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AND THE EQUIVALENCE PRINCIPLE

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**HIGH ENERGY COSMIC NEUTRINOS
AND THE EQUIVALENCE PRINCIPLE**

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ABSTRACT

Observation of ultra-high energy neutrinos, in particular detection of ν_τ , from cosmologically distant sources like active galactic nuclei (AGN) opens new possibilities to search for neutrino flavor conversion. We consider the effects of violation of the equivalence principle (VEP) on propagation of these cosmic neutrinos. In particular, we discuss the two effects: (1) the oscillations of neutrinos due to VEP in the gravitational field of our Galaxy and in the intergalactic space, (2) resonance flavor conversion driven by the gravitational potential of AGN. We show that ultra-high energies of the neutrinos as well as cosmological distances to AGN, or strong AGN gravitational potential allow to improve the accuracy of testing of the equivalence principle by 25 orders of magnitude for massless neutrinos ($\Delta f \sim 10^{-41}$) and by 11 orders of magnitude for massive neutrinos ($\Delta f \sim 10^{-28} \times (\Delta m^2/1eV^2)$). The experimental signatures of the transitions induced by VEP are discussed.

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I. INTRODUCTION

Cosmologically distant objects such as active galactic nuclei (AGN) can be sources of intense high-energy neutrinos [1]. The flux of these cosmic neutrinos is flavor non-symmetric. Models predict τ neutrino flux being at least 3 orders of magnitude smaller than fluxes of electron and muon neutrinos. This opens the possibility of searching for the effects of neutrino flavor transitions over intergalactic distances by detecting cosmic τ neutrinos.

It was suggested recently [2] that large deep underwater neutrino detectors will be able to identify the event induced by τ neutrinos with energies above 1 PeV. The τ neutrinos produce characteristic double-bang events in the DUMAND type detectors. The first bang comes from charged current interaction of a τ neutrino and the second from hadronic decay of the τ lepton. With energies of PeV region the τ tracks have lengths of the order of 100 m and the two bangs are clearly separable. The authors [2] argued that by requiring criterion of greater energy of second bang than the first's the events are essentially background-free, thereby guaranteeing the unambiguous detection of τ neutrinos.

The effects of vacuum oscillations of neutrinos from AGN have been studied [2]. It is estimated that the neutrino flavor mixing and masses can be probed in regions where oscillation probabilities are greater than

$$P \geq (3 - 5) \cdot 10^{-3}. \quad (1)$$

It is expected that the events display the similar anomalous ratio of ν_μ to ν_e as indicated by the atmospheric neutrino observation [3] if it is due to the neutrino oscillation.

In this paper we will consider effects of tiny non-universality in gravitational couplings of neutrinos on the high-energy cosmic neutrinos. Unlike the case of flavor mixing due to masses the expected effects are extraordinarily large thanks to ultra-high neutrino energies and cosmological distances.

Some time ago it has been realized that the non-universality in gravitational couplings of different flavor neutrinos results in neutrino flavor oscillation [4]. It is quite analogous to the well known phenomenon which arises due to neutrino masses and mixing and the effects should be detectable experimentally. It therefore serves as a new tool of exploring possible violation of Einstein's equivalence principle [5-7]. It may also imply nobel mechanism which could solve the puzzles related with astrophysical neutrinos [8-10].

There has been mainly two proposals for appropriate experimental sites for observing such effect; the solar neutrino observation and the long-baseline neutrino oscillation

experiments. The sensitivities expected from the method range from 10^{-14} to 10^{-16} , depending upon how much one can expect for the accuracy of the experiments and how far one can elaborate the analysis. But it appears difficult to go far beyond. It has also been discussed [11] that the arrival time difference between neutrinos and photons from SN1987a gives a bound on violation of the equivalence principle. The authors obtained a modest bound of the order of 10^{-3} .

In this paper we will show that by observing ultra-high energy neutrinos from cosmologically distant sources one can drastically improve the accuracy of testing the equivalence principle. The paper is organized as follows. In Sec. II we will consider gravity-induced oscillations of massless neutrinos and estimate the sensitivity to violation of the equivalence principle (VEP). In Sec. III the gravity effects in the presence of non zero neutrino masses and vacuum mixing are discussed. In particular, we describe the resonance flavor conversion driven by the gravitational potential of AGN. In Sec. IV experimental signatures of the VEP effects are considered. In Sec. V we summarize our results.

