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STOCHASTIC INTERACTION BETWEEN TAE AND ALPHA PARTICLES

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Stochastic interaction between TAE and alpha particles

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Abstract

Toroidicity-induced Alfvén eigenmodes (TAE) interact with thermonuclear alpha particles in the intrinsic stochasticity regime. In this contribution, the investigation of this interaction based on the numerical integration of the equation of motion of alpha particles in the tokamak is presented in more detail. From the first results for ITER parameters and moderate wave amplitudes follows that the strongest stochasticity appears in the region of trapped/passing boundary. Here, alpha particles jump stochastically between these two regime with quite impressive radial excursion ($\approx$ 0.5 m amplitudes). We have found a similar chaotic behaviour also for much lower energies (about 350 keV).

1 Introduction

In a thermonuclear regime, the interaction of the toroidicity-induced Alfvén eigenmodes (TAE) with thermonuclear alpha particles may become important. (For the interaction of TAE with fast ions, see [1], for the interaction of low-frequency waves with plasma in general, see [2] and for the Alfvén continuum, see [3]). In the intrinsic stochasticity regime, this interaction can result in a strong radial diffusion of particles, which might negatively influence the energy output from the tokamak - reactor. This interaction has been discussed in papers [4,5]. Our contribution is aimed at a more detailed study of this interaction in the intermediate regime between the passing and trapped particles ones, which seems to have the key importance.

2 Hamiltonian formalism

For the study of a rather complicated behaviour of particles (both untrapped an trapped) under the effect of RF fields, the use of suitably chosen canonical formalism is effective. Our approach is based on the use of the non-orthogonal coordinate system, determined by the Euler potential [6] (for which, one of the set of coordinate curves is formed by magnetic field lines), and on our previous study [7] and application in [8]. After having determined, by the usual canonical procedure, the conjugated generalized momenta [9], we have found a Hamiltonian, describing in a convenient form the motion of particles in the tokamak magnetic field and in RF fields. Finally, introducing into the coordinate system three types of angles (the angles of the cyclotron, poloidal and toroidal rotations) we have obtained a simplified form of the interaction Hamiltonian (see [10]) for an alpha particle

$$\begin{aligned} H &= \omega_{c0}P_1[1-\frac{r(P_2)}{R_0}\cos\tilde{\beta}] + \frac{P_3^2}{2m_\alpha}[1-2\frac{r(P_2)}{R_0}\cos\tilde{\beta}] + \\ &+\frac{1}{m_p}\sqrt{2e_\alpha P_2 B_0}\,\frac{\delta B}{B_0}\,[v_{alf}-\frac{1}{m_\alpha}(P_3+\frac{P_2}{qR_0})]\times \\ &\times\cos(n_t\frac{Q_3}{R_0}-m_p\tilde{\beta}-\omega t). \end{aligned} \tag{1}$$

The meaning of the canonical coordinates in (1) is the following: $P_1 = \frac{e_\alpha B_0}{2}\rho_c^2$, $P_2 = \frac{1}{2}e_\alpha B_0 r^2$, $P_3^2(1-2\frac{r}{R_0}\cos\tilde{\beta}) = m_\alpha^2 v_\parallel^2$, $Q_1 = \omega_{c0}t$, $\tilde{\beta} = \theta = Q_2 + \frac{Q_3}{qR_0}$. $Q_3 = R_0\phi$, $r = \sqrt{\frac{2P_2}{e_\alpha B_0}}$.

Here, R_0, a are the major and minor radii of the tokamak, $\omega_{c0} = \frac{e_\alpha B_0}{m_\alpha}$ is the cyclotron frequency on the tokamak axis and q is the safety factor. θ and ϕ are the poloidal and toroidal angles, respectively, and e_α is the charge and m_α is the mass of an alpha particle. Parameters δB, m_p, ω, v_{alf} are the parameters of the considered toroidicity-induced Alfvén eigenmode; δB is the perturbation of the magnetic field, caused by TAE. TAE is represented by its scalar potential Φ_{rf} and the parallel component of the vector potential, $A_{\|,rf}$. The usual assumption $E_{\|,rf} = \delta B_{\|,rf} = 0$ allowed the expression of both potentials by means of δB. Parameters n_t, m_p are the toroidal and poloidal wave numbers, respectively, ω is the angular frequency of TAE and v_{alf} is the Alfvén velocity. The toroidal magnetic field B_T has been taken, as usually, as $B_T = \frac{B_0}{1+\frac{r}{R_0}\cos\theta}$.

