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**ON DRAWING COMPLICATED 3D SURFACES**

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**Abstract**

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The problem of constructing images of complicated 3D surfaces — world sheets of strings is considered. The world sheet representation in the form of a frame of points determined with a specified supporting curve is described. The pictures of the world sheets of free string are constructed using various graphical technologies.

**Аннотация**

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Рассмотрена задача построения изображений сложных трёхмерных поверхностей — мировых листов струны. Описано представление мирового листа в виде каркаса точек, определяемых по заданной опорной кривой. Построены изображения мировых листов свободной струны с использованием различных графических технологий.

## INTRODUCTION

The necessity of creating complicated 3D surfaces' graphical representations often arises in various problems of mathematical and physical sciences. For example one can consider the works presented at Visualization '93 International Conference [1] dealing with the visualization of flow fields, molecular surfaces, wave functions, etc.

This work presents the technique of constructing images of the surfaces called world sheets of strings. World sheets of strings are complicated objects in the Minkowski space [2] and we need their comprehensive visual representation for their study. To obtain the desired possibility to create and easily handle "realistic" 3D images we have chosen PHIGS for our purposes. PHIGS -- *Programmer's Hierarchical Interactive Graphics System* -- is the term used to refer to the super set of PHIGS Classic [3] and PHIGS-PLUS [4], [5] an ANSI&ISO standard for 3D graphics which is now as widespread as GKS [6]. We used the highest version of its implementation available for us now -- DEC PHIGS version 2.1 [7]

The description of the physical object of study -- *world sheet of string* will be given in section 1. Section 2 is devoted to a brief introduction to the image constructing technology used in PHIGS. The technique of the obtained image manipulating for its studying will be discussed in Section 3 and the results of the work will be presented in last Section 4 as well as notes on possible ways of future studies.

### 1. PHYSICAL OBJECT OF THE STUDY

World sheets of strings with which string theory deals are surfaces of extreme area in the 4-dimensional Minkowski space. Such surfaces have singularities that

do not disappear in small deformations of surface [8], [9]. These singularities have a form of kinks, propagating on the string with the velocity of light. Visualization of world sheets can be very useful for the studies of singularities.

World sheets of free strings can be classified by their topological type – open, closed, Y-shaped, etc. The world sheet of an open string can be obtained by mapping an infinite band into the Minkowski space, of a closed string – by mapping an infinite cylinder, of Y-shaped string – by mapping three infinite bands glued together along one edge. The requirement of extreme area gives a unique geometrical description of construction for every type of world sheets. For example, an open string of the world sheet can be represented by the geometrical place of the middles of segments, connecting every possible pairs of points on some supporting curve<sup>1</sup> [10], [11].

Such representation was used to produce initial data for further surface drawing. A vector-function  $Q_\mu(\sigma)$ ,  $\mu = 0, 1, 2, 3$  was defined, whose Minkowski square meets the requirement:

$$Q^\alpha = Q_0^\alpha - \vec{Q}^\alpha = 0. \quad (1)$$

This vector-function defines a supporting curve<sup>2</sup>. The coordinates of a point on the world sheet are found as

$$x_\mu(\sigma_1, \sigma_2) = \frac{1}{2}(Q_\mu(\sigma_1) + Q_\mu(\sigma_2)). \quad (2)$$

It can easily be noted that there are such transformations of parameters  $(\sigma_1, \sigma_2)$  that do not change  $x_\mu(\sigma_1, \sigma_2)$ <sup>3</sup>:

$$\begin{aligned} \sigma_1 &\leftrightarrow \sigma_2, \\ \sigma_1 &\rightarrow \sigma_1 + 2\pi, \quad \sigma_2 \rightarrow \sigma_2 - 2\pi. \end{aligned} \quad (3)$$

All this means that the surface defined by equation (2) multiply covers itself. Each point of it has infinite multiplicity: if we shift one point that lays on supporting curve  $N$  periods and another point by  $N$  periods towards the opposite side, the middle of the segment connecting those two points will not shift. It is sufficient to construct only one layer of this multilayer surface (physically it is the very layer that represent the world sheet of the string). Let's limit the parameters  $(\sigma_1, \sigma_2)$  range .

$$\sigma_1 \leq \sigma_2 \leq \sigma_1 + 2\pi. \quad (4)$$

<sup>1</sup>See figure 1, illustrating this representation and the procedure described further.

