WSRC-TR-96-0064
CONF-960706--26

Calculations of Slurry Pump Jet Impingement Loads (U)

by
T. T. Wu
Westinghouse Savannah River Company
Savannah River Site
Aiken, South Carolina 29808

A document prepared for 1996 ASME/ICPVT PRESSURE VESSEL AND PIPING CONFERENCE at Montreal from 07/21/96 - 07/26/96.

DOE Contract No. DE-AC09-89SR18035

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T. T. Wu
Westinghouse Savannah River Co.
Aiken, South Carolina

ABSTRACT

This paper presents a methodology to calculate the impingement load in the region of a submerged turbulent jet where a potential core exits and the jet is not fully developed. The profile of the jet flow velocities is represented by a piece-wise linear function which satisfies the conservation of momentum flux of the jet flow. The adequacy of the predicted jet expansion is further verified by considering the continuity of the jet flow from the region of potential core to the fully developed region.

The jet impingement load can be calculated either as a direct impingement force or a drag force using the jet velocity field determined by the methodology presented.

1.0 INTRODUCTION

The slurry pump vertically immersed in the waste tank serves as a stirrer of the liquid waste. It rotates at the speed of 1/8 to 1/3 rpm and discharges a water jet from each of the two nozzles located at the end of the pump column. The jet flows discharging from the two nozzles will alternatively impinge at the cooling coil support submerged in the tank. As a result, the cooling coil support will undergo a cyclic load caused by the impingement of the jet flows discharged from the slurry pump.

The flow field in an axially symmetric jet may be roughly divided into three regions as shown in Figure 1 (Reference 1). In region I there is a potential core of length L_o in the central part of the jet, in which the flow velocities remain constant. The outer dashed lines represent the nominal boundaries of the jet, that is, the point where the horizontal velocity u is some arbitrarily small fraction of the velocity on the centerline of the jet. A mixing zone lies between the potential core and the surrounding medium. In region II the velocity profile gradually changes until final similarity is established. Finally, in region III, the jet is said to be fully developed and similarity is established among the velocity profiles at the different sections of the jet. In all these three regions, the surrounding fluid is entrained and mixed with the fluid of the jet.

This paper presents a methodology to calculate the jet impingement load in the region of the jet where a potential core exits and the jet is not fully developed. By assuming a piece-wise linear function of the velocity profile, the magnitudes of the jet flow velocities are first determined based on the conservation of momentum flux. The adequacy of the nominal boundary of the jet defined by this piece-wise linear function is then verified by considering the continuity of the jet flow from the region of the potential core to the fully developed region where the existing solution of the velocity profile is available in the literature. The jet impingement loads acting at a 1.5"x1.5"x1/4" steel angle located at a distance of five times the nozzle diameter are calculated as both a direct impingement force and a drag force.

2.0 PROFILE OF JET FLOW VELOCITY

In spite of the fact that the surrounding fluid of liquid waste has higher density and viscosity than the jet flow of water, the fluid medium is assumed to be homogeneous (Reference 1). Thus, the formulations for the turbulent jet given in Reference 2 are applicable.

As shown in Reference 2, the governing equations of conservation of mass and conservation of momentum for a turbulent jet submerged in the same fluid can be reduced to the following equation of constant momentum flux.

$$2\pi\rho\int_0^\infty u^2 r dr = \rho u_0^2\left(\frac{\pi d_0^2}{4}\right) \quad (1)$$

where ρ = Density of fluid
u = Local jet flow velocity in the direction of nozzle axis (x)
u_0 = Velocity of jet flow at nozzle
r = Radial coordinate of jet fluid field
d_0 = Diameter of nozzle

The following subsections discuss the derivations of the jet velocities.

2.1 Function of Flow Velocity

The steel angle that supports the cooling coil is located about $5d_0$ away from the nozzle as shown in Figure 2. Since the length of the potential core, L_o is equal to $7d_0$, the angle is located within the region of the potential core. As indicated earlier, the jet is not yet fully developed in this region and there is no similarity among the flow profiles at different sections along the flow axis. However, the following piece-wise linear function as illustrated in Figure 3 appears to be a good representation:

$$u = u_o \quad \text{for } o \le r \le h \quad (2)$$

and

$$u = u_o\left(\frac{b-r}{b-h}\right) \quad \text{for } h \le r \le b \quad (3)$$

where h = Radius of potential core at the steel angle
b = Radius of nominal jet boundary

As shown in Figure 2, the radius of the potential core and the nozzle diameter are related by the following equation:

$$\frac{h}{\left(d_0/2\right)} = \frac{(7-5)d_0}{7d_0} \quad (4)$$

Since the nozzle diameter is 1.62 inches, the radius of the potential core calculated from the above equation is:

$$h = 0.2314 \text{ inches} \quad (5)$$

The radius of the nominal jet boundary, b is an unknown parameter and will be determined from the conservation condition of momentum flux represented by Equation (1).