It is well known that the banana motion is characterized by ist oscillations in the poloidal and toroidal direction, as well as by its precession velocity. It is supposed that the banana bounce frequency $\omega_B \approx \frac{v}{qR_0}\sqrt{\frac{r_0}{2R_0}}$, satisfies $\omega_B \ll \omega_{c0}$. In this case it is possible to neglect any resonance effects on the cyclotron frequency, and to take average of the Hamiltonian over Q_1. This is the reason why the Hamiltonian (1) is cyclic in Q_1.

3 Numerical results

For the numerical discussion, we use the following set of ITER parameters [11]: $R_0 = 7.75m, a = 2.8m, B_0 = 6T, n_0 = 1.4\times10^{20}m^{-3}, \alpha_n = 0.26, \frac{n_D}{n_T} = 0.4, \Delta q = 2.$

Here, the radial profile of the density $n(r)$ is defined as $n(r) = n_0(1-(\frac{r}{a})^2)^{\alpha_n}$, and the radial profile of q is chosen as $q_r = 1 + \Delta q(\frac{r}{a})^2$.

The results of the numerical discussion is presented in the form of the Poincaré plots. The surface of section is defined by the conditions $\tilde{\beta} = 0$ and $r = r_{max}$. The parallel axis represents the phase $\eta = n_t \frac{Q_3}{R_0} - m_p\tilde{\beta} - \omega t$, the perpendicular axis the parallel component of the particle velocity, $v_{||}$, both taken at the moment the phase line crosses the surface of section. The initial values were chosen as $\omega = 3.41 \times 10^5 s^{-1}, n_t = 3, m_p = 3, \frac{r}{a} = 0.5$, and the energy of alphas 3.52 MeV.

Fig. 1 presents the Poincaré plot for both types of particles, untrapped and trapped, for $\frac{\delta B}{B_0} = 10^{-5}$. The separatrix is in the region $v_{||} \approx 6.8 \times 10^6 ms^{-1}$; trapping appears in the lower half of the picture. Fig. 2 shows the same for the same parameters, but for $\frac{\delta B}{B_0} = 5{\times}10^{-4}$. Figs. 2 already present a stochastic region. This is seen in the next pictures. Fig. 3.a shows the radial excursions of particles in the banana regime for two close parallel velocities, $v_{||} = 5.25; 5.5 \times 10^6 ms^{-1}$. (These points are defined by the conditions $\tilde{\beta} = 0, r = r_{max}$, and are connected by straight lines). These excursions are impressive (in the region of a half meter). Fig. 3.b presents the particle's trajectory in the poloidal plane for the same regime as in the foregoing, but for $v_{||,0} = 5.6 \times 10^6 ms^{-1}$. The picture shows the change of trapped regime into the passing one. The thick line is used for the first banana. Fig. 4 presents a typical maximum radial particle's excursions in the region close to the passing-trapping boundary. Here we choose $\frac{r}{a} = 0.9$, $\frac{\delta B}{B_0} = 5 \times 10^{-4}$ and the other parameters the same as in the foregoing. Starting with $v_{||} = 7.5 \times 10^6 ms^{-1}$, the particle jumps stochastically between the passing and banana regime. Here the particle travels between $r_{max} \approx 2.7m$ and $r_{min} \approx 1.9m$. Fig. 5 shows the banana trajectory for same parameters as in Fig. 2, but for $\frac{\delta B}{B_0} = 10^{-3}$ and $v_{||,0} = 2.5 \times 10^6 ms^{-1}$. The particle traverses radially almost one meter! For lower energies ($\approx 350 keV$) the plots obtained also reveal the stochasticity; this can perhaps help with the ash extraction. Fig.6 presents the Poincaré plot for particles with the energy 350 keV, for $n_t = m_p = 1$, $\omega = 2.54 \times 10^5 s^{-1}$, for $\frac{\delta B}{B_0} = 5 \times 10^{-4}$ and for $\frac{r}{a} = 0.5$. Fig.7 then shows nonnegligible stochastic radial excursions of the particles with the energy 350 keV, with $v_{||,0} = 3.1 \times 10^6 ms^{-1}$ and $3.9 \times 10^6 ms^{-1}$.