II. GRAVITATIONALLY INDUCED OSCILLATIONS OF NEUTRINOS FROM AGN

Let us restrict ourselves to the two-flavor case for simplicity. According to the hypothesis on violation of equivalence principle (VEP) [4] the flavor eigenstates ν_μ and ν_τ are the mixtures of the gravity eigenstates, ν_{2j} , ν_{3j} whose gravitational couplings f_2G and f_3G differ from the Newton constant G ($f_i \neq 1$ at least for one neutrino), and moreover, differ from each other: $f_2 \neq f_3$. By introducing gravitational mixing angle, θ_j , one can write

$$\nu_\mu = \cos\theta_j\nu_{2j} + \sin\theta_j\nu_{3j}, \quad \nu_\tau = -\sin\theta_j\nu_{2j} + \cos\theta_j\nu_{3j} \quad (2)$$

The non-universality of the gravitational couplings can be parametrized as

$$\Delta f = \frac{f_3 - f_2}{\frac{1}{2}(f_3 + f_2)} \quad (3)$$

Thus the second and the third neutrinos feel gravitational fields with slightly different strengths. This leads to the difference in energies of the eigenstates (level energies)

$$V_j \equiv \frac{1}{2}\Delta f E \Phi(x), \quad (4)$$

where E is the energy of neutrino and $\Phi(x) = MG/r$ is the gravitational potential at radius r from an object of mass M in the Keplerian approximation. The difference in

level energies induces a relative phase difference between the second and the third neutrino wave functions which results in neutrino flavor oscillations in the same fashion as in the mass-induced case.

Let us first suggest that neutrinos are massless or have equal masses. As we will show later the matter effect is negligibly small for the task. In this case the propagation of neutrinos has a character of oscillations with the depth determined by θ_j and with the length, l_j , determined by V_j :

$$l_j = \frac{2\pi}{\Delta f E \Phi(x)}. \quad (5)$$

The oscillation probability can be written as

$$P = \sin^2 2\theta_j \sin^2(\Delta f E \int \Phi(x) dx). \quad (6)$$

The distinctive feature of (5) is that the oscillation length is inversely proportional to the energy of neutrinos, in contrast with the case of mass-induced mechanism in which it is proportional to E . The better sensitivity to Δf would be reached if the neutrino energy is higher and the path-length of traversing in gravitational fields is longer. It is for neutrinos from AGN that these conditions are realized. (We note that the gravitational potential $\Phi(x)$ should remain small otherwise the present formulation breaks down). AGN are believed to produce very high energy neutrino spectrum extended to $E \sim 10$ PeV, [1]. They are typically located at the distances $L_{AGN} \sim 100$ Mpc from our Galaxy.

Let us show that in spite of cosmological distances matter effect on the neutrino conversion can be neglected. Consider the $\nu_\mu - \nu_\tau$ system for which matter effect appears in the first order in the weak interactions. The cosmological baryon density estimated by nucleosynthesis is $\rho_B \sim 10^{-31}$ g/cm³. This gives the width of matter in the intergalactic space: $d_{IG} \equiv \rho_B \cdot L_{AGN} \sim 3 \cdot 10^{-5}$ g/cm². According to spheroidal-dark corona models [14] the matter density of our Galaxy is $\rho \sim (1 - 10) \times 10^{-25}$ g/cm³. This leads to the width $d_G \sim 3 \cdot (1 - 10) \times 10^{-3}$ g/cm². Finally, the width of matter in AGN from the region of neutrino production is estimated as $d_{AGN} \sim (10^{-2} - 10^{-1})$ g/cm² [15]. Thus total width, $d_{total} \approx d_{AGN} \sim (10^{-2} - 10^{-1})$ g/cm², is much smaller than the effective width $d_0 \equiv \sqrt{2}\pi m_N/G_F \approx 2 \cdot 10^9$ g/cm² needed for appreciable matter effect. For $\nu_\mu - \nu_\tau$ channel the matter effect appears in high order of perturbation theory and the effective width is even larger.

Let us find the sensitivity to Δf for neutrinos from AGN. As follows from (6) for this we should estimate the integral $I = \int \Phi(x) dx$ along the neutrino trajectory. The integral has three contributions:

$$I = I_{AGN} + I_{IG} + I_G, \quad I_i \equiv \int \Phi_i(x) dx, \quad (i = AGN, IG, G), \quad (7)$$

where $\Phi(x)_{AGN}$, $\Phi(x)_{IG}$, and $\Phi(x)_G$ are the potentials created by AGN itself, by all bodies in intergalactic space between supercluster and us, and by our Galaxy, respectively.