2.2 Determination of Jet Boundary

The radius of the jet boundary, b at the location of the steel angle that supports the cooling coil is determined from the condition of momentum flux conservation in the jet flow as follows.

The momentum flux at the nozzle is

$$M_o = \rho\int_0^{2\pi}\int_0^{d_0/2} u_0^2 (dr)(rd\theta)$$
$$= \pi\rho u_0^2\left(\frac{\pi d_0^2}{4}\right) \quad (6)$$

The momentum flux at the location of the angle is

$$M = \rho\int_0^{2\pi}\int_0^{b} u^2 r dr d\theta$$

$$= 2\pi\rho\left\{\int_0^h u_0^2 r dr + \int_h^b u_0^2\left(\frac{b-r}{b-h}\right)^2 r dr\right\}$$

or

$$M = \pi\rho u_0^2\left[h^2 + \frac{1}{(b-h)^2}\left(\frac{b^4}{6} - b^2h^2 + \frac{4bh^3}{3} - \frac{h^4}{2}\right)\right] \tag{7}$$

Based on the condition of momentum flux conservation expressed in Equation (1), the momentum flux at the nozzle, M_0 must be equal to the momentum flux at the location of the steel angle, M. The radius of the jet boundary at the angle location, b can then be solved from the Equations (6) and (7) by the trial procedure shown in Table 1. The momentum flux at the steel angle is equal to the momentum flux at the nozzle, if b is equal to 1.726 inches. Thus, the radius of the jet boundary is:

b = 1.726 inches (8)

2.3 Verification of Jet Boundary

The jet boundary for the not fully developed region determined in Section 2.2 must correspond to an appropriate jet boundary in the fully developed region. However, the so-called jet boundaries in the fully developed region are not precisely defined; due to the turbulent nature of the flow, the actual jet limits are only statistically meaningful (Reference 2). As shown in Figure 4, the lateral spread of a jet is linear and the angle of spread may be given in terms of an arbitrarily defined radius (Reference 2). Thus, the jet boundary in the fully developed region may be defined as the two sides of the spread angle (α) along which the local jet velocities are equal to a small constant fraction of the maximum jet velocities along the center line of the jet. Based on this definition, the adequacy of the radius of the jet boundary, b, determined in Section 2.2 in relation to the boundaries of the fully developed region can be justified in the following manner.

Based on the configuration of the potential core and jet boundaries shown in Figure 4, the following relations are established:

$$\frac{\left(b - \frac{d_0}{2}\right)}{5d_0} = \frac{y}{7d_0} \tag{9}$$

Substituting the values of b and d_0 into the above equation, we have

y = 1.2824 inches

The radius of the jet boundary at the juncture of the potential core and fully developed regions is:

$$r_1 = y + \frac{d_0}{2} = 2.0924$$

Then the angle of jet spread in the fully developed region is:

$$\tan\alpha = \frac{r_1}{6.4d_0} = 0.20181 \tag{10}$$

The longitudinal variation of the centerline velocity is given in Reference 2:

$$\frac{u_{max}}{u_0} = \frac{6.4d_0}{x} \tag{11}$$

where u_{max} = maximum jet velocity at jet axial coordinate x

u_0 = jet velocity at nozzle

The velocity distribution in the zone of established flow is also given in Reference 2:

$$\frac{u}{u_{max}} = \frac{1}{\left[1 + \frac{r^2}{0.016x^2}\right]^2} \tag{12}$$

The juncture of the potential core and fully developed regions is located at the tip of the potential core as shown in Figure 4. Thus, the axial coordinate of the juncture is:

$$x_1 = 6.4d_0 \tag{13}$$

Combining Equations (11), (12) and (13) together with the value of r_1, we obtain the ratio of the local jet velocities along the sides of the spread angle α to the corresponding maximum velocities:

$$\frac{u}{u_{\max}} = 0.08$$

Since these velocities are only eight percent of the corresponding maximum velocities, we may conclude that the sides of the spread angle α are the boundaries of the fully developed jet. Furthermore, because these boundaries are determined from the parameter b in Equation (3), we may also conclude that the piece-wise linear function of the velocity profile is properly related to the jet boundaries in the fully developed region.