Since we have considered only a single TAE mode, the stochasticity regime appears only in a limited region close to the passing/trapping boundary. There exists, of course, a possibility of further resonant stochastic interaction (e.g on harmonics of alpha particles banana bounce frequency, discussed already in [7]), but for our parameters the effect of the instability in the separatrix region strongly dominates. The radial induced displacement of particles is impressive (even in its quasiperiodical form), and promisses for large amplitudes a sufficiently broad stochastic regime. A global stochasticity can be expected in the case of several TAE modes, as has been alreasy mentioned in [12] (of course, when the radial distribution of such field enables the interaction in the whole plasma volume). Our contemporary results therefore predict only a fast diffusion in the plasma boundary slab with the thickness of a typical banana.

According to recent proposal of Fisch and Rax [13], the radial diffusion of alpha particles, induced by the stochastic interaction with lower hybrid waves (LHW), is generically coupled with the transfer between LHW energy and alpha particles *perpendicular* energy. It can be easily shown that a similar effect appears also in the case of the stochastic interaction of alpha particles with Alfén waves. Considering the resonant condition (for simplicity, for passing particles) $p\omega_2 + q\omega_3 - \omega = 0$, where ω_2 represents the basic frequency of the motion of a particle in the poloidal direction and ω_3 the basic frequency of the motion in the toroidal direction, the following integral can be obtained: $P_2 = \frac{p}{q} P_3 + const.$. Here, the symbolics of the Hamiltonian (1) was used, together with the analogical procedure in [14]. Consequently, there exists a transfer of energy between the wave and the *parallel* energy of a particle, accompanying its radial diffusion. The significance of this effect requires a further study.

4 Conclusion

We have found that the stochastic interaction of TAE with alpha particles in the regime close to passing/trapped boundary results for usually considered wave amplitudes, in large radial excursion of particles. Deeper in the passing or trapped regimes, particles move radially rather quasiperiodically, but also with large amplitudes. The stochasticity appears not only for thermonuclear energies, but also for rather low energies of the ash regime. Our results need to be discussed for a broader regime of parameters and for several simultaneously acting modes. Finally, the effect of these losses on the global reactor efficiency needs to be estimated (for this, see e.g. [15]).

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Figure captions

Fig.1. The Poincaré plot for $\omega = 3.41 \times 10^5 s^{-1}, n_t = m_p = 0.5, \frac{\delta B}{B_0} = 10^{-5}$.

Fig.2. The Poincaré plot for the same parameters, as in Fig. 1, but for $\frac{\delta B}{B_0} = 5 \times 10^{-4}$.

Fig.3.a. The change of the maximal radial excursion of particles with $v_{\|,0} = 5.25; 5.5 \times 10^6 ms^{-1}$ and for other parameters as in Fig. 2.

Fig.3.b. The change of the banana regime into the passing one. The parameters as in the foregoing, but for $v_{\|,0} = 5.6 \times 10^6 ms^{-1}$.

Fig.4. Particle's maximal radial excursions for parameters of Fig. 2, but for $\frac{r}{a} = 0.9$ and for $v_{\|,0} = 7.5 \times 10^6 ms^{-1}$.

Fig.5. The banana trajectory in the poloidal plane the same parameters, as in the foregoing, but for $\frac{\delta B}{B} = 10^{-3}$, and for $v_{\|,0} = 2.5 \times 10^6 ms^{-1}$. Remarkable radial shift.

