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Hadronic interactions from effective chiral Lagrangians of quarks and gluons [a]

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Effective chiral Lagrangians involving constituent quarks, Goldstone bosons and long-distance gluons are believed to describe the *strong* interactions in an intermediate energy region between the confinement scale and the chiral symmetry breaking scale. Baryons and mesons in such a description are bound states of constituent quarks. We discuss the combined use of the techniques of effective chiral field theory and of the field theoretic method known as Fock-Tani representation to derive effective hadron interactions. The Fock-Tani method is based on a change of representation by means of a unitary transformation such that the composite hadrons are redescribed by elementary-particle field operators. Application of the unitary transformation on the microscopic quark-quark interaction derived from a chiral effective Lagrangian leads to chiral effective interactions describing all possible processes involving hadrons and their constituents. The formalism is illustrated by deriving the one-pion-exchange potential between two nucleons using the quark-gluon effective chiral Lagrangian of Manohar and Georgi. We also present the results of a study of the saturation properties of nuclear matter using this formalism.

1 Introduction

The quark-gluon description of the interactions among hadrons and the properties of high temperature and/or density hadronic matter is one of the most central problems of contemporary particle and nuclear physics. Such problems are characterized by processes that involve the simultaneous presence of hadrons and their constituents. The mathematical description of the processes requires approximations where a drastic reduction of the degrees of freedom is unavoidable. In this sense, one would expect simplifications by describing the hadrons participating in the processes in terms of macroscopic hadron field operators, instead of the microscopic constituent ones. At low energies the hadron-hadron interaction can be described by an effective chiral field theory in which the quarks and gluons are "integrated out" in favor of hadrons and Goldstone bosons [1,2]. At higher energies it is very likely that the substructure

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of the hadrons will play a role and another effective field theory involving these degrees of freedom must be introduced.

There is a widespread belief that there exists an intermediate energy region in which it makes sense to describe the strong interactions in terms of an *effective field theory of constituent quarks* subject to weak color forces that become strong only at large separations and keep the quarks confined. The u and d constituent quarks have a mass of $m \sim 300$ MeV, which are believed to be the result of the spontaneous breakdown of the $SU(2) \otimes SU(2)$ chiral symmetry. If this is so, the Goldstone bosons of the spontaneous symmetry breakdown (pions in the case of u and d quarks only) must be included among the degrees of freedom of the effective theory. The lowest order terms of the Lagrangian of such an effective field theory were written down by Manohar and Georgi (MG) [3]. Many of the successes of the simple nonrelativistic quark model can be understood in this framework with a chiral symmetry breaking scale $\Lambda_{\chi SB} \sim 1$ GeV, which is significantly larger than the confinement scale $\Lambda_{conf} \sim 200$ MeV.

The description of the hadron-hadron interaction in such a theory becomes complicated because hadrons are not the basic degrees of freedom of the theory; hadrons are *composites and in general cannot be described by field* operators of the sort used to describe elementary particles. In this talk we discuss a method we believe can be very useful for treating composite hadron interactions at the quark-gluon level. The method is known as Fock-Tani (FT) representation and was invented independently by Girardeau [4] and Vorob'ev and Khomkin [5] to deal with atomic systems where atoms and electrons are simultaneously present in the system and the internal degrees of freedom of atoms cannot validly be neglected. The method is based on a change representation by introducing fictitious elementary hadrons in close correspondence to the real hadrons. The change of representation is implemented by means of a unitary transformation such that the composite hadrons are redescribed by elementary-particle field operators; all field operators representing quarks, antiquarks, gluons and *hadrons* satisfy canonical commutation *relations* and therefore the traditional methods of quantum field theory can be readily applied. *In the new representation the microscopic interquark forces change*, they become weaker, in the sense they cannot bind the quarks into hadrons, and one obtains effective interactions describing all possible processes between hadrons and their constituents.