<sup>2</sup>For linear string it will be  $Q_\mu(\sigma) = (\sigma, \cos\sigma, \sin\sigma, 0)$ .

<sup>3</sup>Supporting curve is a periodical one and we will represent it with such function  $Q_\mu(\sigma)$  that changes on the period of a supporting curve  $2P_\mu$  when  $\sigma$  changes  $2\pi$ . Then in translation (3.2) first term in (2) will be shifted by period to one side and second term – to the another and  $x_\mu$  won't change.

This band and its image in transformations (3) have no common points (except boundaries) that's why in the band (4) there are not any two points that in  $\sigma_1$  corresponds the same point on world sheet. Besides this we may apply the transformations (3) to band (4) one can cover the whole plane  $(\sigma_1, \sigma_2)$ , i.e. every point on world sheet can be represented by a point in band (4) and the world sheet is represented by band (4) one-to-one.

The edges of world sheet correspond to the boundaries of band (4). The edge  $\sigma_2 = \sigma_1$  coincides with a supporting curve, the edge  $\sigma_2 = \sigma_1 + 2\pi$  is a supporting curve shifted by a half of a period  $T_p$ . World sheet is stretched in those two curves and is a periodical surface with a period equal to  $2T_p$ .

We'll deal with a part of this surface. Let's choose an interval  $[0, T_p]$  on the axis of parameter  $\sigma_1$  and divide this interval into  $m$  parts and interval  $[\sigma_1, 2\pi]$  into  $n$  parts. The coordinates of the points in the knots of such frame

$$x_\mu^j = \frac{1}{2}(Q_\mu(\frac{j}{m}T) + Q_\mu(\frac{j}{n}T + \frac{j}{n}2\pi)), \quad (5)$$

will consist of the initial data representing the physical objects of our study.

We can imagine how this complicated surface looks like only in simplest cases (one can look through earlier works [11],[12], [13]). Creation of various images of such surfaces will be one of the objectives of our study.

## 2. IMAGE CONSTRUCTION

PHIGS graphics standard was used for creating images of world sheets of strings. It was chosen due to its strong graphics feature set. It starts at points and goes up through NURBS - non-uniform rational B-spline surfaces, a very general type of parametric surfaces with the ability to assign colors to the positions on it. Also PHIGS gives the user different technologies for obtaining "realistic" 3D images, such as hidden lines' removal algorithms, lighting and shading depending on the properties of drawn areas and active light sources, depth cueing, semi-transparent surfaces etc. A very helpful feature of PHIGS, in which it differs from GKS, is a possibility of image manipulation after its creation. All output primitives with their attributes in PHIGS are stored in hierarchically ordered structures and may be objects of future deleting or changing, while in GKS output primitives are displayed "once and forever". This fact gives the user an opportunity to investigate a created image with the set of standard simple methods.

Below we concisely present the process of creating an image in PHIGS for later discussion of image manipulating methods. For more details refer, for example, [14].

The initial data, describing the physical object, was supplied in the form of a *frame of world sheet of string* —  $m \times n$  set of points in 3D space in the coordinate system called Modelling Coordinates (MC). The data are then mapped into distinguished World Coordinate System (WC) via defining modelling transformation as a sequence of rotation, translation and scaling along each coordinate (x,y,z). Then take place the effects of object lighting and shading depending on object and active light source properties.

Once the object is mapped into WC it is necessary to define how it is viewed by the observer. It is made by defining the viewing transformation from WC into Normalized Projection Coordinates (NPC). NPC space ranges from  $\{0,0,0\}$  to  $\{1,1,1\}$ , while WC space may have arbitrary boundaries. Mutual position of the object and observer is specified by defining in WC a projection plane (in the form of a pair of vectors and a point, simultaneously new coordinate system is being defined — View Reference Coordinates, in which are specified all the rest points) and an eye-point — point, from which the observer is viewing the object. The mapping of the object from WC into NPC is controlled by specifying the view volume in WC and volume in NPC space corresponding to it. Then depth cueing<sup>4</sup> and face culling occur.

