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ABSTRACT

The non-minimal coupling of gravity to a scalar field can be transformed into a minimal coupling through a conformal transformation. We show how to connect the results of a perturbation calculation, performed around a Friedmann-Robertson-Walker background solution, before and after the conformal transformation. We work in the synchronous gauge, but we discuss the implications of employing other frames.

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1 Introduction

Although the existence of an interaction represented by a scalar field is strongly restricted by the evidences coming from local gravitational effects[1], its cosmological consequences remain object of intensive studies during the last decades. It is possible to imagine many coherent scenarios where a scalar field can have played a major role in the primordial Universe, without contradicting the observational data we have today. This is the case of the Inflationary Scenario, with its many different versions [2]. In particular, there was recently a revival of the prototype of scalar-tensorial theories, the Brans-Dicke theory, since it predicts a power-law behaviour for the scale factor, instead of an exponential one, which can avoid many problems that appear in the traditional inflationary model [3, 4]. However, in order to avoid contradictions with observation, the dimensionless coupling ω that appears in the Brans-Dicke theory must be a function of the scalar field itself, so that it can take small values initially, assuming large values after the inflationary phase, leading to acceptable values for this parameter today ($\omega > 500$) [5].

The possible existence of a scalar field in the primordial universe leads to the important question of how cosmological perturbations behave during the period in which such a field play an important role. This has been the object of many studies recently [6, 7, 8]. However, frequently this calculation is performed in the case where the scalar field is coupled minimally to gravity. In principle, this does not exclude a Brans-Dicke type theory, since this theory can be recast in the form of a minimal coupling through a conformal transformation.

The question we would like to answer here concerns the equivalence of perturbation calculations performed in the minimal coupling (so, after the conformal transformation) with those performed in the non-minimal coupling (before the conformal transformation). The answer to this question depends strongly on the perturbation formalism we employ. Today, we can quote three main formalisms: the gauge invariant formalism, first proposed by Bardeen; the covariant formalism, proposed by Ellis and Bruni; finally, the gauge dependent formalism, which was the first to be used, in the classic papers of Lifschitz and Khalatnikov.

In the case of Bardeen's formalism, the basic variables are generally constructed in the minimal coupling; so, theories with non-minimal coupling can be recast in the standard form by employing a conformal transformation. Anyway, the non-standard variables can be obtained by performing the inverse conformal transformation. As any combination of gauge invariant variables is also gauge invariant, the we remain in the context of this formalism. However, care must be take since the solutions in the Bardeen's formalism are not given in terms of the density contrast, but in terms of intermediate quantities.

Concerning the covariant formalism, the fundamental quantity is the spatial variation of the density contrast, projected in the spatial section at constant time. It comes out that this quantity is already conformal invariant. This can be seen as a positive aspect, but we can also ask what is the really physical meaning of this quantity, since the original theory does not possess conformal invariance, that appear in this formalism at the perturbative level.

The density contrast is, on the other hand, the fundamental quantity in the Lifschitz-Khalatnikov formalism (seen as the prototype of the frame-dependent formalism), which can be seen as a positive aspect. So, we will investigate the consequences of solving the perturbative equations before and after a conformal transformation when we work in a specific frame, which will be the synchronous frame in our case. We find that it can be possible to find the results of the later case from the former ones if the coordinate condition choice is compatible with the conformal transformation itself. So, under such condition, it is possible reobtain the perturbation solutions in the non-minimal coupling from the perturbation solutions in the minimal coupling.

Here, in order to present an explicit exemple, we will consider the case of vacuum solutions. It means that our model we contain just gravity and a scalar field, without any phenomenological matter or any other interaction. We will work with the traditional Brans-Dicke theory, but all the discussion can be extended for any other kind of scalar-tensorial gravity theory.