Recently the original FT formalism was extended to hadronic physics for deriving effective hadron Hamiltonians in constituent quark models [6]. Here we discuss its application in the context of the MG effective chiral field theory. In the next section we discuss the main features of the formalism and the properties of the effective Hamiltonians. In section 3 we consider baryons in the MG model and derive an effective chiral Hamiltonian for baryons. One particularly important component present in the effective nucleon-nucleon interaction is the one-pion exchange interaction. We also discuss the saturation properties of nuclear matter in this model. In the last section we present conclusions and discuss future perspectives.

2 Fock-Tani representation: real and ideal particles

In this section we discuss very briefly the main features of the FT representation. We consider a Hamiltonian where quarks and antiquarks interact by two-body forces, and consider mesons and baryons as bound states of a quark-antiquark pair and three quarks respectively. In Fock space $\mathcal{F}$, mesons and baryons can be written in terms of creation operators as:

$$M^{\dagger}_{\alpha}|0\rangle \equiv \Phi^{\mu\nu}_{\alpha} q^{\dagger}_{\mu}\bar{q}^{\dagger}_{\nu}|0\rangle, \qquad B^{\dagger}_{\alpha}|0\rangle = \frac{1}{\sqrt{3!}}\,\Psi^{\mu\nu\sigma}_{\alpha} q^{\dagger}_{\mu}q^{\dagger}_{\nu}q^{\dagger}_{\sigma}|0\rangle, \tag{1}$$

where α represents the hadron quantum numbers (c.m. momentum, internal energy, spin and flavor), and Φ and Ψ are respectively the Fock space meson and baryon amplitudes, where $\mu, \nu, \cdots$ represent spatial, color, spin, and flavor quantum numbers of the quarks and antiquarks. A summation over repeated indices is implied. The quark and antiquark creation and annihilation operators obey standard anticommutation relations. Using the quark anticommutation relations and assuming that the meson wave-functions are orthonormalized one can show that the meson operators satisfy the following *noncanonical* commutation relations:

$$[M_{\alpha}, M^{\dagger}_{\beta}] = \delta_{\alpha\beta} - \Delta_{\alpha\beta}, \qquad [M_{\alpha}, M_{\beta}] = 0, \tag{2}$$

where $\Delta_{\alpha\beta} = \Phi^{*\mu\nu}_{\alpha}\Phi^{\mu\sigma}_{\beta}\bar{q}^{\dagger}_{\sigma}\bar{q}_{\nu} + \Phi^{*\mu\nu}_{\alpha}\Phi^{\rho\nu}_{\beta}q^{\dagger}_{\rho}q_{\mu}$. In addition, one can also show that meson and quark operators do not commute. Baryon operators satisfy similar expressions [6].

One observes that the composite nature of the mesons is manifested by the term $\Delta_{\alpha\beta}$ in Eq. (2), and the nonzero value for the meson-quark commutator indicates that mesons and quarks are not independent degrees of freedom. The presence of these terms complicates the direct application of field theoretic techniques such as Wick's theorem and Feynman graphs for the composite hadron operators, since these techniques are set up for canonical field operators.

The change to the FT representation is implemented by means of a unitary transformation U, such that the *single* composite hadron states are transformed

into *single* ideal-hadron states $m^{\dagger}_{\alpha}|0\rangle \equiv U^{-1}M^{\dagger}_{\alpha}|0\rangle$ and $b^{\dagger}_{\alpha}|0\rangle \equiv U^{-1}B^{\dagger}_{\alpha}|0\rangle$, where the ideal hadron operators satisfy canonical (anti)commutation relations:

$$[m_{\alpha}, m^{\dagger}_{\beta}] = \{b_{\alpha}, b^{\dagger}_{\beta}\} = \delta_{\alpha\beta}, \qquad [m_{\alpha}, m_{\beta}] = \{b_{\alpha}, b_{\beta}\} = [m_{\alpha}, b^{\dagger}_{\beta}] = 0, \tag{3}$$

