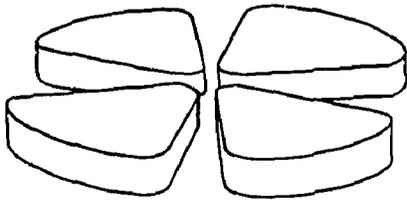


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Does the Ground-State Resonance of ^{10}Li Overlap Neutron Threshold?

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TI: Does the ground-state resonance of ${}^1_0\text{Li}$ overlap neutron threshold?

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Does the Ground-State Resonance of ^{10}Li Overlap Neutron Threshold?

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Abstract: Recent measurements suggest that the ground state of ^{10}Li is a resonance which may well be wide enough to overlap the ($n + ^9\text{Li}$) threshold. In this context we recall some of the curious properties of resonances located near threshold and entered from a non-decay channel, including their asymmetry and the fact that the peak observed in the cross section occurs at neither the R-matrix nor the S-matrix energy, but rather between the two. Because of these and other complications, it does not seem likely that either the ℓ -value of the resonance or the energy of the corresponding state can accurately be determined from the shape of the resonance peak alone.

1. Introduction

A standard experimental technique for measuring properties of a resonant state, if it is not directly accessible from one of its decay channels, is to form it via a 2-body direct reaction (typically transfer) in which the long-lived resonance is itself one of the two final-state partners. In such a “transfer to the continuum”, the resonance is seen as a peak in the spectrum of the other, spectator, particle. A case of current interest is ^{10}Li , all of whose states appear to be neutron-unstable. In particular, its ground state apparently lies less than 500 keV above the ($n + ^9\text{Li}$) threshold^{1,2}). Since it is expected to be a nearly pure single-particle neutron resonance, its width could easily be as large as 500 keV, meaning that the resonance will “overlap threshold” and so have an asymmetric shape. Our purpose in this note is to examine the range of shapes which might be expected, and to ask whether they are sufficiently characteristic to permit a distinction between $\ell = 0$ and $\ell = 1$ states, a question currently under active experimental study.

In analyticity terms, resonances are described by poles in the complex energy plane of the scattering amplitude, and thresholds by \sqrt{E} -type branch points. Thus

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an approximation to the amplitude which describes a resonance near threshold must include both types of singularity, and perhaps the most convenient expression for this purpose is that given by the 1-level R-matrix approximation. If the resonance were entered from a decay channel α , and the resonant term were dominant over any background present, the R-matrix expression for the $\alpha \rightarrow \alpha'$ reaction cross section would be

$$\sigma_{\alpha'\alpha} = \pi\lambda_\alpha^2 \frac{\Gamma_{\alpha'}\Gamma_\alpha}{(E_R - E)^2 + \Gamma^2/4}. \quad (1)$$

Here $\Gamma = \sum_\alpha \Gamma_\alpha$ is the total width, and the energy-dependent partial width Γ_α can conveniently be written as

$$\Gamma_\alpha(k_\alpha) = \Gamma_{\alpha 0} [P_\ell^\alpha(E)/P_\ell^\alpha(E_R)], \quad (2)$$

in which $P_\ell^\alpha(E)$ is the penetration factor, so that $\Gamma_{\alpha 0}$ is the "width at the resonance energy". If E_α is the threshold energy for channel α , $k_\alpha = [2\mu(E - E_\alpha)]^{1/2}/\hbar$ provides the branch point at $E = E_\alpha$, and for S and P waves,

$$P_0^\alpha = x_\alpha, \quad (3a)$$

$$P_1^\alpha = x_\alpha^3/(1 + x_\alpha^2), \quad (3b)$$

where

$$x_\alpha = k_\alpha R_\alpha, \quad (4)$$

and R_α is the channel radius, i.e. the radius of ${}^9\text{Li}$ in the present example. At the energies of the states under consideration here, only the neutron channel is open, so the total width is equal to the neutron width.

The factor of $\lambda_\alpha^2 = 1/k_\alpha^2$ is essentially the inverse of the flux in the entrance channel; it enhances low-energy cross sections and even gives S-wave cross sections a finite (non-zero) value at threshold. However, if the resonance is entered indirectly, as in the case of formation via a transfer reaction, the λ_α^2 factor is not present, and all cross sections vanish at threshold, that for partial wave ℓ rising like $k_\alpha^{2\ell+1}$. Consequently in the present case, with only one channel open, we shall write cross sections for ${}^{10}\text{Li}$ resonances entered indirectly and decaying into $(n + {}^9\text{Li})$ as

$$\sigma(E) = A \frac{\Gamma}{(E - E_R)^2 + \Gamma^2/4}, \quad (5)$$

with

$$\Gamma = \Gamma_\ell(E) = \Gamma_{t0}[P_\ell(E)/P_\ell(E_R)]. \quad (6)$$

Thus just above neutron threshold, $\sigma_0(E) \sim E^{1/2}$ for S-waves and $\sigma_1(E) \sim E^{3/2}$ for P-waves, as they should. The factor A , assumed energy-independent, contains the cross section for the production of ^{10}Li from the entrance channel. The factorization of eq. (5) assumes that the resonance decays only after it is outside the range of other products of the initial reaction. If this distance is taken as, say, 10 fm, this requires the S-matrix width to be less than 20 MeV, which is surely true for the low-lying states considered here.

