

DEMANDE D'AUTORISATION
DE COMMUNICATION OU DE PUBLICATION

PUBLICATION - TITRE : JS 6041 N. FR 960.1.197

Auteur (s) :
Nature de la Publication :
Editeurs (CEA, AIEA, OCDE, ...): J100960

COMMUNICATION - TITRE : CALCULATION OF FOUNDATION RESPONSE TO SPATIALLY VARYING GROUND MOTION BY FINITE ELEMENT METHOD

Auteur (s) : F. WANG - F. GANTENBEIN

Titre de la Conférence : "SMIRT 13"

Lieu et Date : PORTO ALEGRE (Brésil) - 13-18 Août 1995

ORGANISATEURS : UNIVERSITE DE RIO GRANDE DO SUL

Date de remise des textes : Mars 1995

Commentaires :

Ce texte a-t-il été déjà publié ? : non

Si OUI : Référence de la publication antérieure :

Cette publication contient-elle, à votre avis, des informations brevetables OUI NON

Chef de Laboratoire : F. GANTENBEIN *J. Gantenbein*

Chef de Service : A. COMBESURE *A. Combescure*

Date de la demande : 15/3/1995

DEMANDE D'AVIS (éventuellement) :

a) - C.P.I./Saclay OUI NON

b) - Chargé de Mission pour les affaires industrielles OUI NON

c) - SYFRA OUI NON

d) - Partenaires concernés (EDF/ FRA, Dpts de la DRN) :
(joindre photocopie) **Date de la Demande :**

e) - Autres unités opérationnelles : **Date de la Demande :**

f) - Autres avis demandés par le Chef du D.M.T. :

Avis du : (le cas échéant)

Date :
les avis sont à envoyer au DMT/DIR

Date d'Arrivée au D.M.T. : 20/3/95 **Décision D.M.T. n°** **Date :**

Le Chef du Département

P.J. : Un texte complet -
Copie : DMT/DOC
DMT/EA
Nota : Copie autorisation + Résumé + Texte à I.N.S.T.N./MIST/CIRST.

ARRIVÉE - CIRST
26 AVR. 95 002901
Circ. : / Cit: MC

**CALCULATION OF FOUNDATION RESPONSE TO SPATIALLY VARYING
GROUND MOTION BY FINITE ELEMENT METHOD**

F. WANG - F. GANTENBEIN

CEA-CEN/SACLAY - DRN/DMT/SEMT/EMSI - 91191 - GIF-SUR-YVETTE Cedex (France)

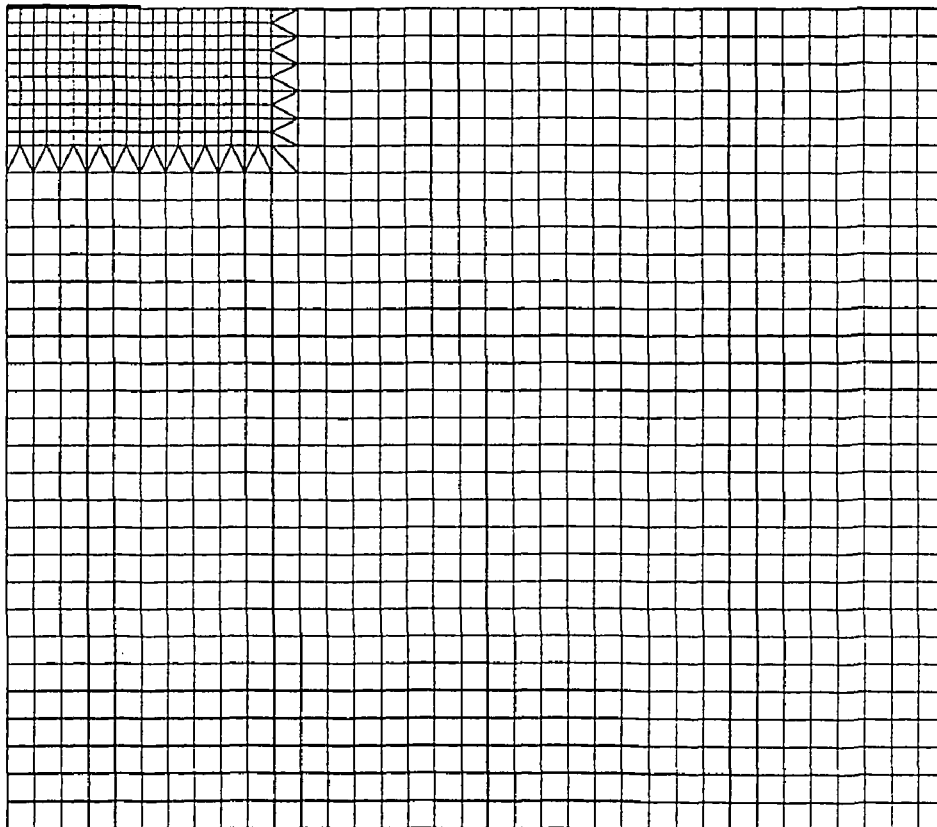
The objective of this work was to validate a general method to compute the response of a rigid foundation of arbitrary shape resting on a homogeneous or multilayer elastic soil subjected to spatially incoherent seismic ground motion.

