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H. van Dam

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Mekelweg 15, 2629 JB Delft
Tel.: 015-785052, telefax: 015-786422

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Abstract—*A two-point reactor kinetic model for the PROTEUS-reactor is developed and the results are described in terms of frequency dependent reactivity transfer functions for the core and the reflector. It is shown that at higher frequencies space-dependent effects occur which imply failure of the one-point kinetic model. In the modulus of the transfer functions these effects become apparent above a radian frequency of about 100* s^{-1}*, whereas for the phase behaviour the deviation from a point model already starts at a radian frequency of 10* s^{-1}*.*

1. Introduction

The experimental zero-power reactor PROTEUS at the Paul Scherrer Institute, Villigen (Switzerland), is a model of a pebble-bed type graphite moderated reactor. It consists of a relatively small core (about 1 cubic metre) surrounded by a thick graphite reflector. For its criticality it is strongly dependent on the reflector effect. The core, by virtue of its small dimensions and the large neutron migration length in a graphite system, has a high neutron leakage and criticality can only be accomplished by the high neutron reflection coefficient of the surrounding reflector. In this sense core and reflector are strongly coupled and one might expect that the system kinetics could be described satisfactorily by a point model. On the other hand, the neutronic properties of core and reflector are rather different. In particular, neutron lifetimes in core and reflector differ strongly: in an infinite core medium the neutron generation time would be a few tenths of a millisecond, whereas in pure graphite neutron lifetime is about 40 milliseconds. This implies that neutrons leaving the core can dwell in the reflector a relatively long time before returning to the core. As a consequence, the effective neutron generation time in PROTEUS is strongly affected by the neutron migration time in the reflector. This raises the question whether a point kinetic model with a single effective generation time for the neutrons is adequate to describe the system kinetics.

At the PROTEUS-project the above mentioned particular properties of the system have been taken into account extensively in the analyses of the kinetic experiments, e.g. in the form of space-dependent correction factors for analyses based on time-dependent neutron flux (ref. 1). For neutron noise studies and reactor oscillator measurements, however, it is often convenient to describe the system response in the frequency domain, i.e. on the basis of a *reactivity transfer function*. In this report the influence of the reflector on the transfer function is analyzed. In order to enable an analytical treatment in favour of physical insight, a relatively simple two-point one energy group model was adopted, as originally introduced by Cohn (ref. 2).

2. Reactor model

The two-point model uses lumped variables and parameters for the core and reflector region, respectively. Omitting delayed neutrons for the time being, the kinetic equations can be stated as follows:

$$\begin{aligned}\frac{dN_c}{dt} &= (\nu\Sigma_f - \Sigma_{ac} - \Sigma_{cr})vN_c + \Sigma_{rc}vN_r \\ \frac{dN_r}{dt} &= \Sigma_{cr}N_c - (\Sigma_{ar} + \Sigma_{\ell r} + \Sigma_{rc})vN_r\end{aligned} \qquad [1]$$

where the subscript '*c*' refers to the core and the subscript '*r*' to the reflector, e.g. N_c = core neutron population and N_r = reflector neutron population.

The other symbols have their usual meaning (mind the difference between ν = average number of neutrons produced per fission and v = neutron velocity). Some of the macroscopic cross sections are, however, of a less conventional type; they are characteristic for a two-point model and describe the transfer of neutrons between core and reflector:

Σ_{cr} = macroscopic cross section for neutron transfer from core to reflector, i.e. multiplied by the spatially integrated core flux (neutron population times an effective velocity) it gives the total neutron current from core to reflector

Σ_{rc} = idem for neutron transfer from reflector to core

$\Sigma_{\ell r}$ = leakage cross section for the reflector; multiplied by the integrated reflector flux it gives the total neutron current from the reflector to the outside (i.e. not to the core).

It must be noted here that the leakage cross sections can be compared to the well-known DB^2-term encountered in diffusion theory. There is, however, an important difference; for instance, Σ_{cr} describes the leakage current from core to reflector, whereas DB^2 is associated with the *net neutron current* from core to reflector i.e. the balance of the currents described by $\Sigma_{cr}N_c$ and $\Sigma_{rc}N_r$.