Fig.6. The Poincaré plot for the total energy of particles 350 keV, for $\frac{\delta b}{B} = 0.5$, for $\omega = 2.54 \times 10^5 s^{-1}$, $n_t = m_p = 1$, and for $\frac{\delta B}{B} = 5 \times 10^{-4}$.

Fig.7. Stochastic radial excursions of the particles with the total energy 350 keV, for the same parameters as in Fig.6, and for $v_{\|,0} = 3.1; 3.9 \times 10^6 ms^{-1}$.

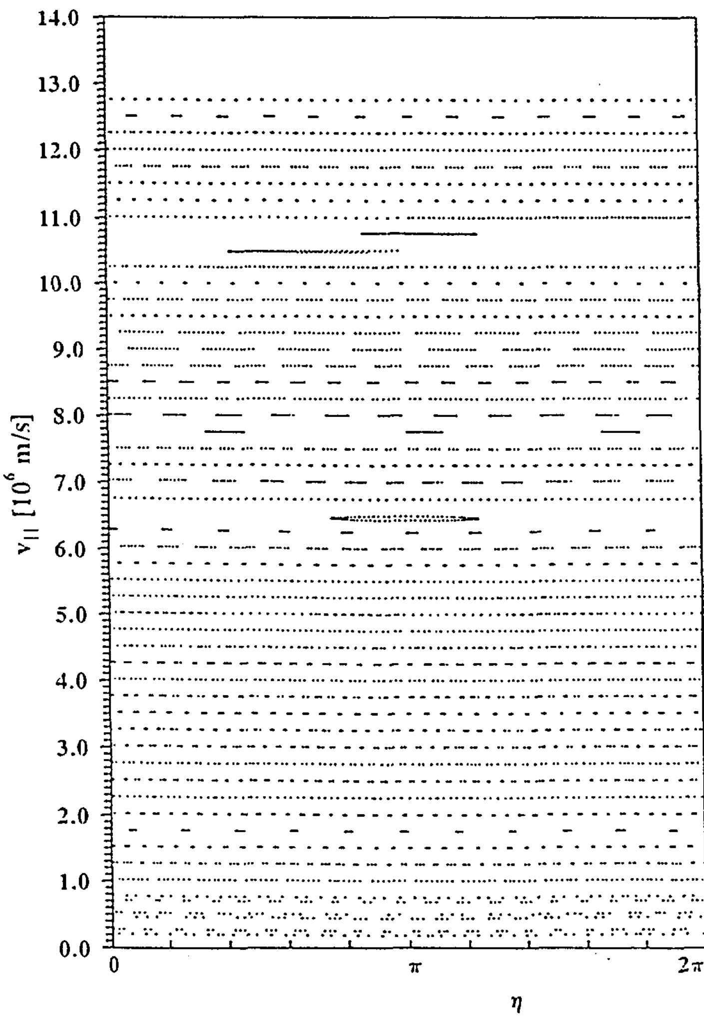


Fig. 1

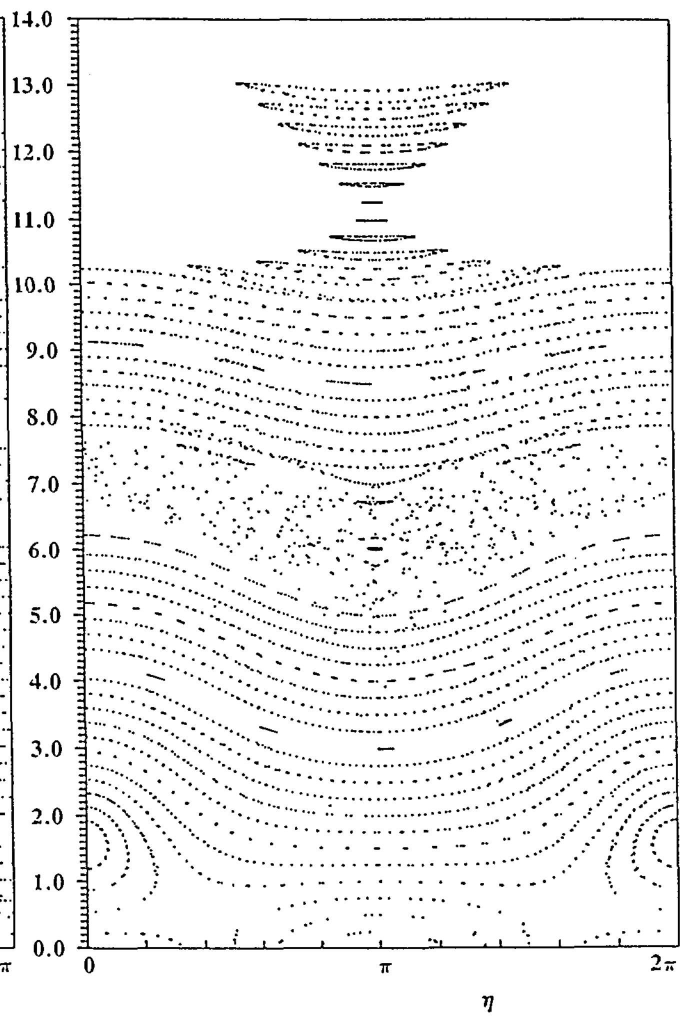
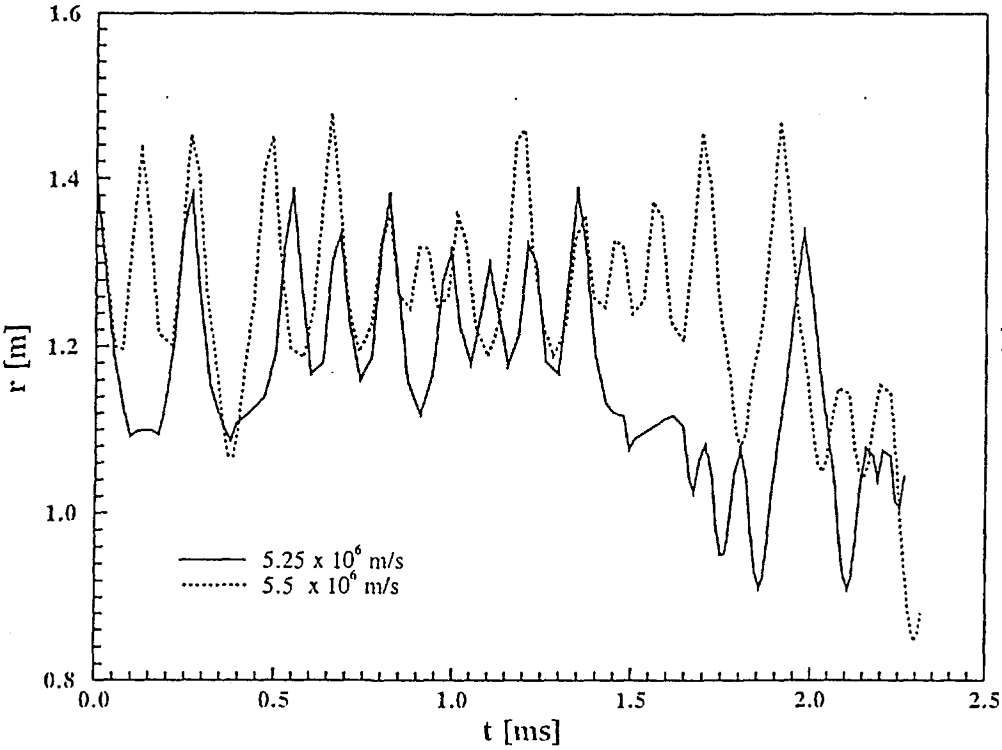
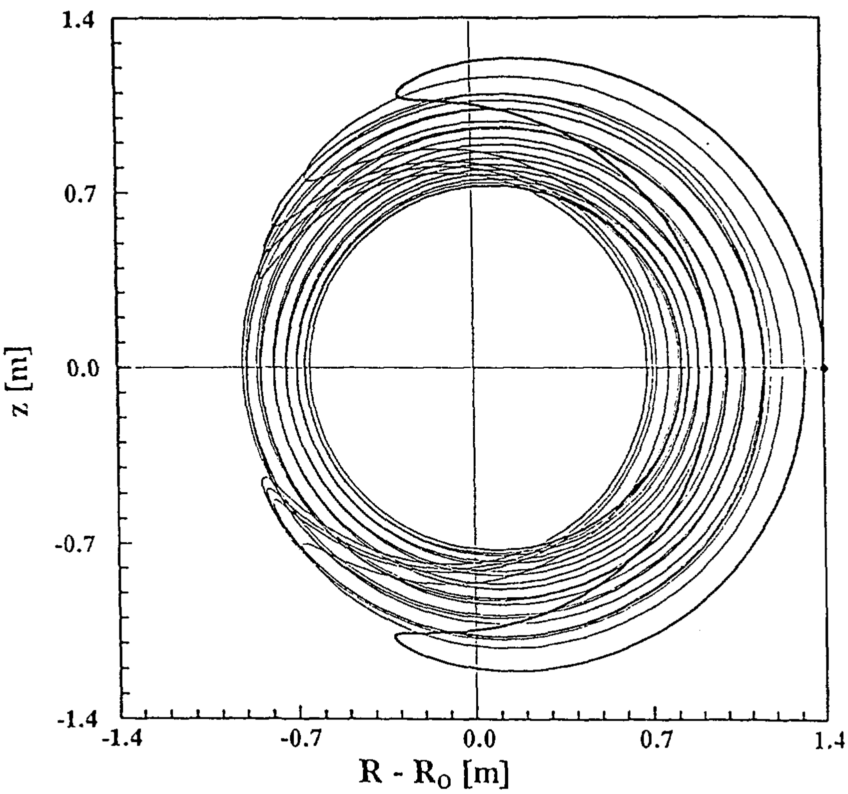