METHOD -

The response of the foundation is represented by an integral expression in terms of the free-field ground motion and of the contact tractions between the foundation and the soil. The spatial variation of the seismic ground motion is introduced by coherence functions. The contact tractions used in this study are obtained numerically using the finite element method in the process of calculating the dynamic compliance of the foundation.

RESULTS -

The response of a rigid circular foundation resting on the surface of a homogeneous elastic half-space is calculated and compared with the analytical results obtained by J.E. LUCO and A. MITA. Another calculation is carried out using a multilayer soil and an empirical coherence function established from the LSST Lotung array records.



Finite element model

**CALCULATION OF FOUNDATION RESPONSE TO SPATIALLY VARYING
GROUND MOTION BY FINITE ELEMENT METHOD**

F.WANG - F.GANTENBEIN

CEA-CEN/Saclay - DRN/SEMT/EMSI - 91191-GIF-SUR-YVETTE Cedex (France)

ABSTRACT

This paper presents a general method to compute the response of a rigid foundation of arbitrary shape resting on a homogeneous or a multilayered elastic soil when subjected to a spatially varying ground motion. The foundation response is calculated from the free-field ground motion and the contact tractions between the foundation and the soil. The spatial variation of ground motion in this study is introduced by a coherence function and the contact tractions are obtained numerically using the Finite Element Method in the process of calculating the dynamic compliance of the foundation. Applications of this method to a massless rigid disc supported on an elastic half space and to that founded on an elastic medium consisting of a layer of constant thickness supported on an elastic half space are described. The numerical results obtained are in very good agreement with analytical solutions published in the literature.

1. INTRODUCTION

Recent observations of strong ground motions over dense instrument arrays like SMART 1 and LSST Lotung [1] have provided new data about significant spatial variations even within relatively short distances. These variations can have significant effects on the seismic response of large structures such as nuclear power plants. Thus methodologies for incorporating spatial incoherence of ground motion in seismic soil-structure interaction analysis become necessary.

It is the purpose of this paper to present a general method to compute the response of a rigid foundation founded on the surface of an elastic soil when subjected to a spatially varying ground motion. The method is based on a probabilistic approach developed by Luco and Wong [3], and uses the Finite Element Method to carry out the analysis. This allows the method to be applicable to rigid surface foundations of arbitrary shape. The elastic soil which supports the foundation can be homogeneous, multilayered or even geometrically irregular. On the other hand, the method described herein can be easily implemented into any standard finite element code capable of performing dynamic analysis.