We organize this paper as follows. In the next section we writte down the field equations in the case of the Brans-Dicke theory before and after the conformal transformation and we determine the background solutions, for a Robertson-Walker metric, showing how to connect them by the inversion conformal transformation. In third section, we study the scalar perturbations in both cases, and we determine the analytical solutions when we employ the synchronous coordinate condition. We show that these solutions can not be connect by a conformal transformation. In the fourth section, we study again the same problem but choosing different coordinate conditions in the minimal and non-minimal coupling cases, which are dictated by the conformal transformation, and we show that in this case we can pass from one case to another by such a transformation. In the fifth section we discuss the question of the residual coordinate freedom, determining which solutions are physical. Finally, in the last section, we discuss the results obtained. In the appendix we consider briefly the case of the tensorial and vectorial modes, which is quite trivial in the exemple considered here.

2 Scalar-Tensorial Gravity Theory and the Conformal Transformation

The Lagrangian of the the Brans-Dicke theory, which can represent with quite of generality the scalar-tensorial theories, is,

$$L = \sqrt{-g}(\Phi R - \omega\frac{\Phi_{;\rho}\Phi^{;\rho}}{\Phi}) \quad . \tag{1}$$

The corresponding field equations are,

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{\omega}{\Phi^2}(\Phi_{;\mu}\Phi_{;\rho} - \frac{1}{2}g_{\mu\nu}\Phi_{;\rho}\Phi^{;\rho}) + \frac{1}{\Phi}(\Phi_{;\mu;\nu} - g_{\mu\nu}\Box\Phi) \quad ; \tag{2}$$

$$\Box\Phi = 0 \quad . \tag{3}$$

Here our conventions are $R_{\mu\nu} = \partial_\lambda\Gamma^\lambda{}_{\mu\nu} - \partial_\nu\Gamma^\lambda{}_{\nu\lambda} + \Gamma^\lambda{}_{\mu\nu}\Gamma^\rho{}_{\lambda\rho} - \Gamma^\lambda{}_{\mu\rho}\Gamma^\rho{}_{\nu\rho}$ and $sig(g) = (+---)$. Inserting in (2,3) the spatially flat Robertson-Walker metric,

$$ds^2 = dt^2 - a(t)^2(dx^2 + dy^2 + dz^2) \quad , \tag{4}$$

we find the following equations of movement for $a(t)$ and $\Phi(t)$:

$$3(\frac{\dot a}{a})^2 = \frac{\omega}{2}(\frac{\dot\Phi}{\Phi})^2 - 3\frac{\dot a}{a}\frac{\dot\Phi}{\Phi} \quad ; \tag{5}$$

$$\ddot\Phi + 3\frac{\dot a}{a}\dot\Phi = 0 \quad . \tag{6}$$

These equations admit the power law solutions,

$$a(t) \propto t^r \quad , \quad r = \frac{1+\omega \pm \sqrt{1+\frac{2}{3}\omega}}{4+3\omega} \quad , \tag{7}$$

$$\Phi(t) \propto t^s \quad , \quad s = 1 - 3r \quad . \tag{8}$$

If we perform a conformal transformation on the metric $g_{\mu\nu}$ [9] such that,

$$g_{\mu\nu} = \Phi^{-1}\tilde g_{\mu\nu}, \tag{9}$$

and suprimming the tilde, we obtain the new Lagrangian:

$$L = \sqrt{-g}(R - (\frac{3}{2}\omega + 1)\frac{\Phi_{;\rho}\Phi^{;\rho}}{\Phi^2}) \quad . \tag{10}$$