The process of creating 2D image on some screen from 3D data is ended by workstation transformation from NPC to a drawable coordinate system. It is specified by defining a 3D viewport in NPC and its corresponding 2D workstation window and is followed by approximation of pixels' colors.

### 3. IMAGE MANIPULATION

As it has already been mentioned above all PHIGS output primitives with drawn area properties and view definitions may be objects of a consequent manipulation. Operations that can be done on created image can be conditionally divided into two classes. The first one consists of actions that do not change the image orientation and position, such as changing various area properties — color, transparency, ambient, diffuse and specular reflection coefficients and so on. Special attention should be payed to the work with light sources, on whose properties lighting and shading of the object depend. Several types of light sources are defined in PHIGS: ambient light source, source of directional light, point light source and a source in the form of a light spot. By activating and/or deactivating light sources one can obtain the desired lighting of the parts of the studied object.

<sup>4</sup>We did not use this technique because we have no color printer at hand while, the black- and slate one gives poor effects

The second class consists of actions that enable one to look at the object from different sides and points of view. In the process of image creating, described in Section 2, the image positioning takes place twice. The first object defined in MC is positioned in WC via modelling transformation. Varying the parameters of the modelling transformation, one can rotate and translate the object while light sources and point of view remain immovable. Then positioning takes place while mapping the object from WC into NPC. This time object is orientated relatively to the fixed point of view. Varying the coordinates of the eye point and the parameters of the projection plane one can look at the object from different points of view<sup>5</sup>. A detailed discussion of what may happen during such actions can be found in, for example, [15]. We used both ways of image manipulating to study our objects.

#### 4. OBTAINED RESULTS AND FUTURE RESEARCH

The results of the study are presented on Figs.2-6. A representation of the world sheet of string with a surface constructed on a set of points (2) was found very suitable considering the future directions of work. As it was shown such world sheet of string representation was obtained with a simple procedure using the supporting curve specification as initial info. So one can consider special output primitive that represents 3D surface and is defined by a curve parametrized in some way to reduce the amount of input data necessary for the image construction. Besides this, a surface specified in such way has a number of features that are useful for its visualization. For example, taking into account the presented procedure of finding a point on the world sheet one can see how easily a normal vector to this surface can be found. This feature may help to optimize the method used for the surface lighting and shading because the determination of a normal in a point of surface is its significant part.

It should be noted that we have concentrated our efforts on drawing only the world sheets of free strings. The topological type of string is known to change in the processes of fision and fusion of the string [12]. Such world sheets have branches. So of special interest is the study of processes mentioned above in their dynamics. It can be done by drawing the world sheet projections from the 4D Minkowski space to 3D space through sectioning by constant time planes. World sheet section by constant time plane in the Minkowski space gives a string at that instant of time. This section is a curve in 3D space. The surface

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<sup>5</sup>Imagine that you are looking at a small cube under a table lamp. You can rotate the cube under the lamp to light its different sides or move your head to look on it from up or down. Two ways described above.

spanned by that curve during its motion is a projection of world sheet to  $3D$  space.<sup>6</sup> Creating the sequences of such sections in time like sequences of the film (time scanning) one can study the motion of string, fission and fusion processes, creation/annihilation and movement of singularities on the string. Such a film as a representation of the world sheet of string can most adequately fit the needs of physical problem.

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<sup>6</sup>It has the sence similar to the trajectory of pacticle.



