containers (MPC’s). Currently, the containers will be comprised of a high-nickel alloy inner shell surrounded by an outer shell consisting of mild steel. A neutron absorber will be added to the design of the MPC to ensure low k$_{eff}$ values. Unfortunately, the neutron poisons may seep from the canisters before the fuel matrix material; thus, enhancing the possibility for a critical event.
Over time, the MPC shells will corrode and fail. Any remaining neutron poisons and other water soluble nuclei will leak from the MPC site. Once the canister has degraded, UO, and PuO, will adsorb in the vicinity of the MPC, allowing the site to function as a slab reactor. The Oklo natural reactors indicate that the spent nuclear fuel at Yucca Mountain will function like a reflected slab reactor. Groundwater flow may cause migration of the lighter nuclei from the MPC site. Heavy metals and fission products may collect within original drifts or within fractures at the site.

The moderator will be water. Since the worst possible conditions must be taken into account, a saturated state will be assumed at the Yucca Mountain burial site.

The dispersion of fission products is considered the most important issue. If criticality is attained at the Yucca Mountain site, a wide range of radionuclides will be created. Using Oklo as a model, some of the fission products would remain immobilized within the uranium ore. These include ruthenium, palladium, silver, neodymium, and lanthanum. Water soluble fission products such as cadmium, strontium and cesium will leak from the site. This dispersion of fission products can be a significant health hazard to the food chain and consequently, man.

Since a critical reactor event in a geologic setting has occurred, the experience may be analogous to Yucca Mountain. A critical event occurring at Yucca Mountain is dependent upon various factors, as previously mentioned.
4. Mathematical Model of Oklo

A computer generated analysis was used to simulate the fission chain reaction which occurred approximately 2 billion years ago at the Oklo site. To display the accuracy of the program, it correctly followed the decomposition of uranium 235. The program consisted of various modules. The first module defined the reactor. Successive modules contained the diffusion, energy, and criticality model. The last module calculated the radionuclide inventory and displayed the results.

4.1 Definition of the Reactor

Defining the reactor required a literature search which described the geological composition of Oklo. The distribution of fertile and fissile uranium was obtained. Refer to Table 1 (Curtis, et.al).[8] The initial distribution of water was assumed to be in a saturated state. The average temperature of the site was 400 degrees celsius.
Table 1. Detailing of the Location and Uranium Composition in Reactor Zone 9.

<table>
<thead>
<tr>
<th>Sample Identification</th>
<th>Reactor Zone 9</th>
<th>Relative Coordinate (m from 0,0)</th>
<th>U (g/g)</th>
<th>$^{235}$U (atom%)</th>
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<td></td>
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4.2 Diffusion Model

The diffusion model, based on temperature, was developed to calculate the distribution of water as a function of time. The model was limited to a one dimensional model. The model accounted for the periodic nature of the reactor. The influences of air were ignored.

4.3 Energy Model

The energy model computed the amount of energy created by
fission and the loss of energy by convection to surrounding water. This was accomplished by conduction through the rock and water. This caused created the phase change of water to steam.

4.4 Criticality Model

The power generated by fission is a function of position due to changes in the neutron flux. In the one dimensional model, the neutron flux was determined using the criticality model. As a preliminary study, an analysis of an infinite slab reactor was done to determine the required thickness of the core to sustain a chain reaction.

The formation of the slab reactors at Oklo are as previously mentioned (refer to Figure 1). In the criticality model, the Oklo reactors were presumed to be infinite slab reactors. An infinite slab reactor will not have leakage in the y or z directions. So, there will be a flux gradient and neutron flux in the x direction exclusively. Refer to Figure 2 for the dimensions of the infinite slab reactor. The equations to determine the critical thickness of the core were developed from this model and the diffusion equation,

\[ \nabla^2 \phi + B^2 \phi = 0 \]  

The two boundary conditions for the diffusion equation are as follows:

1) At the extrapolated periphery, the flux drops to zero. Thus, when

\[ x = \pm x_c / 2, \ \phi = 0 \]
2) The flux is symmetrical and finite about the origin. Thus, when 
\[ x = 0, \frac{d\phi}{dx} = 0 \]

The values, \( B^2 \) and \( \nabla^2 \phi \), represent the buckling of the reactor and the leakage of neutrons from the core boundaries, respectively. The neutron flux, \( \phi \), is expressed in units of neutrons/cm\(^2\)-s.

Buckling is defined by equation 2.

\[
B^2 = \frac{k_{\infty} - 1}{L^2}
\] (2)

The values for the infinite multiplication factor, \( k_{\infty} \), and the thermal diffusion length, \( L \), are dependent upon material properties. Such properties include concentrations and cross sections.