From (10) we derive the field equations:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{C(\omega)}{\Phi^2}(\Phi_{;\mu}\Phi_{;\rho} - \frac{1}{2}\Phi_{;\rho}\Phi^{;\rho}) \quad ; \tag{11}$$

$$\Box\Phi - \frac{\Phi_{;\rho}\Phi^{;\rho}}{\Phi} = 0 \quad . \tag{12}$$

We have written $c(\omega) = \frac{3}{2}\omega + 1$. Inserting in these field equations the metric (4), we obtain the movement equations,

$$3(\frac{\dot a}{a})^2 = \frac{\omega}{2}(\frac{\dot\Phi}{\Phi})^2 \quad ; \tag{13}$$

$$\ddot\Phi + 3\frac{\dot a}{a}\dot\Phi - \frac{\dot\Phi^2}{\Phi} = 0 \quad . \tag{14}$$

which admit the solutions,

$$a(t) \propto t^{\frac{1}{3}} \quad , \tag{15}$$

$$\Phi(t) \propto t^n \quad , \quad n = \frac{3}{2}\frac{1}{\sqrt{1+\frac{2}{3}\omega}} \quad . \tag{16}$$

If we call t, a the proper time and the scale factor in the minimal coupling form, and τ, $\bar a$ the proper time and the scale factor in the non-minimal coupling form, we have the relations,

$$\tau = t^{-\frac{n}{2}+1} \quad , \tag{17}$$

$$\bar a = \Phi^{-\frac{1}{2}}a \tag{18}$$

So, using (15,16) and (17,18), we reobtain (7,8).

3 Scalar Perturbations

Now, we will evaluate an analytic expression for the scalar perturbations around the background solutions found above. We will calculate them both in the case of minimal coupling (which is also called Einstein's frame), as in the case of the non-minimal coupling (also called Jordan's frame). All these calculations will be performed in the synchronous gauge [10, 11]. We could also have performed them in another formalism, like the so-called guage invariant formalism, first proposed by Bardeen[12], or the covariant formalism, proposed by Ellis[13]. But, all the problems we would like to point out, can be studied essentially in the the frame-dependent formalism of Lifschitz- Khalatnikov, employing the synchronous coordinate condition. Since the analysis is more simple and direct in this case, we will from the begining fix the synchronous coordinate condition.

In this computation, we will consider that the perturbed quantities behave spatially as plane waves,

$$\delta(t,\vec{x}) = \delta(t)e^{i\vec{q}.\vec{x}} \tag{19}$$

where $\vec{q}$ is the wavenumber of the perturbation. So, all laplacian operatores acting on the three-dimensional spatial section can be replaced by $-q^2$.

3.1 Perturbations in the Minimal Coupling

We write the field equations as,

$$R_{\mu\nu} = C(\omega)\frac{\Phi_{,\mu}\Phi_{,\nu}}{\Phi^2} \quad , \tag{20}$$

$$\Box\Phi - \frac{\Phi_{;\rho}\Phi^{;\rho}}{\Phi} = 0 \quad . \tag{21}$$

We introduce now the perturbed quantities,

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} + h_{\mu\nu} \quad , \tag{22}$$

$$\tilde{\Phi} = \Phi + \delta\Phi \quad , \tag{23}$$

where $g_{\mu\nu}$ and Φ are the background solutions (15, 16) and $h_{\mu\nu}$ and $\delta\Phi$ are small perturbations around them.

Fixing the synchronous coordinate condition $h_{0\mu} = 0$, and defining $h = \frac{1}{2}\frac{h_{kk}}{a^2}$ and $\lambda = \frac{\delta\Phi}{\Phi}$, we obtain, after a quit long calculation, the following expressions linking h and λ:

$$\ddot{h} + 2(\frac{\dot{a}}{a})\dot{h} = 2C\frac{\dot{\Phi}}{\Phi}\dot{\lambda} \quad ; \tag{24}$$

$$\ddot{\lambda} + 3\frac{\dot{a}}{a}\dot{\lambda} + \frac{q^2}{a^2}\lambda = \frac{\dot{\Phi}}{\Phi}\dot{h} \quad . \tag{25}$$

The integration of this equation is not very difficult[14]. First of all, we change for the conformal time, $qx = \int \frac{dt}{a}$. Using (25), we eliminate the terms in h' and h'' in (24), where the primes mean derivative with respect to the conformal time. So, we obtain a third order equation for λ:

$$\lambda''' + \frac{5}{2}\frac{\lambda''}{x} + \Big(1 - \frac{5}{2}\frac{1}{x^2}\Big)\lambda' + \frac{3}{2}\frac{\lambda}{x} = 0 \quad . \tag{26}$$