Fig. 2



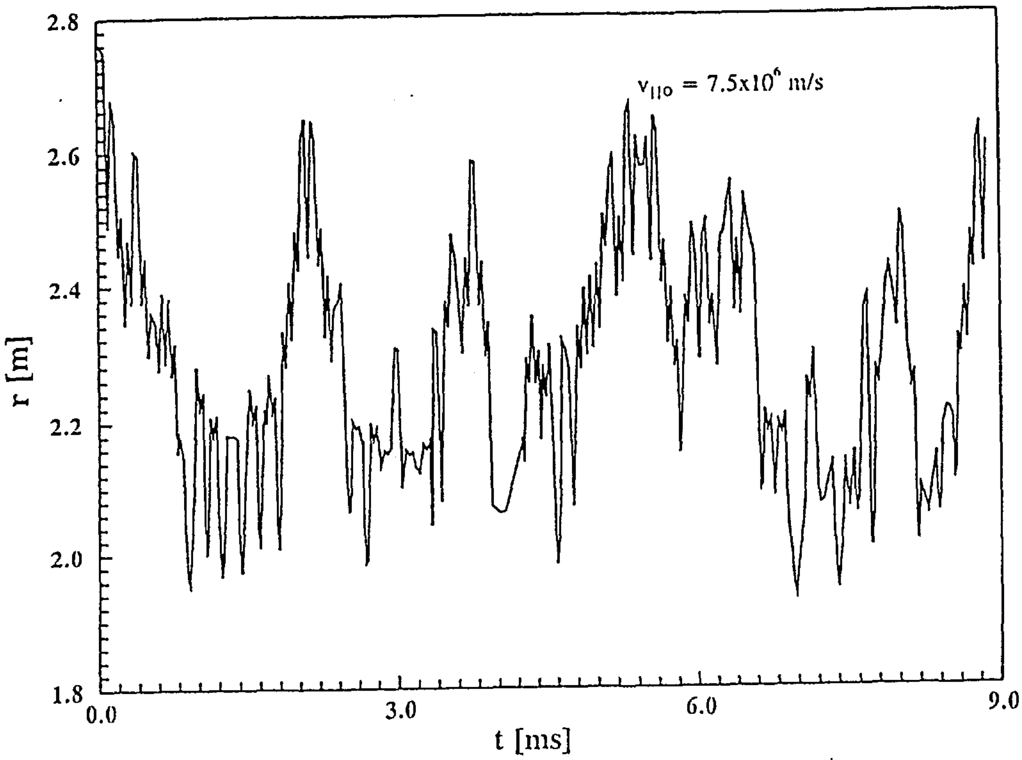
1.6
1.4
1.2
1.0
0.8
r [m]
0.0
0.5
1.0
1.5
2.0
2.5
t [ms]
5.25 x 10^6 m/s
5.5 x 10^6 m/s