We get $I_{AGN}(r) \sim -\frac{1}{2}R_S \log(r/R_\epsilon)$ for a radial trajectory, where R_S is the Schwarzschild radius of AGN and R_ϵ is the radius of neutrino emission. Using $M_{AGN} \sim 10^8 M_\odot$ as a typical mass of AGN, the former can be estimated as $R_S \sim 3 \times 10^{11}$ m. Thus I_{AGN} is of the order

$$I_{AGN} \sim 10^{-10} \text{Mpc}. \quad (8)$$

The effect in the intergalactic space is dominated by the gravitational field of the so called great attractor [12]. This supercluster is located about $(43.5 \pm 3.5) h_0^{-1}$ Mpc from us, where h_0 is in the range 0.5-1.0. Its mass is about $M_{sc} \sim 3 \times 10^{16} h_0^{-1} M_\odot$, where M_\odot is the solar mass. [13]. The gravitational potential $\Phi(x)$ of supercluster of mass M_{sc} and at distance R can be estimated in the Keplerian approximation as $-5.2 \times 10^{-6} (R/100 \text{Mpc})^{-1} (M_{sc}/10^{16} M_\odot)$. Therefore, the weak-field approximation still applies. The integral I_{IG} satisfies the inequality

$$|I_{IG}| \gtrsim 5.2 \times 10^{-4} \left(\frac{L}{100 \text{Mpc}} \right) \left(\frac{M_{sc}}{10^{16} M_\odot} \right) \text{Mpc} \quad (9)$$

for any trajectories of path length L within radius of 100 Mpc.

For our Galaxy we get, assuming radial trajectory, $I_G = -GM_G \log(L_{AGN}/r)$ under the Keplerian approximation. For the mass $M_G \sim 10^{11} M_\odot$, $L_{AGN} = 100$ Mpc and $r = 10$ kpc this formula leads to $I_G \approx -10^{-7}$ Mpc.

Therefore, the supercluster dominates the gravitational effect: $I \approx I_{IG}$. The reason is that the path-length is of the order of linear dimension of region where gravitational field is effective and it tends to cancel the inverse distance dependence of the Keplerian potential. Therefore, the oscillation probability is essentially governed by the mass of the source of the gravitational field and we have $M_{sc} \gg M_G \gg M_{AGN}$.

Now it is straightforward to make an order-of-magnitude estimation of the sensitivity. For $E = 1$ PeV and I_{IG} ($\Phi = 10^{-5}$, and the distance L of 100 Mpc), one gets

$$|I_{IG} E| > |\Phi E L| = 1.5 \times 10^{41}, \quad (10)$$

which means according to (6) that the phases of oscillation of order unity will be obtained for $\Delta f > 10^{-41}$.

If the great attractor is a fake object then the dominant effect would be due to the gravitational field of our Galaxy: $I \approx I_G$. In this case the expected sensitivity to Δf is

$\sim 10^{-37}$ which still implies a great improvement of sensitivity by more than 20 orders of magnitude. This Δf can be considered as the conservative estimation of the sensitivity.

III. GRAVITATIONALLY INDUCED TRANSITIONS IN THE PRESENCE OF NEUTRINO MASSES

Most probably neutrinos are massive and mixed. The gravitational effects itself can generate via the nonrenormalizable interactions the neutrino masses of the order $v^2/M_P \sim 10^{-5}$ eV [16], where v is the electroweak scale and M_P is the Planck mass. Moreover, there are some hints from solar, atmospheric, cosmological as well as accelerator data that neutrino masses are even larger than that value. Forthcoming experiments will be able to check these hints.

In this connection we will consider the gravitational effects in the presence of neutrino masses and mixing, suggesting that the latter will be determined from these forthcoming experiments with rather good accuracy.

In the presence of vacuum and gravity mixing the effective Hamiltonian of neutrino system in the flavor basis $\nu = (\nu_\mu, \nu_\tau)$ is (see e.g. Ref. [7]):

$$H(x) = \delta \begin{pmatrix} -c & s \\ s & c \end{pmatrix} + V_J \begin{pmatrix} -c_J & s_J \\ s_J & c_J \end{pmatrix}, \quad (11)$$

where $\delta \equiv \Delta m^2/4E$ with $\Delta m^2 \equiv m_3^2 - m_2^2$, $c \equiv \cos 2\theta$, and $c_J \equiv \cos 2\theta_J$, etc. As we have shown in Sec. II the matter effects can be neglected.

From (11) one gets for the mixing angle in medium, θ_m :

$$\tan 2\theta_m = \frac{s\delta + V_J s_J}{c\delta + V_J c_J}. \quad (12)$$

Evidently, for $V_J \gg \delta$ the mixing is determined by gravity mixing: $\theta_m \approx \theta_J$. And for $V_J \ll \delta$ the mixing angle is approximately equal to that of the vacuum mixing: $\theta_m \approx \theta$.