To derive these equation we have also expressed the solutions (15,16) in terms of the conformal time. To solve (26), we proceed as follows. We write $\lambda = x^{-\frac{3}{2}}\Psi$. With this

definition, the equation (26) reduces to a second order equation. Then, writting $\Psi' = x^{\frac{3}{2}}\gamma$, we reduce this equation to a Bessel's equation of first kind. The final solution, now in terms of the proper time t, is:

$$\lambda = \frac{1}{t}\Big(\int t^{-\frac{2}{3}}[c_1J_1(t^{\frac{2}{3}}) + c_2N_1(t^{\frac{2}{3}})]dt + c_3\Big) \quad , \tag{27}$$

where J_1 and N_1 are Bessel's and Neumann's functions of first kind, and c_1, c_2 and c_3 are constants. As we will see later, the solution represented by the integration constant c_3 can be eliminated by a coordinate transformation and has no physical meaning.

The solution for h can be obtained from (27) and (25). The result is:

$$\begin{aligned} nh = & \int_0^t v^{-2}\Big(\int_0^v (u^{\frac{2}{3}} + \frac{2}{3}u^{-\frac{2}{3}})[c_1J_1(u^{\frac{2}{3}}) + c_2N_1(u^{\frac{2}{3}})]du + c_3\Big)dv + \\ & + \int_0^t \Big(\frac{2}{3}v[c_1J_1'(v^{\frac{2}{3}}) + c_2N_1'(v^{\frac{2}{3}})] - \frac{1}{3}[c_1J_1(v^{\frac{2}{3}}) + c_2N_1(v^{\frac{2}{3}})]\Big)dv + \\ & + c_4 \quad . \end{aligned} \tag{28}$$

3.2 Perturbations in the Non-Minimal Coupling

The field equations (2,3) can be rewritten as

$$R_{\mu\nu} = \frac{\omega}{\Phi^2}\Phi_{,\mu}\Phi_{,\nu} + \frac{1}{\Phi}\Phi_{;\mu;\nu} \quad , \tag{29}$$

$$\Box\Phi = 0 \quad . \tag{30}$$

We will proceed exactly as before in order to calcule the perturbed equations and solve them. But, in order to distinguish the calculations performed before the transformal conformation from those performed after the conformal transformation, we note now τ as the proper time, $H = \frac{1}{2}\frac{h_{kk}}{a^2}$ where now all quantities are calculeted in the Jordan's frame, and a is now given by (7).

Introducing the calculations as in the preceding case, we find the following differential equations linking H and λ:

$$\ddot{H} + 2\frac{\dot{a}}{a}\dot{H} = \ddot{\lambda} + 2(1+\omega)\frac{\dot{\Phi}}{\Phi}\dot{\lambda} \quad , \tag{31}$$

$$\ddot{\lambda} + (3\frac{\dot{a}}{a} + 2\frac{\dot{\Phi}}{\Phi})\dot{\lambda} + (\frac{q}{a})^2\lambda = \frac{\dot{\Phi}}{\Phi}H \quad . \tag{32}$$

We solve these coupled equations in the same way as before. The final results are:

$$\lambda = \frac{1}{\tau}\Big(\int_0^\tau u^{1-r}[b_1J_1(qu^{1-r})b_2N_1(qu^{1-r})]du + b_3\Big) \quad ; \tag{33}$$

$$\begin{aligned} (1-3r)H = & \int_0^\tau \Big([\frac{3r}{1-r}u^{r-2} + (1-r)u^{-r}]\int_0^u v^{1-r}[b_1J_1(qv^{1-r}) \\ & + b_2N_1(qv^{1-r})]dv + b_3\Big)du + \\ & + \int_0^\tau \Big((1-4r)[b_1J_1(qu^{1-r}) + b_2N_1(qu^{1-r})] + \\ & + (1-3r)[b_1J_1'(qu^{1-r}) + b_2N_1'(qu^{1-r})]\Big)du \quad . \end{aligned} \tag{34}$$