Fig. 3.a



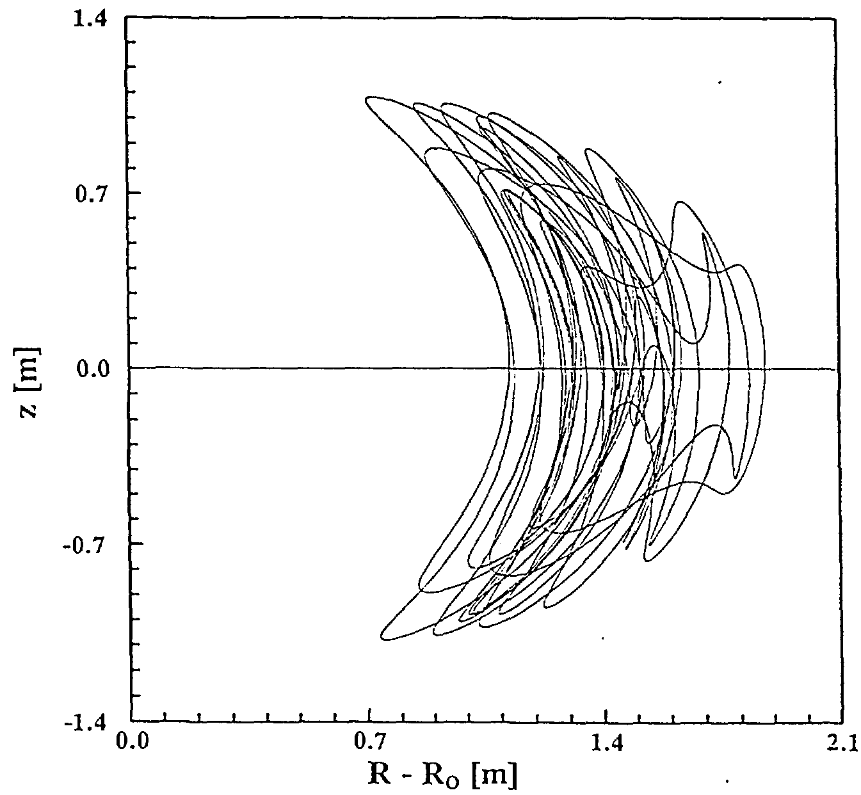
1.4
0.7
0.0
-0.7
-1.4
z [m]
-1.4
-0.7
0.0
0.7
1.4
R - R0 [m]