3. 0 JET IMPINGEMENT LOAD

The hydrodynamic load generated by the slurry pump jet impingement at the cooling coil support angle is calculated by two alternative methods as discussed in the following two subsections.

3.2 Load Calculated as Direct Impingement Force

The hydrodynamic load is assumed to be generated by the direct impingement of the jet at the steel angle. Consider the control volume in the immediate vicinity of the frontal surface of the steel angle as shown in Figure 5. The horizontal velocities of the flow entering the control volume are given by Equations (2) and (3). On the other hand, the velocities of the flow leaving the control volume have no horizontal components. Thus, the impingement load can be determined from the equation of momentum conservation:

$$F = \rho \int_{S_{in}} u^2 dA - \rho \int_{S_{out}} u^2 dA$$

$$= \rho \int_{S_{in}} u^2 dA \qquad (14)$$

Figure 6 illustrates the projection of the jet flow over one quarter of the steel angle frontal surface and the functions of the jet velocities in different regions of the angle. Due to the complexity of the geometry and flow distribution, it is difficult to carry out the integration in Equation (14) directly. Therefore, the area of the steel angle frontal surface that is impinged by the jet flow is divided into three different regions where integration can be performed with relative ease; the total impingement force is then obtained by combing those forces.

The impingement force in the region ABC is:

$$F_{BCED} = \rho \int_0^{\alpha} \int_h^{a\sec\theta} u_0^2 \left(\frac{b-r}{b-h}\right)^2 r dr d\theta$$

$$= \frac{\rho u_0^2}{(b-h)^2} \int_0^{\alpha} \left[\frac{(ab)^2}{2}\sec^2\theta - \frac{2a^3 b}{3}\sec^3\theta + \frac{a^4}{4}\sec^4\theta \right.$$

$$\left. - \frac{(bh)^2}{2} + \frac{2bh^3}{3} - \frac{h^4}{4} \right] d\theta$$

$$= \frac{\rho u_0^2}{(b-h)^2} \left\{ \frac{(ab)^2}{2}\tan\alpha \right.$$

$$- \frac{a^3 b}{3}\left[\tan\alpha\sec\alpha + \log\tan\left(\frac{\pi}{4} + \frac{\alpha}{2}\right)\right]$$

$$+ \frac{a^4}{12}\left(\sec^2\alpha + 2\right)\tan\alpha$$

$$\left. - \alpha\left[\frac{(bh)^2}{2} - \frac{2bh^3}{3} + \frac{h^4}{4}\right]\right\} \qquad (15)$$

The impingement force in the region ECGH is:

$$F_{ECGH} = \rho \int_{\alpha}^{\frac{\pi}{2}} \int_h^b u_0^2 \left(\frac{b-r}{b-h}\right)^2 r dr d\theta$$

$$= \frac{\rho u_0^2}{(b-h)^2}\left(\frac{\pi}{2} - \alpha\right)\left[\frac{b^4}{12} - \frac{(bh)^2}{2} + \frac{2bh^3}{3} - \frac{h^4}{4}\right] \qquad (16)$$

The impingement force in the region ADH is

$$F_{ADH} = \rho \int_0^{\frac{\pi}{2}} \int_0^h u_0^2 r dr d\theta$$

$$= \rho u_0^2 \frac{\pi h^2}{4} \quad (17)$$

The volumetric flow rate, Q_0 discharged from the nozzle, the diameter of the nozzle outlet, d_0 and one half of the width of the of the steel angle are given:

$$Q_0 = 1.4 \ ft^3/sec$$

$$d_0 = 1.62 \text{ inches}$$

$$a = \frac{1.5}{2} = 0.75 \text{ inches}$$

The value of angle α is calculated from the values of a and b:

$$\alpha = \cos^{-1}\left(\frac{a}{b}\right)$$

$$= \cos^{-1}\left(\frac{0.75}{1.726}\right)$$

$$= 64.2446 \text{ degrees or } 1.1213 \text{ radians} \quad (18)$$

The velocity of the jet leaving the nozzle can then be calculated as follows:

$$u_0 = Q_0 / \left(\frac{\pi d_0^2}{4}\right)$$

$$= 1.4 \frac{ft^3}{sec} \times \left(12 \frac{in}{ft}\right)^3 \times \frac{1}{\frac{\pi}{4}(1.62in)^2}$$

= 1173.686 in/sec or 97.807 ft/sec

The density of jet liquid (water) is:

$$\rho = 62.4/g$$

$$= 0.000093455 \frac{lb - sec^2}{in^4} \quad (19)$$

where g is the gravitational constant.