4 Connecting the Perturbations

It is possible to verify that the solutions (27,28) do not correspond to the solutions (33,34). The proper time t in the Jordan's frame is related to the proper time τ in the Einstein's frame by the relation,

$$t \propto \tau^{\frac{3}{2s}} \tag{35}$$

where $s = \frac{3}{2}(1 - \frac{r}{2})$. Inserting this relation in (27,28) we see that they correspond to a different solution from (33,34).

This problem arises from our choice of coordinate conditions. To the obtain the above solutions we have imposed the synchronous coordinate condition in the Jordan's frame and in the Einstein's frame. But, this is not compatible with the conformal transformation. Considering the conformal transformation (9), we have for the perturbations,

$$H_{\mu\nu} = -\frac{\lambda}{\Phi}g_{\mu\nu} + \frac{1}{\Phi}h_{\mu\nu} \tag{36}$$

where $H_{\mu\nu}$ is the metric perturbation in the Jordan's frame and $g_{\mu\nu}$ and $h_{\mu\nu}$ are the metric and its perturbation in the Einstein's frame respectively. So, if we impose the synchronous coordinate condition in the Jordan's frame, in order to retain a coherence with the conformal transformation, we find that in the Einstein's frame, we must impose the conditions(which, for simplicity, we call conformal frame, in contrast with the synchronous frame),

$$\begin{aligned} h_{00} &= \lambda \quad , & (37)\\ h_{i0} &= 0 \quad . & (38)\end{aligned}$$

With these conditions, we find the following equations linking h, defined as before, and λ:

$$\begin{aligned} \ddot{h} + 2(\frac{\dot{a}}{a})\dot{h} &= (2C\frac{\dot{\Phi}}{\Phi} - \frac{3}{2}\frac{\dot{a}}{a})\dot{\lambda} + \frac{1}{2}(\frac{q}{a})^2\lambda \quad ; & (39)\\ \ddot{\lambda} + (3\frac{\dot{a}}{a} - \frac{1}{2}\frac{\dot{\Phi}}{\Phi})\dot{\lambda} + (\frac{q}{a})^2\lambda &= \frac{\dot{\Phi}}{\Phi}\dot{h} \quad . & (40)\end{aligned}$$

To solve this equation, we follow the same procedure as before: we pass to the conformal time, and with the help of (40), we eliminate $\dot{h}$ and $\ddot{h}$ in (39). We obtain a third order equation for λ, which, with the help of the background solutions (15,16), can be written as,

$$\lambda''' + (s+1)\frac{\lambda''}{x} + \Big(1 - (\frac{1+s}{x^2})\Big)\lambda' + s\frac{\lambda}{x} = 0 \quad . \tag{41}$$

To solve this equation, first we define $\lambda = x^{-s}\omega$, getting a second order differential equation. Then, defining $\omega' = x^s\gamma$, this equation takes the form of a Bessel's equation of the first kind. The final solution for λ in terms of the proper time t is:

$$\lambda = \frac{1}{t^{\frac{2s}{3}}}\Big(\int_0^t t^{\frac{2s-1}{3}}[c_1J_1(qt^{\frac{2}{3}}) + c_2N_1(qt^{\frac{2}{3}})]dt + c_3\Big) \quad . \tag{42}$$

Using (35) we can now obtain from (42) the solution (33). We can also obtain (34), but it is more easy to show that the equation (39) reduces to (32). This can be done

using the relations,

$$\begin{aligned} H &= \frac{3}{2}\lambda + h \quad , & (43)\\ \tilde{a} &= \Phi^{-\frac{1}{2}}a \quad . & (44)\end{aligned}$$

Under these transformations, which are induced by the conformal transformation itself, we can rewritte the perturbed equations in the Jordan's frame into the perturbed equations in the Einstein's frame and vice-versa.