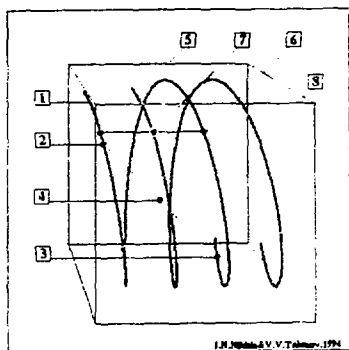


Fig.1. It is an illustration to the procedure of creating the representation of world sheet of string in the form of a set of points. Numbers on the figure correspond to:  $\boxed{1}$  - supporting curve  $Q_\mu(\sigma)$  — one edge of the world sheet;  $\boxed{2}$  and  $\boxed{3}$  - points on supporting curve corresponding to two different values  $\sigma_1$  and  $\sigma_2$ ;  $\boxed{4}$  - the middle of segment connecting points  $Q(\sigma_1)$  and  $Q(\sigma_2)$  — the point on the world sheet and the knot in the frame;  $\boxed{5}$  and  $\boxed{6}$  - points corresponding to  $Q(\sigma)$  and  $Q(\sigma + 2\pi)$  respectively;  $\boxed{7}$  - the middle of segment connecting these two points;  $\boxed{8}$  - the curve that consists of the points obtained similar to  $\boxed{7}$  — it is supporting curve shifted on a half of its period  $2P_\mu$ , the another edge of the world sheet. The box in which these two curves are enclosed here and further is the one that bounds the frame  $\{x_\mu^{ij}\}$ .

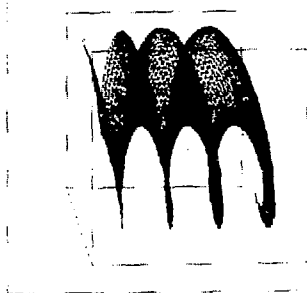


Fig. 2. The simplest picture of the world sheet of linear string: the segments  $[x^{ij}, x^{i+1,j}]$ ,  $[x^{ij}, x^{i,j+1}]$ ,  $[x^{ij}, x^{i+1,j+1}]$ ,  $i = 1, \dots, m - 1$ ,  $j = 1, \dots, n - 1$  connecting the points of frame are drawn. One can see that the parts of this surface are partially overlapping in view but it is hard to understand its exact shape and the style of its deformations.

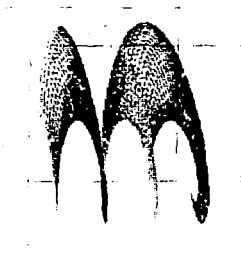


Fig. 3. This picture was drawn similar to Fig.2 (the points of frame are connected with segments) except that we set the distinguishing of front and back planes of the surface, drawing them in different colors. Two planes of the surface were distinguished according to following condition: if a scalar product of  $\vec{n}$  and  $\vec{r}$  (where  $\vec{n}$  is a normal vector to a facet on plane, calculated according to clockwise or counterclockwise rule, and  $\vec{r}$  is a vector from facet to the eye-point) is greater than zero, the plane is front or back in the opposite case. It can be seen how the surface is twisted in horizontal direction (the axis of first coordinate  $\mu = 0$ , which is physically the time) and where we are observing front or back plane of it.



Fig. 4. To confirm the shape of the surface we filled the areas between the segments, connecting points of frame. Front and back planes of the surface are still drawn in different colors but now they look like ordinary 2D planar objects. It is due to the fact that now only one light source of ambient type is activated. It uniformly lights over all parts of the surface so both more and less distant parts of the surface have the same color.

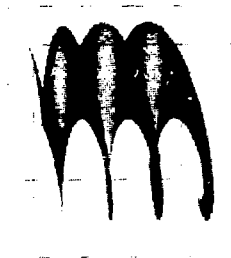


Fig. 5. Final picture — at last the directional light source that lights inside the bounding box from its front plane (we tried different directions of light and have found this the most suitable) is activated. Also the necessary surface properties are set such as the ambient, diffuse and specular coefficients of light reflection as well as the reflectance equation for the surface is specified.

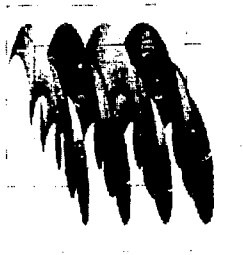


Fig. 6. One additional picture. Here using all the techniques described above we presentate the world sheet of free string with a kink on it. Sample supporting curve that specifies such world sheet is:  $Q_\mu(\sigma) = (\sigma + \frac{\alpha}{2}\sin\sigma, \frac{1}{2}\sin 2\sigma + \frac{\alpha}{2}(\frac{1}{3}\sin 3\sigma + \sin\sigma), -\frac{1}{2}\cos 2\sigma - \frac{\alpha}{2}(\frac{1}{3}\cos 3\sigma + \cos\sigma), 0)$ ,  $\alpha = 0.5$ . Bold line on the picture is the line along which kink is propagating on the surface. We consider such world sheets as the objects of our future study.

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