Calculation of Foundation response - - -
F. WANG 1

2. METHOD

2.1. Free-field ground motion

The free-field ground motion without foundation is assumed to be a zero-mean-value random field that is stationary in time and homogeneous in space. The second order properties of the field are described by the 3×3 covariance matrix $[B(\bar{x}, \bar{x}', \omega)]$ whose components B_{mn} are assumed to have the form :

$$B_{mn}(\bar{x}, \bar{x}', \omega) = D_{gmn}(\omega) f_{mn}(|\bar{x} - \bar{x}'|, \omega) \quad (m, n = 1, 2, 3)$$

in which $D_{gmn}(\omega)$ is the power ($m=n$) or the cross power ($m \neq n$) spectral density which is independent of the position of the point \bar{x} on the ground surface. The spatial coherence function $f_{mn}(|\bar{x} - \bar{x}'|, \omega)$ for points \bar{x} and \bar{x}' is generally a decreasing function of the distance $|\bar{x} - \bar{x}'|$ and depends on the frequency ω .

2.2. Foundation response

Based on the work of Bycroft [2], Luco and Wong [3], the 6×6 covariance matrix $[D_o^*(\omega)]$ of the response of a rigid massless foundation can be obtained by the following integral representation:

$$D_o^*(\omega) = \iint_{S S} [\Gamma(\bar{x}, \omega)]^T [B(\bar{x}, \bar{x}', \omega)] [\tilde{\Gamma}(\bar{x}', \omega)] dS(\bar{x}) dS(\bar{x}')$$

in which S denotes the contact area, $[\Gamma(\bar{x}, \omega)]$ is a 3×6 contact traction matrix and $[\tilde{\Gamma}(\bar{x}, \omega)]$ its complex conjugate. Each column of the matrix $[\Gamma(\bar{x}, \omega)]$ corresponds to the traction vector at a point \bar{x} on the contact area between the foundation and the soil for unit generalized harmonic forces applied to the rigid foundation at its center in the order $(F_1, F_2, F_3, F_4, F_5, F_6)$ where (F_1, F_2, F_3) and $(F_4, F_5, F_6) = (M_1/a, M_2/a, M_3/a)$ (a is the dimension of the foundation) represent forces and normalized moments, respectively.

The components of the covariance matrix $[D_o^*(\omega)]$ correspond to the power and cross-power spectral density of the foundation response and can be written in the form:

$$D_{opq}^*(\omega) = \sum_{m=1}^3 \sum_{n=1}^3 A_{mn}^{pq}(\omega) D_{gmn}(\omega) \quad , \quad (p, q = 1, 2, \dots, 6; m, n = 1, 2, 3)$$

in which the coefficients A_{mn}^{pq} are defined by

$$A_{mn}^{pq}(\omega) = \iint_{S S} \Gamma_{mp}(\bar{x}, \omega) \tilde{\Gamma}_{nq}(\bar{x}', \omega) f_{mn}(|\bar{x} - \bar{x}'|, \omega) dS(\bar{x}) dS(\bar{x}')$$

It has been shown that the square root of the coefficient $A_{mm}^{pp}(\omega)$ can be interpreted as the amplitude of the transfer function between the m -component of the free-field ground motion and the p -component of the foundation response.

2.3. Calculation of contact tractions $[\Gamma(\bar{x}, \omega)]$ by Finite Element Method

The components of the contact traction matrix $[\Gamma(\bar{x}, \omega)]$ represent the stress distributions at the soil-foundation interface under unit generalized forces and serve as weighting functions in the calculation of the foundation response. They are generally

obtained in the process of calculating the foundation compliance functions. For this purpose, analytical method for circular foundation and Green function method for foundations of arbitrary shape have been successfully used by Luco and Mita [4] and Luco and Wong [3]. In the current study, a rather straight forward procedure is adopted which consists of calculating the contact tractions by Finite Element Method.