5 Residual Coordinate Freedom

In spite of the fact that we have already fixed a coordinate condition, it remains a residual coordinate freedom so that some of the modes found above can have no physical meaning[16]. We will consider this problem both in the case of the *no-minimal coupling*, with the synchronous coordinate condition, as in the case of the minimal coupling, with the coordinate conditions that is compatible with the conformal transformation.

5.1 Residual Coordinate Freedom in the Synchronous Frame

By an infinitesimal coordinate transformation of the kind,

$$x^\mu \rightarrow x^\nu\mu + \chi^\mu \tag{45}$$

the perturbed metric changes as,

$$h_{\mu\nu} \rightarrow h_{\mu\nu} + \chi_{\mu;\nu} + \chi_{\nu;\mu} \quad . \tag{46}$$

Defining $\chi^0 = \chi$ and $\chi^i = \theta^{,i}$, employing the synchronous coordinate condition and imposing that this transformation preserves the coordinate condition, we obtain, that

$$h \rightarrow h - \theta_{,k,k} - 3\frac{\dot{a}}{a}\chi. \tag{47}$$

where $h = \frac{1}{2}$ and θ and χ are time-independent functions. A solution for h is non-physical it it can be written as,

$$h = \theta_{,k,k} + 3\frac{\dot{a}}{a}\chi \quad . \tag{48}$$

Inserting this in (32), we obtain,

$$\ddot{\lambda} + (3\frac{\dot{a}}{a} + 2\frac{\dot{\Phi}}{\Phi})\dot{\Phi} + (\frac{q}{a})^2 = -\frac{\dot{\Phi}}{\Phi}\Big((\frac{q}{a})^2 + 3\frac{\dot{a}}{a}\Big)\chi \quad . \tag{49}$$

The general solution for this equation is,

$$\lambda = t^{\frac{2p}{3}}\Big(a_1J_p(qt^{\frac{2}{3}}) + a_2J_{-p}(qt^{\frac{2}{3}})\Big) - \frac{(1-3r)\chi}{t} \quad . \tag{50}$$

So, the mode represented by the integration constant c_3 in (33) can be eliminated by a coordinate transformation and has no physical meaning.

5.2 Residual Coordinate Freedom in the Conformal Frame

If we employ the same coordinate transformation, with respect to the coordinate condition employed in the last section in the conformal frame, we have,

$$h_{00} \rightarrow h_{00} + 2\dot{\chi}_0 \quad , \tag{51}$$

$$h_{0i} \rightarrow h_{0i} + \chi_{0;i} + \chi_{i;0} \quad , \tag{52}$$

$$h_{ij} \rightarrow h_{ij} + \chi_{i;j} + \chi_{j;i} \quad . \tag{53}$$

Remembering that $h_{00} = \lambda$, we obtain the following solution for $h = \frac{1}{2}\frac{h_{kk}}{a^2}$:

$$h = \int \frac{\chi_{,k,k}}{a^2} dt + 3\frac{\dot{a}}{a}\chi \quad , \tag{54}$$

where $\dot{\chi} = -\frac{\lambda}{2}$.

Inserting this in the equation (25) we can determine a general solution for χ:

$$\chi = t^{\frac{r}{2}}\left(a_1 I_{\frac{1}{2}}(qt^{\frac{2}{3}}) + a_2 K_{\frac{1}{2}}(qt^{\frac{2}{3}}) + a_3\right) \quad , \tag{55}$$

where I and K are the modified Bessel's function.

In the same way, returning to λ, we can see that the solution represented by c_3 in (27) can also be eliminated by a coordinate transformation, and it has also no physical meaning. So, all modes calculated in one frame are directly related with the modes calculated in the other frame.

6 Conclusion

A non-minimal coupling between gravity and a scalar field, like in the Brans-Dicke theory, can be put generally in the form of a minimal coupling through a conformal transformation. The motivations to do this can be twofold: we can perform such a transformation for technical reasons, looking for a frame in which the analysis of the problem can be more transparent; or because the physical content of the theory is in the metric field of the minimal coupling and not of the non-minimal coupling, or vice-versa.