Note that according to (12) mixing angle is zero at

$$V_J^0 = -\delta \frac{s}{s_J}. \quad (13)$$

In the exceptional case $\theta_J = \theta$ one has from (12) $\tan 2\theta_m = \tan 2\theta$. The evolution of the neutrinos is then reduced again to oscillations with the depth $\sin^2 2\theta$ and the oscillation length

$$l_\nu = \frac{2\pi}{\delta + V_j}. \quad (14)$$

The phases due to mass difference and the gravitational effects add up and the latter dominates if $V_j > \delta$, or explicitly, $\Delta f > \Delta m^2/(2E^2\Phi)$. For supercluster potential $\Phi_{sc} \sim 10^{-5}$ and $E = 1$ PeV we get $\Delta f > 5 \times 10^{-36}, 5 \times 10^{-28}, 5 \times 10^{-25}$ for $\Delta m^2 = 10^{-10}, 10^{-2}, 10$ eV², respectively. However, in all these cases the oscillations are averaged and even if Δm^2 will be known it will be impossible to identify the gravity effects.

If $\theta \neq \theta_j$ the Hamiltonian (11) can lead to the resonant flavor conversion due to the change of V_j with distance, the gravity version of the Mikheyev-Smirnov-Wolfenstein (MSW) mechanism [17]. Note that apart from a brief remark in [7] the consideration in the existing literatures is restricted to the level crossing driven by matter density change with distance. Here we describe the level crossing driven by gravitational potential change rather than matter density change. The resonance condition is

$$V_j = -\frac{c\delta}{c_j}. \quad (15)$$

and the resonance value of the potential equals

$$\Phi_R = -\frac{\Delta m^2 \cos 2\theta}{2E^2 \Delta f \cos 2\theta_j}. \quad (16)$$

Correspondingly, V_j in the resonance is $V_j^R = \Delta f E \Phi_R / 2$.

The adiabaticity condition in the resonance reads [7,10]

$$\left| \frac{(\delta \sin 2\theta + V_j \sin 2\theta_j)^2}{\frac{d}{dr} V_j \cos 2\theta_j} \right|_{\text{resonance}} \gg 1. \quad (17)$$

The adiabaticity condition simplifies under the Keplerian approximation. Substituting $dV_j/dr = -V_j/r$ and using the resonance condition (15) we can rewrite (17) as

$$(\tan 2\theta - \tan 2\theta_j)^2 \cos 2\theta_j \gg \frac{1}{r_R \delta}, \quad (18)$$

where r_R is the radius at which the resonance condition is fulfilled: $r_R = r_R(\Delta f)$.

For fixed Δm^2 and $c \approx c_j \approx 1$ (namely, for not very small angle factors in (18)), the minimal and maximal values of the potential Φ determine, via the resonance condition, the range of Δf for which the resonance conversion may take place. In turn, Δf and the corresponding r_R give the lower bound on mixing angles through the adiabaticity condition (18).

If vacuum mixing angle satisfies the adiabaticity condition alone, i.e. $\sin^2 2\theta \approx \tan^2 2\theta \gg (r_R \delta)^{-1}$, then θ_j can be arbitrarily small. The role of the gravitational effect is reduced in this case just to level splitting. On the contrary, for $\theta = 0$ one has the lower bound on the gravitational mixing

$$\sin^2 2\theta_j \gg \frac{1}{r_R \delta}. \quad (19)$$

For fixed mixing angles the adiabaticity condition can be rewritten as the lower bound on Δf . Indeed, substituting in (18) δ from the resonance condition (15) we find

$$\Delta f \gg \frac{2 \times 10^{-33}}{(\tan 2\theta - \tan 2\theta_j)^2 \cos 2\theta_j} \left(\frac{E}{1 \text{ PeV}} \right)^{-1} \left(\frac{M_{AGN}}{10^8 M_\odot} \right)^{-1}. \quad (20)$$

If the adiabaticity condition is fulfilled then the transition probability is determined by the initial and final mixing:

$$P_a = \frac{1}{2} (1 - \cos 2\theta_i \cos 2\theta_f). \quad (21)$$

In this connection let us consider a dependence of mixing angle on the potential Φ or level splitting $V_j = \Delta f \Phi E / 2$. The θ_m as the function of Φ crucially depends on the sign of $\Delta m^2 \Delta f$, and on whether $\theta_j > \theta$ or $\theta_j < \theta$. We focus first on the resonant channel, $\Delta m^2 \Delta f > 0$, and consider these two cases separately.

1). $\theta_j < \theta$. In this case $|V_j^0| > |V_j^R|$. At $|V_j^i| \gg |V_j^R|$, the gravity mixing dominates and $\theta_m \approx \theta_j + \pi/2$. With diminishing $|V_j|$, θ_m decreases and becomes zero at $V_j = V_j^0$. It crosses the resonance at $\theta_m = \pi/4$ and then approaches $\theta_m \approx \theta$ at $|V_j| \ll |\delta|$.