Substituting the values of ρ , u_0, a, b and α into Equations (15) through (17), we obtain:

$$F_{BCDE} = 43.1093 \text{ lbs}$$

$$F_{ECGH} = 17.444 \text{ lbs}$$

$$F_{ADH} = 5.4134 \text{ lbs}$$

Based on the geometrical configuration of the impinged area and the distribution of the jet flow, the total impingement force is calculated as follows:

$$F_I = 4 \times (F_{BCDE} + F_{ECGH} + F_{ADH})$$

$$= 264.0 \text{ lbs} \quad (20)$$

3.3 Load Calculated as Drag Force

In a uniform flow field, the drag force acting at an immersed body is expressed in the following equation (Reference 3).

$$F_D = C_D \frac{\rho V_0^2}{2} A \quad (21)$$

where F_D = Drag force
C_D = Drag coefficient
V_0 = Uniform upstream flow velocity
A = Area of the projection of the body on a plane normal to V_0

In the present analysis, the above equation is extended to the non-uniform field of the jet flow. The drag force for the jet flow is defined in the following equation and the same drag coefficient of a uniform flow is assumed to be applicable to the jet flow.

$$F_D = C_D \frac{\rho}{2} \int_{A_I} u^2 dA \qquad (22)$$

where A_I is the area of the steel angle covered by the jet flow. The drag coefficient for an angle given in Reference 4 is:

$$C_D = 1.98 \qquad (23)$$

By comparing Equations (21) and (22) and also using Equation (23), it is apparent that the following equation is valid:

$$\rho \int_{A_I} u^2 dA = F_I = 264.0 \text{ lbs} \qquad (24)$$

Substituting Equations (23) and (24) into Equation (22), we have the drag force acting at the steel angle:

$$F_D = 1.98 \times \frac{1}{2} \times 264 = 261 \text{ lbs} \qquad (25)$$

Thus, the impingement load calculated as a drag force is 261 lbs.

4.0 CONCLUSION

A methodology was developed to calculate the impingement load of a submerged turbulent jet in the region where the jet flow is not yet fully developed. The profile of the jet velocity developed by using this methodology satisfies both the condition of constant momentum flux in the potential core region and the continuity condition imposed by the boundaries of the jet in the fully developed region.

The jet impingement load acting at a 1.5"x1.5"x1/4" steel support angle of the cooling coil is calculated both as a direct impingement force and a drag force. The magnitudes of the impingement load either treated as a direct force or a drag force are very close in this case.

5.0 REFERENCES

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2. Daily, J. W. and Harleman, D. R F., Fluid Dynamics, Addison-Wesley Publishing Co., Inc., 1966.

3. Hansen, A. G., Fluid Mechanics, John Wiley and Sons, Inc. 1967.

4. "Drag and Lift Coefficients," NACA Report.

Table 1. Trial Solution of Jet Boundary Radius

b (in)	M (lb-sec/sec)	M_0 (lb-sec/sec)
1.4	$0.4615 \times \pi \rho u_0^2$	$0.6561 \times \pi \rho u_0^2$
1.6	$0.5769 \times \pi \rho u_0^2$	$0.6561 \times \pi \rho u_0^2$
1.7	$0.6396 \times \pi \rho u_0^2$	$0.6561 \times \pi \rho u_0^2$
1.8	$0.7056 \times \pi \rho u_0^2$	$0.6561 \times \pi \rho u_0^2$
1.726	$0.6561 \times \pi \rho u_0^2$	$0.6561 \times \pi \rho u_0^2$