Regarding just the background solution, this question is quite trivial, since we can easily pass from one frame to the other. We have showed here that, on the other hand, at the perturbative level, there is no clear equivalence when we perform the calculation in one frame or in the other. We have exemplified this problem in the context of the Brans-Dicke theory. This choice was made due to the great generality of the Brans-Dicke theory, and in fact it can easily be transposed to other scalar-tensorial theories.

We have showed that, in order to permit an equivalence of the calculation in the minimal frame in respect to the non-minimal frame, the coordinate conditions employed must respect the conformal transformation itself. If we choose a coordinate condition in one frame, we must choose another coordinate condition in the other frame, which is related to the first one by a conformal transformation. If we do this properly, then it is possible to pass from one frame to the other coherently at the perturbative level. In particular, physical modes are transported into physical ones, and the non-physical modes are also transported into non-physical modes, as it could be expected.

We have performed our analysis using a particular coordinate condition. But this problem can be also analysed in the case of the so-called gauge invariant formalism[7]. We observe that in [6] it is claimed that the density perturbations calculated in the Ellis formalism is conformal invariant. But we note that the Einstein theory is not gauge invariant. So, it is reasonable to expect that the physical quantities defined in the context of this theory must not also be coformal invariant.

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Appendix: The Tensorial and Vectorial Modes

We will consider now how the tensorial and vectorial modes behave under a conformal transformation. In a matter of fact, when we consider the case of gravity coupled to a scalar field, the solutions are quite trivial for this case: the scalar field has no direct influence on the tensorial and vectorial modes, as we shall see.

a second order tensor can be decomposed into a pure tensorial term, a vectorial term and two scalar terms. The decomposition has the form,

$$h_{ij} = T_{ij} + V_{(i;j)} + S_{;i;j} + \tilde{S}\gamma_{ij} \quad , \tag{56}$$

where T_{ij} is a traceless and transverse tensorial term, V_i is a divergence free term and S, $\tilde{S}$ are two different scalar modes. The derivatives are with respect to the homogenous space section and γ_{ij} is the metric on this homogenous space. This is equivalent to a division into spin components 2. 1 and 0.

To perform the calculations we calculate the perturbed components of the Ricci tensor $\delta R_{\mu\nu}$, and decompose all the functions into tensorial, vectorial and scalar eigenfunctions. Taking the components of the tensorial mode only, considering the properties described above, we obtain the following equation:

$$\ddot{T}_{ij} - \frac{\dot{a}}{a}\dot{T}_{ij} + ((\frac{q}{a})^2 + 4(\frac{\dot{a}}{a}^2)T_ij = 0 \quad . \tag{57}$$

Note that the influence of the scale factor enters only in the behaviour of the scale factor. The solution for this equation is,

$$T_{ij} = x^p(c_1 J_\nu(x) + c_2 J_{-\nu}(x))Q_{ij} \tag{58}$$

where x is the conformal time, c_1 andc_2 are constants, $p = r + \frac{1}{2} + r$, $a \propto x^r$ and Q_{ij} is the tensorial eigenfunction. The result above is valid both in synchronous ($h_{0\mu} = 0$) or conformal ($h_{0i} = 0, h_{00} = \lambda$) gauge .

Concerning the vectorial modes, we have the equation

$$\dot{F}_i - 2\frac{\dot{a}}{a}F_i = 0 \quad , \tag{59}$$

where $F_i = \nabla V_i$, where the operator $\nabla = \gamma^{ij}\partial_i\partial_j$ is defined in the homogenous space. The solution is

$$F_i = ca^2 Q_i \tag{60}$$

c being a constant, a the scale factor and Q_i a vectorial eigenfunction. Again this result is valid in both gauges, the difference of working in the Einstein's or Jordan's frame comming from the behaviour of the scale factor.

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