2). $\theta_j > \theta$. Now $|V_j^0| < |V_j^R|$. At $|V_j^i| \gg |V_j^R|$, $\theta_m \approx \theta_j - \pi/2$. Then, with diminishing $|V_j|$ θ_m increases and crosses resonance when $\theta_m = -\pi/4$. At $V_j = V_j^0$ the angle θ_m vanishes so that $\sin^2 2\theta_m = 0$, and then θ_m approaches vacuum value θ .

In the nonresonant channel, movement of the angle θ_m is simpler. Under the same variation of V_j as above it starts from $\theta_m = \theta_j$ and ends up with θ without crossing zero irrespective of the relative magnitudes of θ_j and θ .

Suppose that the initial potential (the potential at the production point) is much larger than the one at resonance, $\Phi_i \gg \Phi_R$, and the final potential is much smaller than the value at resonance, $\Phi_f \ll \Phi_R$. In this case the transition probability in the adiabatic approximation (21) becomes:

$$P_a = \frac{1}{2} (1 \pm \cos 2\theta_j \cos 2\theta), \quad (22)$$

where the plus sign is for resonant channels ($\Delta m^2 \Delta f > 0$), and the minus sign is for nonresonant channels ($\Delta m^2 \Delta f < 0$).

Let us mark one interesting feature related to zero in mixing at $V_j = V_j^0$ (13). If the initial (final) potential is such that $V_j = V_j^0$ for $\theta_j < \theta$ ($\theta_j > \theta$), then the transition

probability in the resonance channel reduces to $P = \cos^2 \theta$ ($P = \cos^2 \theta_j$) as in the case of conversion in matter.

Let us consider the flavor transitions of the neutrinos from AGN using the results (15) - (22). Following the scenarios described in [1], we assume that neutrinos are produced within the region located at the distance $R_e = (10 - 100)R_S$ from the center of AGN, here R_S is the Schwarzschild radius: $R_S \simeq 3 \times 10^{11} (M_{AGN}/10^8 M_\odot)$ m. For radii larger than the neutrino production point we may use the Keplerian approximation for the potential of AGN. The total potential probed by neutrinos on the way to the Earth is

$$\Phi(r) \approx \Phi_{AGN}(r) + \Phi_{IG} = \Phi_{AGN}^0 \left(\frac{R_e}{r} \right) + \Phi_{IG}. \quad (23)$$

Here

$$\Phi_{AGN}^0 \simeq -5 \times 10^{-3} \left(\frac{M_{AGN}}{10^8 M_\odot} \right) \quad (24)$$

is the AGN potential at the neutrino production point, and for simplicity we take the potential in the intergalactic space to be constant: $\Phi_{IG} = 10^{-5}$. Therefore, at the neutrino production point Φ_{AGN}^0 dominates over the supercluster and the galactic effects: For AGN located about 100 Mpc from us it is about 3 and 7 orders of magnitude larger than the potentials of Great attractor and the Milky way galaxy, respectively. We will neglect the potential of our Galaxy.

For fixed Δf the dependence of the transition probability on the neutrino energy (or $E^2/\Delta m^2$) is the following. For small energies the mass-induced vacuum oscillation effect dominates:

$$P \approx \frac{1}{2} \sin^2 2\theta \quad \text{for} \quad \frac{E^2}{\Delta m^2} < \frac{1}{2\Phi_{AGN}^0 \Delta f}. \quad (25)$$

For larger energies the transition probability is described by P_a in (22), if the adiabaticity condition is fulfilled:

$$P \approx P_a \quad \text{for} \quad \frac{E^2}{\Delta m^2} > \frac{1}{2\Phi_{AGN}^0 \Delta f}. \quad (26)$$

Moreover, in the region where AGN potential dominates over IG potential the resonance conversion takes place (in resonant channels). This corresponds to

$$\frac{1}{2\Phi_{AGN}^0 \Delta f} < \frac{E^2}{\Delta m^2} < \frac{1}{2\Phi_{IG} \Delta f}. \quad (27)$$

Here the transition probability $P > 1/2$ and can be close to 1. Note that with diminishing the potential of supercluster the region of the resonance effect increases. Moreover, if the

$\Phi_{IG} < 10^{-7}$ the resonance conversion may take place in the gravitational field of our Galaxy.