2. Specific examples for ^{10}Li

Presuming the ^9Li ground state (which is particle-stable) to fill the $1p_{3/2}$ neutron shell, the last neutron in the ^{10}Li ground state should be in either the $1p_{1/2}$ or the $2s_{1/2}$ state, raising the interesting question of whether the ^{10}Li ground state is an S-wave or P-wave resonance^{1,2}). Two recent experiments^{1,2}) attempting to identify low-lying states in ^{10}Li have come to very different conclusions regarding this ground state. Kryger *et al.*¹) produce ^{10}Li by fragmentation of ^{18}O . By detecting ($n + ^9\text{Li}$) coincidences and measuring their relative-energy spectrum, they could, in principle, measure decay energies without the necessity of a threshold-mass determination. Unfortunately, energy-resolution limitations prevented the actual identification of individual resonances, and only permitted the determination of an upper limit on $[E_R + \Gamma_0]$. Assuming the observed ($n + ^9\text{Li}$) events to be decay products of the ground state of ^{10}Li , and that ^9Li is produced in *its* ground state, two possible descriptions of the ^{10}Li ground state which Kryger *et al.* find to be consistent with their data are:

$$\ell = 0, E_{00} = 150 \text{ keV}, \Gamma_{00} = 400 \text{ keV}; \quad (7a)$$

$$\ell = 1, E_{10} = 50 \text{ keV}, \Gamma_{10} = 100 \text{ keV}. \quad (7b)$$

The resonance shapes given by eq. (5) for these two cases are shown in fig. 1. Both resonances seriously overlap threshold. In consequence, each curve peaks well below its corresponding E_R , and each FWHM is only 60% or so of the corresponding Γ_0 .

To understand this behavior, and recall some of the curious features of resonances near a threshold, we show in fig. 2 a series of curves calculated with eq. (5) for a P-wave resonance. $E_R = 400 \text{ keV}$ in all 4 cases; they differ only in having $\Gamma_0 = E_R/10$,

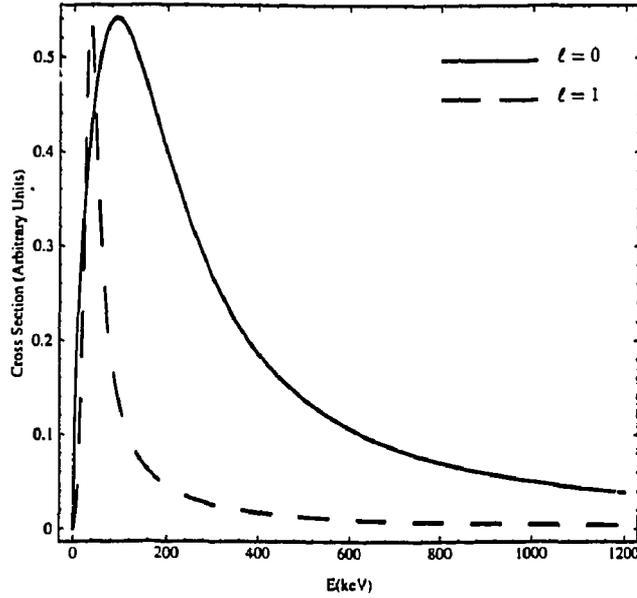


Figure 1: Two resonance shapes for the ground state of ^{10}Li which provide equivalent fits to the data of Kryger *et al.*¹⁾. The solid curve is for $\ell = 0$ and the dashed for $\ell = 1$. $E = 0$ is at the ($n+^9\text{Li}$) threshold.