The equations necessary to obtain the thickness, the flux, and the power generated in a steady state critical infinite slab reactor were obtained using the variables from Figure 2.[9] The definition for each variable follows.
\( x_s/2 \) -- the extrapolated half thickness of the core, where the flux falls to zero.

\( x_s/2 \) -- finite distance from the origin to either slab face, where the flux is also finite.

\( x_o \) -- the extrapolated thickness of the core.

\( x_g \) -- the critical thickness of the core.

\( d \) -- the extrapolated distance by which the core is conceptually extended for the flux to drop to zero.

The critical thickness is equivalent to:

\[
x_g = x_o - 2d
\]  

(3)

The neutron flux computed for a slab reactor is \( \phi(x) = A \cos(Bx) \). For the derivation refer to Foster and Wright.[9] The buckling, \( B \), is given by equation (2) as eigenvalues of the equation:

\[
B_n = \frac{n \pi}{x_o}, \quad n = 1, 2, 3, \ldots
\]  

(4)

Where \( n = 1 \), corresponds to the stable reactor flux distribution derived as:

\[
\phi(x) = \phi_{\text{max}} \cos \left( \frac{\pi x}{x_o} \right)
\]  

(5)
The value for $\phi_{\text{max}}$ is equal to the maximum flux in the slab reactor. The maximum flux occurs at the centerline of the slab. The definition for the differential power generated by the reactor is

$$dP = G (\Sigma_{fis} \phi) dV$$ (6)

Thus,

$$P = \int dP = \int G \Sigma_f \phi_{\text{max}} \cos\left(\frac{\pi x}{x_0}\right) dx$$ (7)

$$P = G \Sigma_f \phi_{\text{max}} \int \cos\left(\frac{\pi x}{x_0}\right) dx$$ (8)

The integrated power is given by:

$$P = \left[ 2 \Sigma_f \phi_{\text{max}} \left(\frac{x_0}{\pi} \sin\frac{\pi x}{x_0}\right) \right]_{0}^{x_o}$$ (9)

$$P = 2 G \Sigma_f \phi_{\text{max}} \left(\frac{x_0}{\pi}\right) \sin\frac{\pi x_o}{2x_0}$$ (10)

Each variable is specified as follows:

$$dV = dx \, dy \, dz = \text{differential volume} = dx(1)$$ (1)

$$G = 191 \text{ MeV/fission} = 8.9 \times 10^{-18} \text{ kWh/fission}$$
\[ \phi = \phi_{\text{max}} \cos(\pi x / x_0) \]

\[ \Sigma_{\text{fs}} = \text{macroscopic cross section} = N_{235} \sigma_{\text{fs}} \]

Where \( N_{235} \) is the number density of U-235 atoms and \( \sigma_{\text{fs}} \) is the microscopic fission cross section. The power is used to compute the temperature distribution and water phase as a function of \( x \). If \( T(x) \) is greater than \( T_{\text{sat}} \), the saturation temperature, then the water is a vapor.

To obtain critical conditions in the reactor, it is necessary to obtain a minimum of \( k_{\text{eff}} = 1 \). The infinite multiplication factor, \( k_{\text{eff}} \), is the ratio of the number of neutrons created in the new generation to the number of neutrons created in the previous generation. If a slab reactor has a minimum thickness of \( x_g = \pi / B + d \), the reactor will sustain a chain reaction since \( k_{\text{eff}} = 1 \).

4.5 Radionuclide Inventory

The radionuclide inventory module was used to calculate the amount of each radionuclide within the ore. Losses due to radioactive decay were considered for each radionuclide. The amounts were computed within each increment of time as a position of \( x \). The depletion of fissile elements and the accumulation of fertile elements or compounds were also computed. Finally, the concentration of each radioisotope was displayed as a function of \( x \) and time. The program repeated the calculations from \( t = 0 \) to \( t = 2 \) billion years.
5. Conclusion

The Oklo phenomenon reveals various factors which affect high level radioactive waste disposal. In a high level radioactive waste repository, the desire is to maintain a neutron flux which will remain below $k_{\text{eff}} < 1$. Hence, criticality will not occur. If criticality is attained, the dispersion and concentration of the radionuclides is of significant interest. The insoluble radionuclides at the Oklo site demonstrated retentivity within the rock. The water soluble radionuclides were absent; thus, they were carried away from the reactor to the environment. The effects to the environment, by the dispersion of both soluble and insoluble radionuclides at Oklo, is assumed to be minimal. The ramifications of a critical event occurring at Yucca Mountain or another possible permanent waste repository site cannot be overlooked. The risk remains that the environment could be dramatically affected, even to the point of destruction of life.
REFERENCES