Fig. 3.b



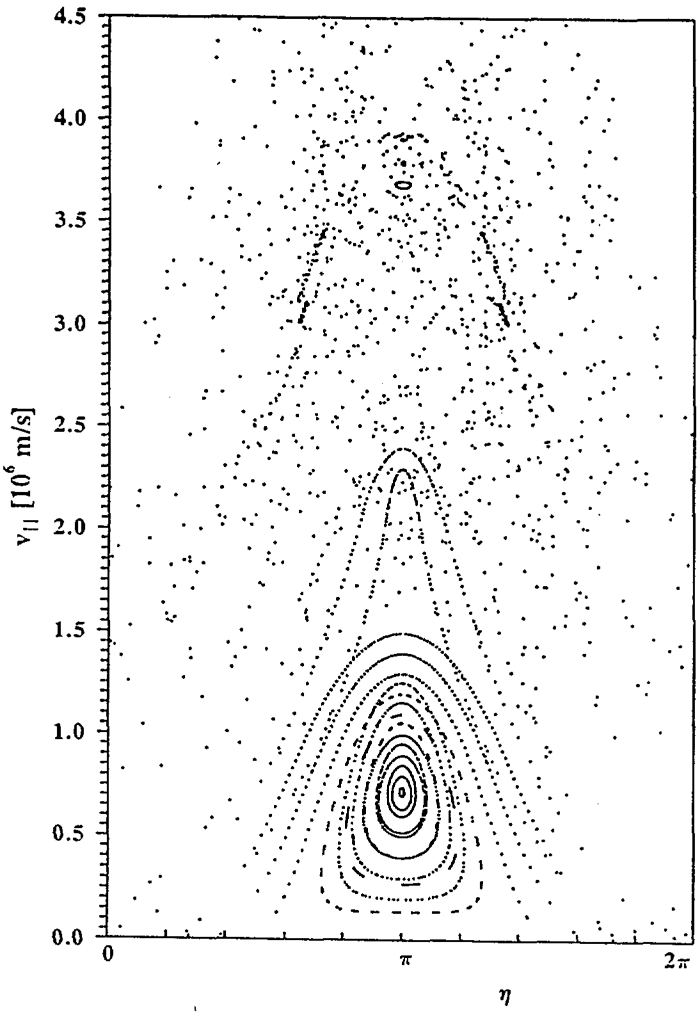
$v_{||0} = 7.5x10^6$ m/s
r [m]
t [ms]
2.8
2.6
2.4
2.2
2.0
1.8
0.0
3.0
6.0
9.0

Fig. 4



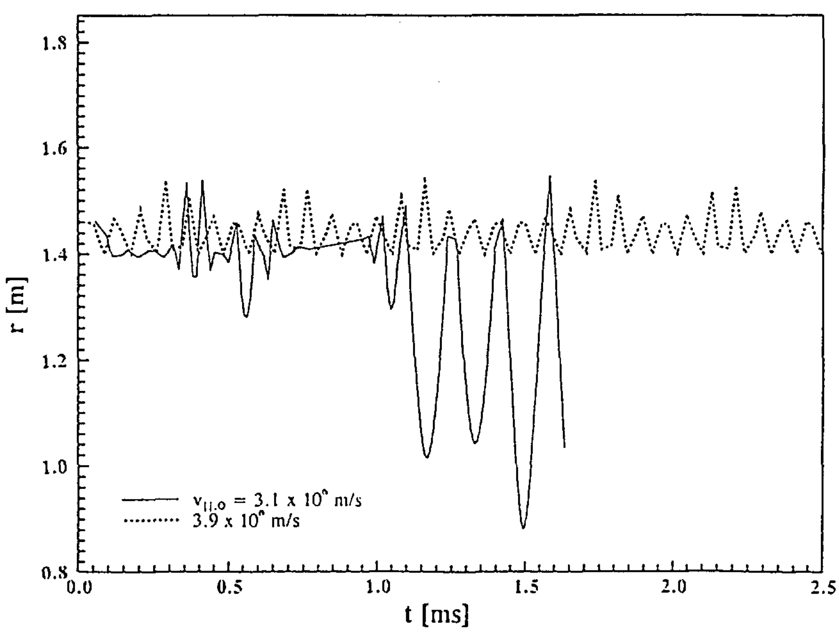
z [m]
R - Ro [m]
1.4
0.7
0.0
-0.7
-1.4
0.0
0.7
1.4
2.1

Fig. 5



4.5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
$v_{||}$ [10^6 m/s]
0
π
2π
η

Fig. 6



1.8
1.6
1.4
1.2
1.0
0.8
r [m]
0.0
0.5
1.0
1.5
2.0
2.5
t [ms]
$v_{||,o}$ = 3.1 x 10^6 m/s
3.9 x 10^6 m/s

Fig. 7