A rigid foundation of arbitrary shape and its surrounding elastic soil are modeled by finite elements. The element mesh of the soil and that of the foundation should be identical in the contact area so that contact conditions can be realized by setting displacement identity relations between the corresponding nodes on each side of the interface. Suppose that N is the number of pairs of the corresponding nodes, and \bar{x}_i ($i = 1, 2, \dots, N$) their position. If one can obtain the reaction forces at these nodes $R(\bar{x}_i, \omega)$ for unit generalized forces applied to the foundation, the coefficients A_{mn}^{pq} will be written in the simple form:

$$A_{mn}^{pq} = \sum_{i=1}^N \sum_{j=1}^N f_{mn}(|\bar{x}_i - \bar{x}_j|, \omega) R_{mp}(\bar{x}_i, \omega) \tilde{R}_{nq}(\bar{x}_j, \omega)$$

For the computation of the reaction forces $R(\bar{x}_i, \omega)$ in the frequency domain, Consistent Transmitting Boundary can be used to represent the far field soil which is not included in the finite element soil mesh.

In the time domain, if one is not satisfied with the existing absorbing boundaries which are unfortunately not perfect, accurate reaction forces can be found using the following method: a short impulse of a generalized force $F(t)$ is applied on the foundation and dynamic analysis is performed on the finite element model with free boundary. To do this, the soil model should be large enough to let the foundation response $U(t)$ die out before the waves reflected by the free soil boundary return to the foundation. The computation is stopped when the foundation response vanishes. If the impulse excitation $F(t)$ contains all the required frequencies, the discrete Fourier transform ratio of the displacement $U(t)$ and that of the reaction forces $R'(\bar{x}_i, t)$ to $F(t)$ give the dynamic compliance function $C(\omega)$ and the reaction forces due to a unit generalized harmonic force :

$$C(\omega) = \frac{U(\omega)}{F(\omega)} \qquad R(\bar{x}_i, \omega) = \frac{R'(\bar{x}_i, \omega)}{F(\omega)}$$

3. NUMERICAL RESULTS AND CONCLUSIONS

For the validation of the proposed finite-element-based method, the covariance coefficients A_{mn}^{pq} for a massless rigid disc of radius $a = 10$ m resting on a homogeneous elastic half space have been calculated for a range of frequencies. The half space is characterized by a Poisson's ratio $\nu = 1/3$, a shear wave velocity $V_s = 400$ m/s and a mass density $\rho = 1875$ kg/m³. The coherence function are selected to have the form [3]:

$$f_{mn}(|\bar{x} - \bar{x}'|, \omega) = \exp[-(\gamma \omega |\bar{x} - \bar{x}'| / V_s)^2] \qquad (m, n = 1, 2, 3)$$

where γ is a dimensionless spatial incoherence parameter. The case $\gamma = 0$ corresponds to a motion perfectly coherent with respect to space. The dynamic compliance function

$C(\omega)$ and the reaction forces $R'(\bar{x}_i, t)$ were determined by time domain resolution with the general finite element computer program CASTEM2000. The axisymmetric model used for the analysis is shown in figure 1, where the foundation is divided into 30 elements of equal length. To illustrate the time resolution method described above, the horizontal displacement response to a horizontal force impulse is shown in figure 2. The compliance functions thus obtained (see figure 3) are rather close to analytical solutions found by Veletsos and Wei [5] under relaxed contact conditions (no shear stress at the interface for overturning moment and no normal stress for horizontal force). Numerical values of the amplitude of some of the transfer functions ($\sqrt{A_{11}^{11}} = \sqrt{A_{22}^{22}}, \sqrt{A_{11}^{66}} = \sqrt{A_{22}^{66}}, \sqrt{A_{33}^{33}}, \sqrt{A_{33}^{44}} = \sqrt{A_{33}^{55}}$) with $\gamma = 0, 0.2, 0.5$ are shown in figure 4 versus the dimensionless frequency $a_0 = a\omega/V_s$. We can note the close agreement with the analytical results presented by Luco and Mita [4].