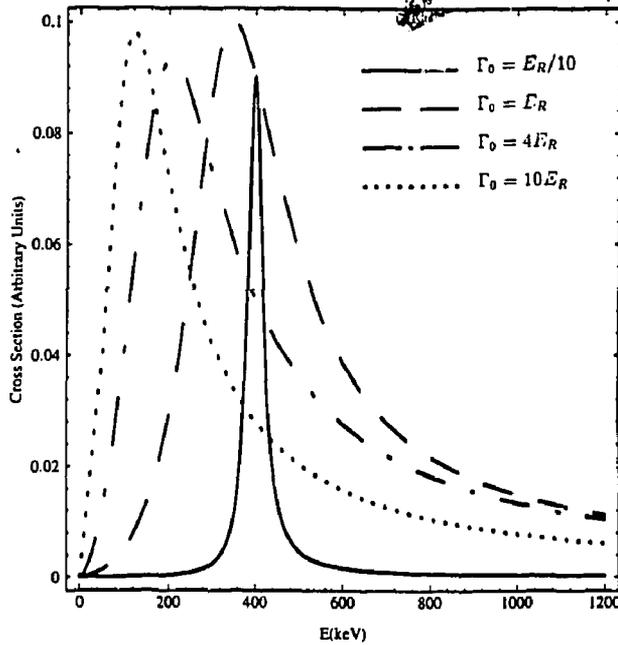


Figure 2: Examples of the way in which the energy of the peak of a P-wave resonance curve depends on Γ_0 as well as on E_R of the R-matrix parametrization of eq. (5). All 4 curves have the same $E_R = 400$ keV, but different values for Γ_0 . Solid curve: $\Gamma_0 = E_R/10$; dashed: E_R ; dash-dot: $4E_R$; dots: $10E_R$.

Table 1: S-matrix pole energies for $\ell = 1$ and $E_R = 400$ keV

Γ_0	Pole Energy (keV)
$E_R/10$	399. – 19.9 <i>i</i>
E_R	313. – 133.4 <i>i</i>
$4E_R$	144. – 133.0 <i>i</i>
$10E_R$	74.7 – 90.2 <i>i</i>

E_R , $4E_R$ and $10E_R$. If $\Gamma_0 < E_R$, the width of the peak measured at half-maximum is approximately Γ_0 , as expected for a narrow resonance, but when Γ_0 is substantially greater than E_R , the resonance overlaps threshold. Since the cross section must vanish at threshold, the curve becomes asymmetric, its FWHM drops well below Γ_0 , and its peak, curiously enough, moves *below* E_R .

It may be of help in understanding this behavior to recall that the shape of the curve is largely determined by the position of the nearest pole of the scattering amplitude (S-matrix pole) in the complex energy plane. For an isolated resonance far from threshold, this pole occurs close to $E = E_R - i\Gamma_0/2$, and produces a cross section of normal Lorentzian shape. However, in general the pole energy is that solution of

$$E = E_R - i\Gamma_0[P_\ell(E)/P_\ell(E_R)]/2 \quad (8)$$

which occurs nearest $E_R - i\Gamma_0/2$. If the E -dependence of $P_\ell(E)$ is significant, this pole appears substantially to the “northwest” of $E_R - i\Gamma_0/2$ in the complex E -plane, as is seen in table 1 for the 4 cases of fig. 2.

What then is the “energy of the state” to be entered in the data tables? Clearly it is the real part E_S of the S-matrix pole energy, which equals the bound-state energy for a state below threshold—but this can only be obtained from a fit of eq. (5) to the data if the ℓ -value is also determined by the fit, since solving eq. (8) for E clearly requires knowing ℓ . What is essential to note is that for resonances near threshold, this E_S is *neither* the E_R of the R-matrix parametrization *nor* the energy of the peak in the cross section. In fact, comparing fig. 2 with the numerical values in table 1, we see that in general the cross section peaks at an energy which is less than the R-matrix energy E_R but greater than the S-matrix energy E_S .

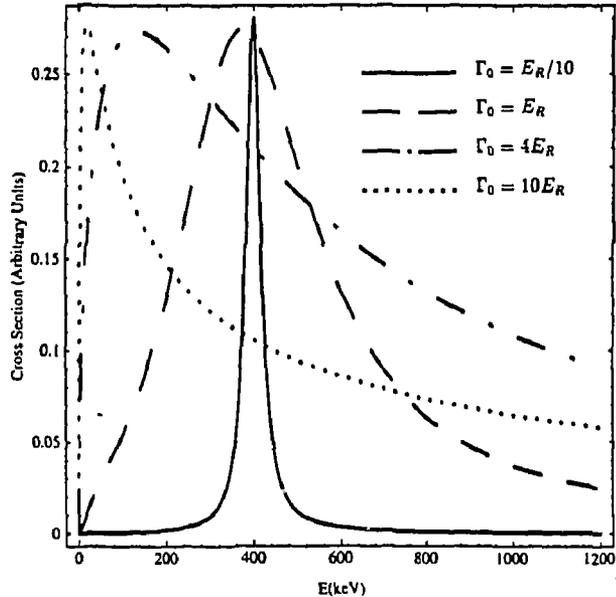


Figure 3: Repeat of fig. 2, for an $\ell = 0$ resonance. $E_R = 400$ keV, and the curves are identified by the same values of Γ_0 as in fig. 2.