For much larger energies the mass splitting can be neglected and the dominant effect is due to gravitationally induced oscillations:

$$P \approx \frac{1}{2} \sin^2 2\theta_j \quad \text{for} \quad \frac{E^2}{\Delta m^2} \gg \frac{1}{2\Phi_{IG} \Delta f}. \quad (28)$$

Once the adiabaticity condition is satisfied in the AGN gravitational field the transition probability (22) applies also to the nonresonant channels.

We note that the resonance flavor conversion is effective thanks to the fact that the mass of AGN is concentrated in a small region of space since it is dominated by its central black hole. Since our Galaxy is more massive than the typical AGN one might suspect that once converted neutrinos at around AGN could be reconverted by our Galaxy's gravitational field. It does not occur for $\Phi_{IG} \sim 10^{-5}$: the gravitational potential of the Galaxy is at most $\Phi_G \sim 10^{-7}$ because its mass is extended to ~ 10 kpc.

Let us consider the sensitivity to Δf suggesting as in [2] that future underwater experiments will be sensitive to flavor transition with probability as small as in (1). (This corresponds to mixing parameter in the region of averaged oscillations $\sin^2 2\theta \sim 10^{-2}$). Let us denote the average energy of the detected neutrinos \bar{E} . Then, we can distinguish three ranges of Δf corresponding to three energy regions defined in (25), (27), and (28). *Region I:*

$$\Delta f < \Delta f_{AGN}, \quad (29)$$

where

$$\Delta f_{AGN} = 10^{-30} \times \left(\frac{\Delta m^2}{10^{-2} eV^2} \right) \left(\frac{\bar{E}}{1 \text{ PeV}} \right)^{-2} \left(\frac{R_e}{100 R_S} \right) \left(\frac{M_{AGN}}{10^8 M_\odot} \right)^{-1}. \quad (30)$$

The gravity does not play role and the effect of mass-induced vacuum oscillation dominates.

Region II:

$$\Delta f_{AGN} < \Delta f < \Delta f_{IG}, \quad (31)$$

where

$$\Delta f_{IG} = 5 \cdot 10^{-28} \times \left(\frac{\Delta m^2}{10^{-2} eV^2} \right) \left(\frac{\bar{E}}{1 \text{ PeV}} \right)^{-2} \left(\frac{10^{-5}}{\Phi_{IG}} \right). \quad (32)$$

Here neutrinos undergo resonance conversion driven by a potential of the AGN and the transition probability can be close to 1 if both θ and θ_j are small.

Region III:

$$\Delta f \gg \Delta f_{IG} . \quad (33)$$

Here gravitational effect dominates and neutrinos oscillate due to VEP. The probability converges to $P = \frac{1}{2} \sin^2 2\theta_j$.

Suppose that the angle factor in (20) is of order unity. The region of Δf which satisfies the adiabaticity condition is then given by $\Delta f \gg 2 \times 10^{-33}$. If $\Delta m^2 \gtrsim 10^{-4} \text{ eV}^2$ the adiabaticity holds in whole region where the resonance condition is met, as one can see from (30). If $10^{-7} \text{ eV}^2 \lesssim \Delta m^2 \lesssim 10^{-4} \text{ eV}^2$ the adiabaticity region partially overlaps with the resonance region. If $\Delta m^2 \lesssim 10^{-7} \text{ eV}^2$ there is no overlapping between two regions and the resonance conversion does not occur.

IV. OBSERVATIONAL SIGNATURE OF GRAVITY EFFECTS

Let us now discuss possibility of identifying the gravity effect. If neutrinos are massless or degenerate, an excess signals for τ neutrinos in underwater installations would imply evidence for neutrino flavor transition not due to mass-induced neutrino oscillation. The gravity effect discussed in this paper would serve as a good candidate mechanism for such a transition; the observation of ν_τ may testify for violation of the equivalence principle with parameter $\Delta f > 10^{-41}$ and $\sin^2 2\theta_j \gtrsim 0.01$.

The problem is: how to prove this? There exists a clear signature for that the gravity is responsible for such ν_τ events. In fact, underwater experiments will give also the ratio of number of events induced by ν_τ and ν_μ : we will denote it ν_τ/ν_μ . Also ratios involving events induced by electron neutrinos like ν_e/ν_μ will be measured. We note that the dominant gravitational effect is due either to the supercluster potential if it exists, or to our Galaxy if it does not. Then, one can expect asymmetry of the ν_τ/ν_μ (as well as ν_e/ν_μ) ratio either between toward and away the supercluster, or with respect to the right ascension. If observed, it would provide a strong evidence in favor of the gravitational oscillation effect.