In a similar way, the current procedure is further applied to the same disc foundation supported by a multilayered elastic soil shown in figure 5. The results of this application are presented in figures 4, 6 and 7. As compared with the homogeneous soil, the compliance functions of the disc have changed (see figure 6), but the transfer functions remain almost the same (see figure 4), except for $\sqrt{A_{11}^{55}} = \sqrt{A_{22}^{44}}$ which are shown in figure 7 and seem to be greater for the multilayered soil.

From the above comparison of results, it is concluded that the finite element approach proposed in this study can be used to compute the foundation response to spatially incoherent ground motions. The foundation response thus obtained is the result of the kinematic interaction and can be used as input motion in the analysis of the inertial interaction. The total response of the soil-structure system can then be obtained by the combination of the kinematic and inertial responses.

The work described here was carried out as part of a CEA-EDF-Framatome cooperative joint study.

REFERENCES

- [1] N.A. Abrahamson, J.F. Schneider, J.C. Stepp (1991), "Empirical spatial coherency function for application to soil-structure interaction analysis" Earthquake Spectra, Vol.7, No 1.
- [2] G.N. Bycroft (1980), "Soil-foundation interaction and differential ground motions", Earthquake eng. struct. dyn. Vol 8.
- [3] J.E. Luco, H.L. Wong (1986), "Response of a rigid foundation to a spatially random ground motion", Earthquake Eng. Struct. Dyn. Vol 14.
- [4] J.E. Luco, H.L. Mita (1987), "Response of circular foundation to a spatially random ground motion", Journal of Engineering Mechanics. Vol.133, No 1.
- [5] A.S. Veletsos, Y.T. Wei (1971), "Lateral and rocking vibration of footings", Journal of Soil Mechanics and Foundation Division. ASCE SM9.

Calculations of Foundation response
F. WANG

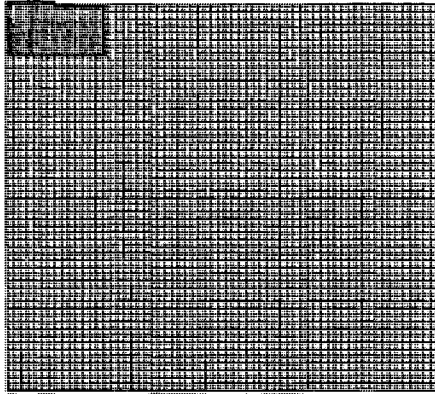


Figure 1. Axisymmetric soil model
disc radius $a = 10$ m, model dimension $= 9a \times 8a$