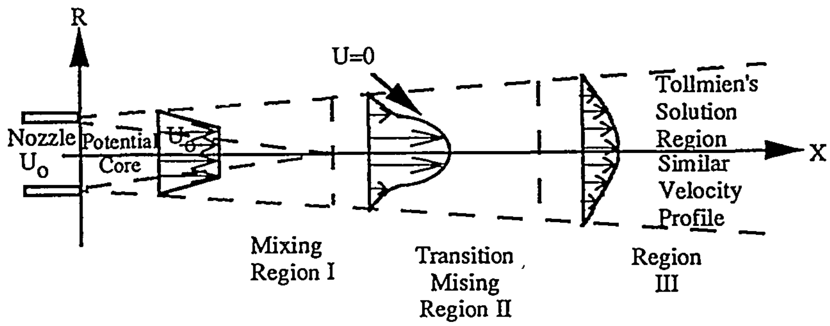

Figure 1. Axially Symmetric Jet in a Medium at Rest

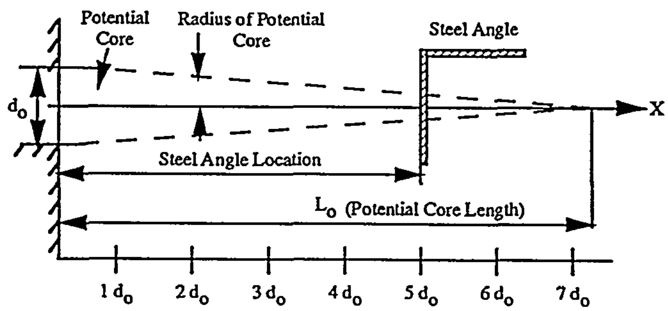

Figure 2. Potential Core Length and Steel Angle Location

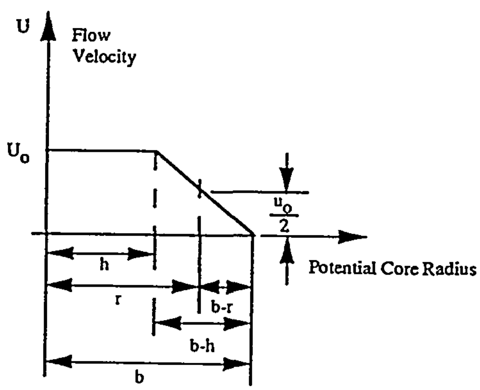

Figure 3. Velocity Porfile of Jet Flow at Location of Steel Angle

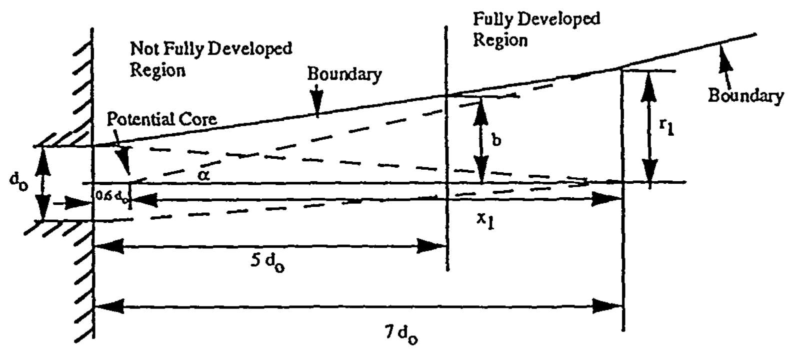

Figure 4. Relationship of Jet Boundaries in Not Developed and Fully Developed Regions

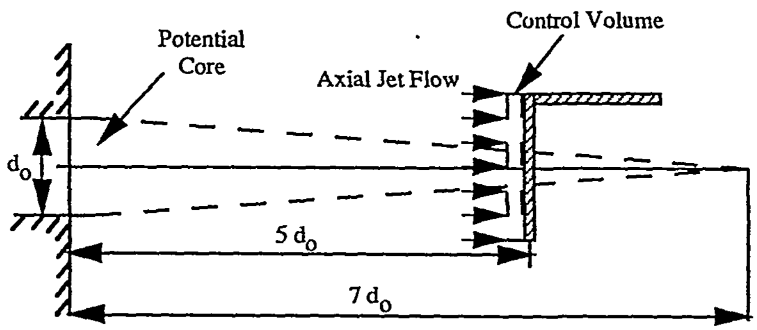

Figure 5. Control Volume for Jet Impingement Load Calculation

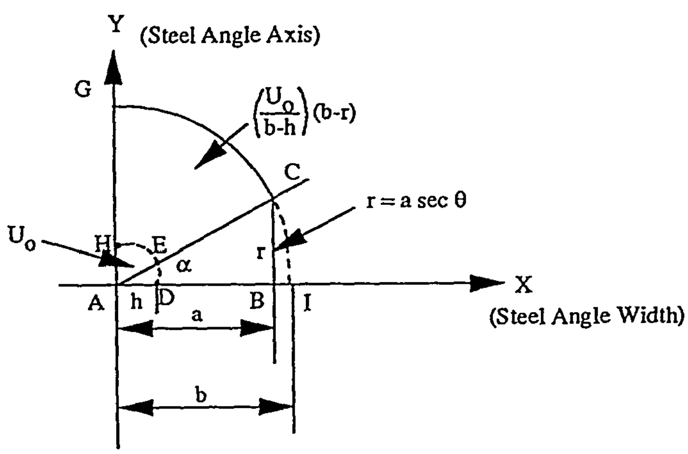

Figure 6. Projection of Jet Flow Over One Quarter of Steel Angle Frontal Surface