One might wonder if the asymmetry can be completely wiped out by superposing the contributions from various AGN. It is not the case. Suppose that the oscillation length is, for example, of the order of 100 Mpc. Roughly speaking, neutrinos from AGN at much far from the distance do not contribute to the asymmetry because of averaging of oscillations. Neutrinos from AGN much closer to us also do not contribute because there is no enough space to oscillate. Therefore, only the AGN's located at the distance comparable with oscillation length contribute to asymmetry.

In the case of Galaxy, the asymmetry may appear if supercluster potential is weaker than the galactic one and if the oscillation length is comparable with galactic scale.

If an excess of ν_τ events will not be found, then this will allow one to exclude the region of parameters $\sin^2 2\theta_j > 10^{-2}$, $\Delta f > \Delta f_{AGN}$ (see Eq. (30)), where for Δm^2 in Δf_{AGN} one should take the upper experimental bound on neutrino mass difference.

As we have discussed in Sec. III most probably neutrinos are massive and mixed, and moreover there is a good chance that forthcoming experiments will measure Δm^2 and $\sin^2 2\theta$. In the massive neutrino case the signature of the gravity effect is quite different from that of the massless case. Basic feature of the signal is the deviation of the observed ratio ν_τ/ν_μ 's (as well as ν_e/ν_μ) from the one stipulated by vacuum oscillations. Gravity induced mixing can both suppress and enhance ν_τ -signal.

If Δf takes the value in the Region II (31) the gravitational MSW effect occurs in the resonant channel. The resonant conversion of ν_μ to ν_τ will have the probability $P > 1/2$. Therefore, observation of the ratio ν_τ/ν_μ 's larger than 1 would provide clear evidence for the gravitational MSW effect.

The asymmetry between toward and away the supercluster does not prevail the case of gravitational MSW effect occurring for AGN neutrinos.

In nonresonant channel the gravity effect is described by the transition probability (22) with minus sign. For $\theta_j > \theta$ ($\theta_j < \theta$), the gravity effect enhances (suppresses) the transition. The modification is largest if the vacuum angle θ is small and the gravity angle is large, $\theta_j \sim \frac{\pi}{4}$.

If Δf falls into Region III (33) the signal would mimic the one due to vacuum flavor oscillation. But, since we assume that the masses and vacuum mixing angles will be determined by future experiments, the difference between θ_j and θ should show up in the measured ratio of ν_τ to ν_μ and would be observable. In Region III the dominant gravitational effect is due to the supercluster potential. Then one can expect asymmetry of the $\nu_\tau - \nu_\mu$ ratio between toward and away the supercluster if the averaging effect is weak.

Let us discuss experimental signatures and estimate the sensitivity region for VEP parameters for two probable scenarios of neutrino masses and mixing which will be checked by forthcoming experiments.

1. The heaviest neutrino (which practically coincides with ν_τ) has the mass in the cosmologically interesting region: $m_3 = (3 - 7) \text{ eV}$, so that $\Delta m^2 = (10 - 50) \text{ eV}^2$ in suggestion of mass hierarchy. For these values of masses the experimental bound on mixing angle is

$\sin^2 2\theta < 5 \cdot 10^{-3}$. We will suggest also that mixing with electron neutrinos is negligibly small.

In this case the effect of mass-induced vacuum oscillation can be neglected. Measuring the ratio $\nu_\tau/\nu_\mu > 0.01$ in the detectors would then be an indicator of the mechanism of flavor conversion. Observation of the large ratio, $\nu_\tau/\nu_\mu \gtrsim 1$, would provide a clear evidence for the gravitational MSW effect. From the resonance condition we find the resonance region for Δf :

$$\Delta f = (10^{-27} - 5 \times 10^{-25}) \left(\frac{\Delta m^2}{10 \text{ eV}} \right)^{-1}. \quad (34)$$

Then for fixed Δf the adiabaticity condition gives the bound on mixing angles

$$(\tan 2\theta - \tan 2\theta_j)^2 \cos 2\theta_j \gg 5 \times 10^{-9} \left(\frac{\Delta f}{5 \times 10^{-25}} \right)^{-1} \left(\frac{\Delta m^2}{10 \text{ eV}} \right)^{-1}. \quad (35)$$

If lepton mixing is similar to quark mixing then one expects $\sin^2 2\theta > 10^{-3}$ for $\nu_\tau - \nu_\mu$. In this case the adiabaticity condition is satisfied by vacuum mixing alone and gravitational angle can be arbitrarily small.

Non-observation of such signal will allow one to exclude the whole region of parameters (34) and θ_j not too close to θ .