In fact, this can become even more problematical in the case of S-wave resonances near threshold, because of the occurrence of “virtual states”, of which the singlet state of the deuteron is the classic example. In the unique case of $\ell = 0$, $\Gamma(E) \sim E^{1/2}$ changes less rapidly than E itself, making it very ‘easy’ for S-states to overlap threshold. The details can be found in ref. ³⁾, but the essential point is that if $\Gamma_0 > 4E_R$, the state becomes “virtual”. The S-matrix pole then lies on the (negative) imaginary axis in the complex k -plane, so its energy $E \sim k^2$ is *negative*, i.e., below threshold (actually between $-E_R$ and 0), even though the state is unbound. In analyticity terms, this pole is on the second Riemann sheet of the energy surface. If the force responsible for the resonance is made more attractive, the energy of the state *increases*, reaching $E = 0$ as the state becomes bound, then decreases to negative (bound) values, but now on the first sheet of the Riemann surface.

Figure 3 shows the shapes produced by eq. (5) for four S-wave resonances with the same E_R and Γ_0 's as used in fig. 2. The corresponding S-matrix pole positions are given in table 2; the last two cases represent virtual states, with energies below threshold, even though the states are unbound and the cross sections peak at positive energies.

Finally, to return to ^{10}Li , the Bohlen group produced it via the transfer reaction

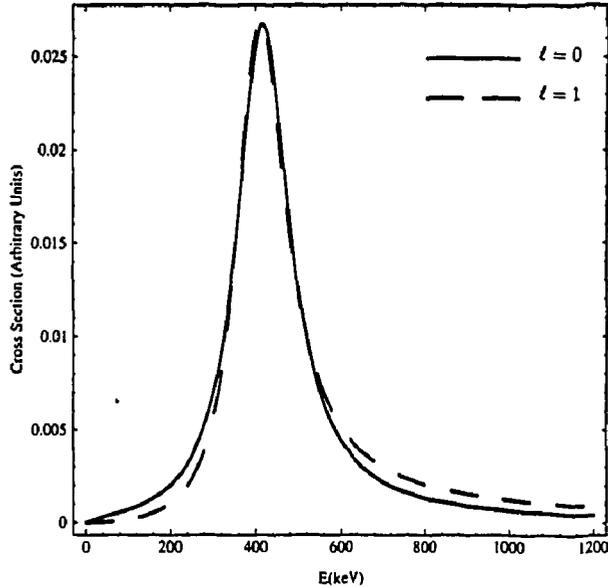


Figure 4: The dashed curve is the resonance given by the parameters of Bohlen *et al.*²⁾ for the ground state of ^{10}Li : $E_R = 420$ keV, $\Gamma_0 = 150$ keV, $\ell = 1$. The solid curve has the same E_R and Γ_0 , but $\ell = 0$.

$^9\text{Be}(^{13}\text{C}, ^{12}\text{N})^{10}\text{Li}$, and they measured the spectrum of the spectator ^{12}N . They observed an asymmetric peak centered at approximately 420 keV, which they interpret as two overlapping resonances: $E_R = 420$ keV, $\Gamma_0 = 150$ keV (ground state of ^{10}Li) and $E_R = 800$ keV, $\Gamma_0 = 300$ keV. Though they have no experimental evidence on the ℓ -values, they favor $\ell = 1$ for both states on shell-model grounds. Figure 4 shows the two curves given by eq. (5) for a state with $E_R = 420$ keV, $\Gamma_0 = 150$ keV, the solid curve being for $\ell = 0$ and the dashed curve for $\ell = 1$. Since in the Bohlen experiment the resonance rides on a 3-body background whose shape and magni-

Table 2: S-matrix pole energies for $\ell = 0$ and $E_R = 400$ keV

Γ_0	Pole Energy (keV)
$E_R/10$	$400. - 20.0i$
E_R	$350. - 194.i$
$4E_R$	$-E_R$
$10E_R$	-17.4

tude can only be conjectured, it would seem very difficult to determine the ℓ -value from the resonance shape alone. A third recent experiment⁴⁾ suggests that the ^{10}Li ground state may lie only slightly higher than 100 keV above neutron threshold, with a width less than 230 keV, of unknown angular momentum.

3. Summary and conclusions

In summary, we have used a number of examples, stimulated by recent experiments on ^{10}Li , to recall the asymmetric shapes and other unusual properties of resonances near a threshold. On the basis of these examples, it would appear to be very difficult to settle the current controversy over the S or P nature of the last neutron in the ^{10}Li ground state on the grounds of line shape alone—or even to know how close the energy of the state is to the energy of the observed peak. If this state does turn out to be an $\ell = 0$ state very close to threshold, it could be extremely interesting, as Thompson and Zhukov have recently pointed out⁵⁾, for this would make ^{11}Li the only known physical example of the Efimov effect, with a large number of S-states clustered near zero excitation energy.

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