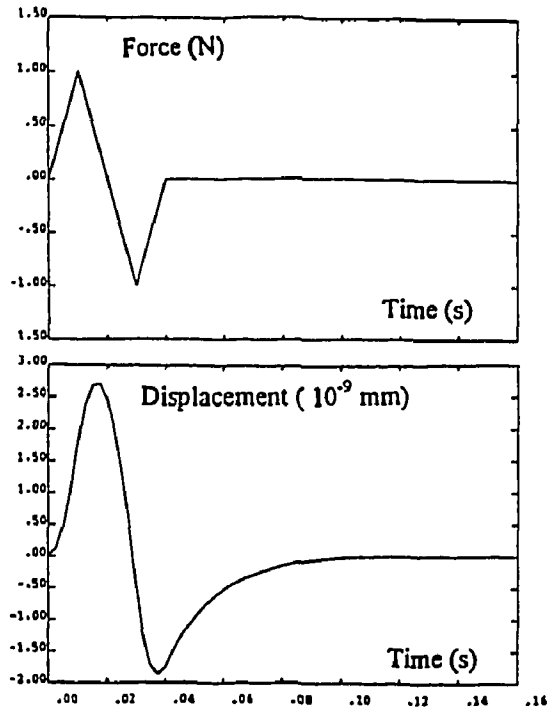


Figure 2. Horizontal excitation and response

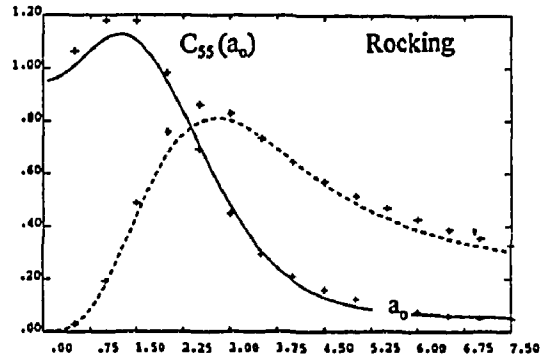
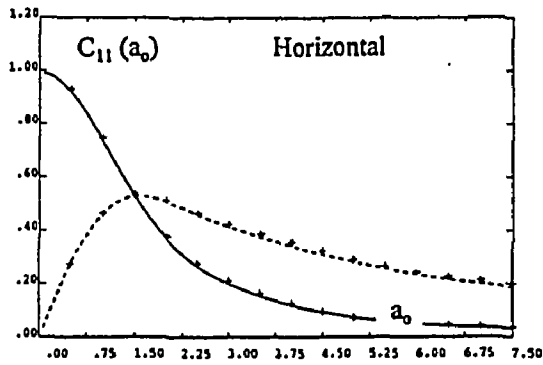
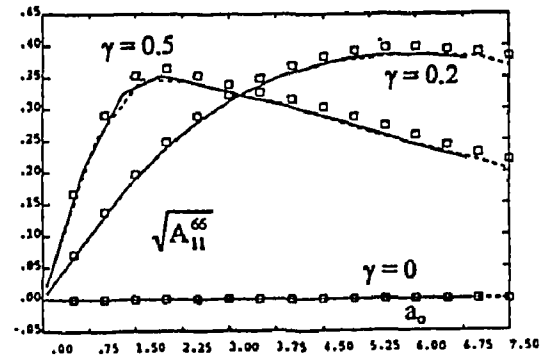
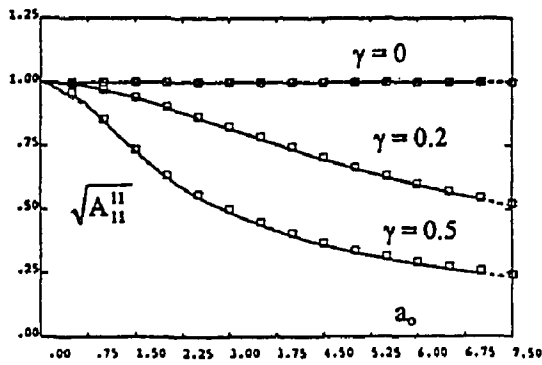


Figure 3. Normalized compliance functions
— Real part, - - - Imaginary part, +++ Veltsos and Wei (1971)



4 (a) Horizontal-horizontal

4 (b) Horizontal-torsion

Figure 4. Transfer functions (to be continued)
□ Luco and Mita, — homogeneous soil, — multilayered soil