Observation of a moderately-large ratio, $0.01 < \nu_\tau/\nu_\mu < 1$ would be the signal for one of the following three possibilities; the nonadiabatic gravitational MSW mechanism (in resonant channel), or the nonresonant adiabatic conversion with transition probability $P_a \approx \sin^2 \theta_j$ (in nonresonant channel), or oscillations in the intergalactic gravitational field with $\sin^2 2\theta_j > 0.01$. In the first case, Δf should take the value around (34) and $\theta_j \approx \theta$ to violate the adiabaticity which is well satisfied by vacuum mixing alone. In the other two cases, the region $\sin^2 2\theta_j \gtrsim 0.01$ will be probed.

2. The heaviest mass is $m_3 \sim 0.1 \text{ eV}$ and $\nu_\mu - \nu_\tau$ mixing is large: $\sin^2 2\theta \sim 0.5 - 1$, so that $\nu_\mu - \nu_\tau$ oscillations solve the atmospheric neutrino problem.

For neutrinos from AGN one predicts then the ratio $\nu_\tau/\nu_\mu \leq 1$. The observation of larger ratio: $\nu_\tau/\nu_\mu \gtrsim 1$, signaling an almost complete adiabatic conversion of ν_μ to ν_τ will be clear signature for the gravitational MSW effect. Typical interval of the VEP parameter are

$$\Delta f = 10^{-30} - 5 \times 10^{-28}. \quad (36)$$

The adiabaticity condition is satisfied by large vacuum mixing angle, and θ_j can be arbitrarily small.

In the nonresonant channel the gravitational effect manifests in a different manner. If Δf is in Region II (31): $\Delta f > 10^{-30}$, and the gravitational mixing angle θ_j is smaller than the vacuum angle θ , the gravitational effect suppresses neutrino transition. Namely, the ν_τ signal will be smaller than the one expected for mass-induced vacuum oscillations. Using Eq.(22) with $\theta_j \ll 1$ we find transition probability $P = \sin^2 \theta$ instead of $P = \frac{1}{2} \sin^2 2\theta$ in the absence of VEP. In such a way the transition probability is reduced by 40% and 30% for $\sin^2 2\theta = 0.6$ and 0.8, respectively.

In Region III (33) the averaged transition probability equals $P = \frac{1}{2} \sin^2 2\theta_j$ and for small θ_j suppression can be much stronger, so that ν_τ signal will not be observable at all.

V. CONCLUSION

1. Cosmological distances ($\sim 100 \text{ Mpc}$) and ultrahigh energies ($\sim 1 \text{ PeV}$) of neutrinos from AGN open unique possibility to improve an accuracy of testing the equivalence principle by 11 - 25 orders of magnitude.
2. For massless neutrinos VEP can induce the oscillations $\nu_\mu - \nu_\tau$ of cosmic neutrinos which may lead to an observable ν_τ signal in the large underwater installations. The sensitivity to the parameters of the VEP can be estimated as $\Delta f \gtrsim 10^{-41}$ and $\sin^2 2\theta_j > 2 \cdot 10^{-2}$. In the case of the nonaveraged oscillations (Δf at the lower bound) one can expect an anisotropy of the ν_τ signal correlated to the position of the Great Attractor or with orientation of our Galaxy.
3. In the case of massive neutrinos the gravitational effects due to VEP can modify vacuum oscillations. For certain values of the parameters neutrinos may undergo the resonance flavor conversion driven by the gravitational potential of AGN (or our Galaxy, if the intergalactic potential is sufficiently small). We have found that the gravitational effects become important if $\Delta f \gtrsim \Delta f_{AGN} \sim 10^{-28} (\Delta m^2 / 1 \text{ eV}^2)$. For $\Delta f \sim (1 - 10^3) \Delta f_{AGN}$ one may expect the resonance conversion - almost complete transition of ν_μ to ν_τ . In this region a strong observable effect may exist for arbitrarily small $\sin^2 2\theta_j$. For $\Delta f \gg (1 - 10^3) \Delta f_{AGN}$ neutrinos undergo the gravity induced oscillations and vacuum mixing effect can be neglected.
4. The VEP effects can be identified if the gravitational mixing differs appreciably from vacuum mixing. The ratio $\nu_\tau/\nu_\mu > 1$ is the clear signature of the resonance conversion. In general, VEP effects will manifest itself as deviation of the observed signals like ratios ν_τ/ν_μ and ν_μ/ν_e from those expected by vacuum oscillations. Of course, the latter can be predictable only if neutrino masses and mixing are determined by forthcoming neutrino

experiments. For small vacuum mixing VEP can enhance the $\nu_\mu - \nu_\tau$ transition and the ν_τ -signal. On the contrary, for large vacuum mixing VEP can lead to suppression of the ν_τ -signal.

5. If deviation from vacuum oscillation effects will not be found then one will be able to exclude very large new region of the VEP - parameters.

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