and, by definition, the $m^{\dagger}$ and m ($b^{\dagger}$ and b) commute (anticommute) with the quark and antiquark operators. U is an unitary operator, which can be constructed by an iterative procedure as a power series in the Φ's and Ψ's. Details on the derivation of the generator can be found in Ref. [6]; here we simply present the general form of the effective Hamiltonian in the new representation. From the microscopic quark-antiquark Hamiltonian H, one obtains

$$H_{\rm FT} \equiv U^{-1}HU = H_q + H_h + H_{hq}\,, \tag{4}$$

where the subindices identify the operator content of each term. The quark Hamiltonian H_q involves only quark and antiquark operators. In general it has a similar structure to the one of the microscopic quark Hamiltonian H, except that it cannot produce the hadronic bound states. This feature leads to the same effect of curing the bound state divergencies of the Born series as in Weinberg's quasi-particle method [7]: the modified quark interaction is unable to form hadrons, they are redescribed by the H_h part of the effective Hamiltonian.

H_{hq} describes quark-hadron processes as hadron breakup into quarks and hadron-quark scattering. In models where quarks are truly confined, these terms contribute to free-space meson-meson processes only as intermediate states. However, in high temperature and/or density systems hadrons and quarks can coexist and the breakup and recombination processes can play an important role.

The term involving only ideal hadron operators H_h represents effective meson-meson, baryon-baryon, and baryon-meson processes. We exemplify this in the next section for the baryon-baryon interaction using the Manohar and Georgi Lagrangian density.

3 Effective chiral Hamiltonian for baryon-baryon interactions

Now we discuss the application of the FT formalism to the effective QCD field theory of MG [3]. As discussed in the Introduction, since the effects of dynamical chiral symmetry breaking are included in the constituent quark mass the interquark forces become weaker in the effective theory. This allows to identify the low-lying hadrons with nonrelativistic bound states of the constituent quarks. Of course, there remains the problem of the confining forces which are difficult to access in such an approach. At this stage probably the best attitude is simply to postulate a phenomenological effective confining interaction. We then proceed deriving from this quark-gluon theory an effective quark-quark interaction up to a certain order in the chiral expansion. Once one has the microscopic quark-quark potential and the "bare" hadron bound-states obtained with the phenomenological confining interaction, one implements the FT transformation to obtain effective hadron-hadron interactions. Corrections to the bare hadron bound-states can be calculated perturbatively in the new representation.

Let us consider just the lowest order pion-quark interaction piece of the MG model. At tree level, this corresponds to the standard pseudovector coupling. Given the quark-quark interaction and the baryon wave-functions Ψ, one obtains through the lowest-order FT transformation an effective baryon Hamiltonian H_b of the general form[6]:

$$H_b = E_\alpha \, b^\dagger_\alpha b_\alpha + \frac{1}{2} V_{bb}(\alpha\beta;\delta\gamma)\, b^\dagger_\alpha b^\dagger_\beta b_\gamma b_\delta, \tag{5}$$

where E_α is the total energy (internal plus c.m.) and V_{bb} is the effective baryon-baryon interaction, which we can write as a sum of direct and quark-exchange parts, $V_{bb} = V^{dir}_{bb} + V^{exc}_{bb}$. The direct term V^{dir}_{bb}, represented in Figure 1 below, is given by:

$$V^{dir}_{bb}(\alpha\beta;\delta\gamma) = 9\, V_{qq}(\mu\nu;\sigma\rho)\Psi^{*\mu\mu_2\mu_3}_\alpha \Psi^{*\nu\nu_2\nu_3}_\beta \Psi^{\rho\nu_2\nu_3}_\gamma \Psi^{\sigma\mu_2\mu_3}_\delta. \tag{6}$$

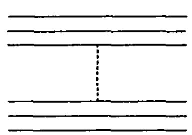

Figure 1: Graph of V^{dir}_{bb}.