Calculations of foundation response ...
F. WANG

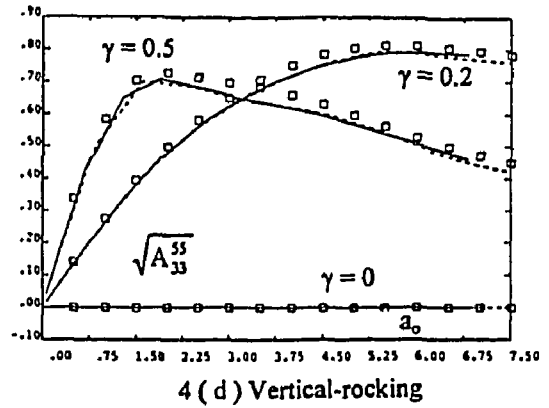
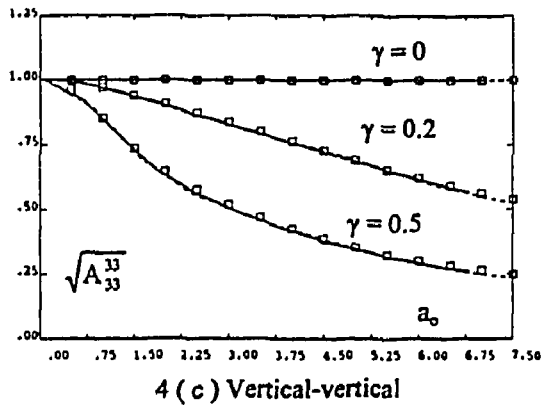


Figure 4. Transfer functions (continued)
 □ Luco and Mita, — homogeneous soil, —multilayered soil

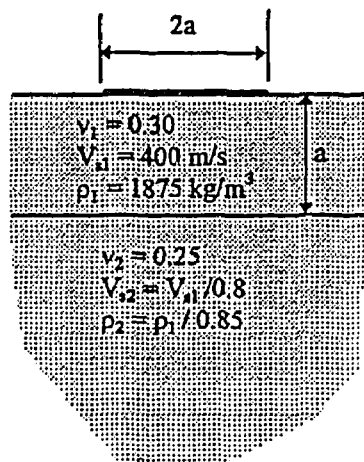


Figure 5. Multilayered soil

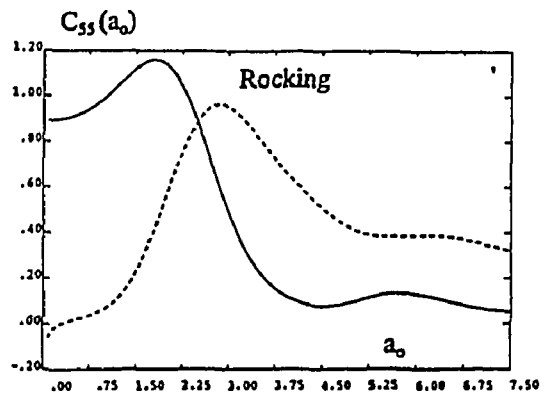
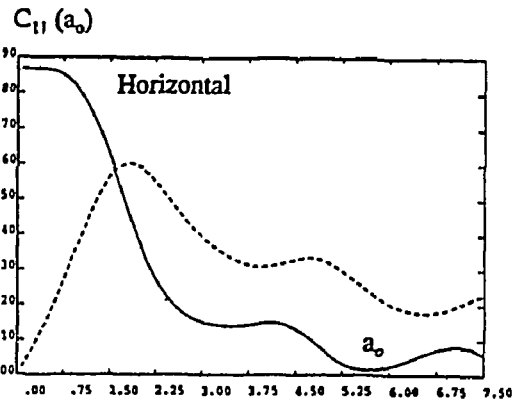


Figure 6. Normalized compliance functions
 — Real part, — Imaginary part
 Multilayered soil

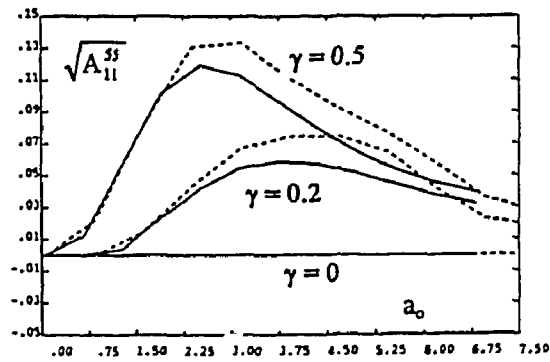


Figure 7. Transfer function : Horizontal-rocking
 — homogeneous soil, —multilayered soil