When the baryons have the quantum numbers of nucleons and the microscopic quark-quark interaction is the one-pion-exchange, the explicit form of this term is the familiar *one-pion-exchange potential in momentum space:*

$$V^{dir}_{NN}(q) = -9\,\frac{25}{81}\left(\frac{g_A}{f_\pi}\right)^2 \tau^{a(2)}_N \tau^{a(1)}_N\, F(q^2)\frac{\sigma^{(1)}_N\cdot \mathbf{q}\, \sigma^{(2)}_N\cdot \mathbf{q}}{\mathbf{q}^2 + m^2_\pi} F(q^2), \tag{7}$$

where the $\tau^a_N, a = 1, \cdots, 3$ and σ_N are respectively the nucleon isospin and spin Pauli matrices, and $F(q^2)$ is the nucleon (matter) form factor.

The exchange term V^{exch}_{bb} involves the simultaneous exchange of pions between two quarks and the exchange of quarks between two baryons. For illustrative purposes we show explicitly only one of such terms:

$$V^{exch}_{bb}(\alpha\beta;\delta\gamma) = -9\, V_{qq}(\mu\nu;\sigma\rho)\Psi^{*\mu\mu_2\mu_3}_\alpha \Psi^{*\nu\nu_2\nu_3}_\beta \Psi^{\sigma\nu_2\nu_3}_\gamma \Psi^{\rho\mu_2\mu_3}_\delta. \tag{8}$$

This is schematically represented in Figure 2. Because exchange terms involve quark exchange between the baryons, they are of shorter range than the one of V^{dir}_{bb}.

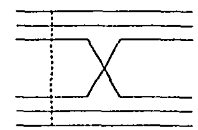

Figure 2: Graph of V^{exch}_{bb}.

The multidimensional integral over the quark coordinates in V^{exch}_{bb} cannot in general be done explicitly. However, when the quark wave functions are nonrelativistic gaussians, V^{exch}_{bb} can be expressed in terms of a single integral:

$$\begin{aligned} V^{exch}(\mathbf{p},\mathbf{p}') &= \mathcal{O}^{ij}(\sigma,\tau)\left(\frac{g_A}{f_\pi}\right)^2 \left(\frac{3b^2}{4\pi}\right)^{3/2} e^{-b^2[5(\mathbf{p}^2+\mathbf{p}'^2)/12-\mathbf{p}\,\mathbf{p}'/2]} \\ &\times \int d^3q\, \frac{q^i q^j}{\mathbf{q}^2+m^2_\pi}\, e^{-b^2[\mathbf{q}^2-\mathbf{q}\,(\mathbf{p}'-\mathbf{p})]}, \end{aligned} \tag{9}$$

where b is the r.m.s. radius of the nucleon and $\mathcal{O}^{ij}(\sigma,\tau)$ is given by:

$$\begin{aligned} \mathcal{O}^{ij}(\sigma,\tau) &= \frac{1}{9}\left\{\left[25 + \frac{1}{3}\tau^{a(2)}_N \tau^{a(1)}_N \left(1 + 18\sigma^{(1)}_N\cdot\sigma^{(2)}_N\right)\right]\delta^{ij}\right. \\ &+ \left.\left(1 + 19\tau^{a(2)}_N\tau^{a(1)}_N\right)\sigma^{i(1)}_N \sigma^{j(2)}_N\right\}. \end{aligned} \tag{10}$$

An interesting feature of a potential derived from a quark model is that when interated to obtain the scattering amplitude, there is no need for regularization and renormalization of the scattering equation, because of the form factors that come from the baryon wave functions. Because of this one can expect that the study of the baryon-baryon scattering at low energies in the present approach will be much easier than in the approach at the hadronic level, where the renormalization of the Lippman-Schwinger equation must be performed[8].

Beyond tree-level, loops of quarks, pions and gluons introduce divergencies and higher dimensional operators must be included to cancel the divergencies. The new coefficients that come along with the higher dimensional operators are fitted to experiments. Work is in progress where the four-point interactions and the one gluon-exchange at the tree level are summed to the tree-level pion exchange just discussed. The aim is to calculate phase-shifts of the nucleon-nucleon scattering and compare with the traditional one-boson exchange models.

4 Nuclear matter

Once one has an effective NN interaction, the traditional nuclear many-body techniques are readily applicable because all field operators in the new representation satisfy canonical (anti)commutation relations. We illustrate this by outlining a calculation of the equation of state of symmetric nuclear matter in the Hartree approximation[9].

If one neglects quark-exchange contributions to the effective NN interaction, and considers only the Hartree approximation, the relevant terms in the MG Lagrangian[3] are:

$$\mathcal{L} = \mathcal{L}_{conf} + \bar{\psi}\left(i\gamma^\mu\partial_\mu - m_q\right)\psi - \frac{C_s}{2f_\pi^2}(\bar{\psi}\psi)^2 + \frac{C_v}{2f_\pi^2}(\psi^\dagger\psi)^2, \qquad (11)$$

since the one-pion and one-gluon exchanges, as well as the other four-point interactions involving γ_5 and $\sigma^{\mu\nu}$, average to zero. In Eq. (11), f_π is the pion decay constant.

Using the Fermi-Breit approximation for the effective four-quark interactions, one obtains an effective NN interaction which gives an energy density $E/A - M_N = \mathcal{E}/\rho - M_N$ for symmetric nuclear matter of the form[9]:

$$\frac{E}{A} - M_N = 3(m_q - m_q^*) + \frac{3}{5}\frac{k_F^2}{6m^*} + \frac{f_\pi^2}{18C_s\rho}(3m_q - 3m_q^*)^2 + \frac{9C_v}{2f_\pi^2}\rho + \frac{3}{2m_q b^2}\left(\frac{b^{*2}}{b^2} + \frac{m_q}{m_q^*}\frac{b^2}{b^{*2}} - 2\right), \qquad (12)$$

where M_N is the nucleon mass (calculated within the model), $\rho = 2k_F^3/3\pi^2$ is the nuclear density and k_F is the nuclear Fermi momentum, and the effective

quark mass m_q^* satisfies the self-consistent equation:

$$3m_q^* = 3m_q - \frac{9C_s}{f_\pi^2}\left[\left(1 - \frac{1}{2m^{*2}b^{*2}}\right) - \frac{3}{5}\frac{k_F^2}{2(3m_q^*)^2}\right]\rho. \qquad (13)$$

The confining interaction is taken to be an harmonic oscillator. Note that the in-medium size of the nucleon b^* is allowed to be different from b, its value is determined self-consistently with m_q^* by requiring that E/A be a minimum also with respect to variations on b^*.

Nuclear matter can be saturated with $E/A - M_N = -15.64$ MeV at $k_F = 1.364\ \mathrm{fm}^{-1}$ using the following set of parameters: $9C_s/2f_\pi^2 = 8.09$, $9C_v/2f_\pi^2 = 0.56$, $b = 0.6 fm$, $m_q = 350$ MeV. At the saturation point, we find $m_q^*/m_q = 0.81$ and $b^*/b = 1.05$. The compressibility is found to be 150 MeV.

5 Conclusions and future perspectives

The combined use of the techniques of effective chiral field theory and the Fock-Tani representation seems to provide great opportunities to study the role of quarks and gluons in hadronic interactions. The fact that in the Fock-Tani representation all operators satisfy canonical (anti)commutation relations allows the direct use of the known field theoretic techniques such as Feynman diagrams and Green's functions which have proven to be very useful in the study of processes involving elementary particles. Particularly interesting applications of these techniques are the study of short-range hadron-hadron interactions and the problem of hadron properties in a hot and/or dense medium.

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