Re-arrangement of terms in [1] and introducing some characteristic system parameters leads to a different form of the same equations:

$$\begin{aligned}\frac{dN_c}{dt} &= \frac{\rho_\infty - \alpha_{cr}}{\Lambda_c}N_c + \frac{\alpha_{rc}}{\ell_r}N_r \\ \frac{dN_r}{dt} &= \frac{\alpha_{cr}}{\Lambda_c}N_c - \frac{N_r}{\ell_r}\end{aligned} \qquad [2]$$

where

$\rho_\infty = 1 - \dfrac{1}{k_\infty}$ = infinite core reactivity

$k_\infty = \nu\Sigma_f/\Sigma_{ac}$ = infinite core multiplication factor

$\Lambda_c = \dfrac{1}{v\nu\Sigma_f}$ = neutron generation time for an infinite core

$\ell_r = \dfrac{1}{(\Sigma_{ar}+\Sigma_{\ell r}+\Sigma_{rc})v}$ = neutron lifetime in the reflector

$a_{cr} = \dfrac{\Sigma_{cr}}{\nu\Sigma_f}$ = fraction of fission neutrons produced that escapes to the reflector (and partly returns to the core!)

$a_{rc} = \dfrac{\Sigma_{rc}}{\Sigma_{ar}+\Sigma_{\ell r}+\Sigma_{rc}}$ = fraction of reflector neutrons flowing to the core, being equal to the reflection coefficient or *albedo* of the reflector; mind that all neutrons entering the reflector are either absorbed or leak out or return to the core, the fractional part of the latter being equal to the albedo.

All parameters in the equations [2] should be inferred from some type of neutron transport calculations. From [2] we deduce for the neutron population ratio in the stationary (critical) situation:

$$\frac{N_c}{N_r} = \frac{\Lambda_c}{\ell_r}\cdot\frac{a_{rc}}{a_{cr} - \rho_\infty} = \frac{\Lambda_c}{\ell_r a_{cr}} \qquad [3]$$

which gives us a relation between three system parameters, the *criticality condition*:

$$\rho_\infty = a_{cr}(1 - a_{rc}) \qquad [4]$$

This criticality condition is equivalent to stating that the determinant of the coefficient matrix, if we write [2] in matrix and vector form, should be equal to zero in order to have a non-zero solution.

The effective neutron generation time of a system can be strictly defined with the help of adjoint function theory (ref. 3). Applying this to our two-point model, the adjoint functions are the solutions of the next equation, put into matrix form:

$$\begin{bmatrix} \dfrac{\rho_\infty - a_{cr}}{\Lambda_c} & \dfrac{a_{cr}}{\Lambda_c} \\ \dfrac{a_{rc}}{\ell_r} & -\dfrac{1}{\ell_r} \end{bmatrix} \begin{bmatrix} \psi_c \\ \psi_r \end{bmatrix} = \vec{0} \qquad [5]$$

where the coefficient matrix is the transpose of the matrix in the "forward' equation. The solutions ψ_c and ψ_r can be interpreted as the neutron importance in core and reflector, respectively. From [5] we can deduce the ratio between the neutron importances, which becomes particularly simple if we take into account the criticality condition:

$$\frac{\psi_r}{\psi_c} = \frac{a_{cr} - \rho_\infty}{a_{cr}} = a_{rc} \qquad [6]$$

which means that the importance ratio between neutrons in reflector and core is equal to the reflector albedo, which is in accordance with physical considerations.
The general expression for the neutron generation time in terms of neutron velocities, cross sections, fluxes and adjoint functions (see e.g. ref. 3) leads to the two-point one-group theory form:

$$\Lambda = \frac{\psi_c N_c + \psi_r N_r}{v\nu\Sigma_f \psi_c N_c} = \Lambda_c \left[1 + \frac{\psi_r N_r}{\psi_c N_c}\right] = \Lambda_c + a_{cr} a_{rc} \cdot \ell_r \qquad [7]$$

3. Reactivity transfer functions

In order to deduce the reactivity transfer functions of the system the equation set [1] is extended to include delayed neutrons; for ease of notation we assume one group of delayed neutron precursors, extension to more groups being trivial:

$$\begin{aligned} \frac{dN_c}{dt} &= \{\nu(1-\beta) - \Sigma_{ac} - \Sigma_{cr}\} vN_c + \Sigma_{rc} N_r + \lambda C \\ \frac{dC}{dt} &= \beta\nu\Sigma_f v N_c - \lambda C \\ \frac{dN_r}{dt} &= \Sigma_{cr} v N_c - (\Sigma_{ar} + \Sigma_{\ell r} + \Sigma_{rc}) N_r \end{aligned} \qquad [8]$$

where C denotes the total number of delayed neutron precursors. This set can be written in a form analogous to eq. [2]:

$$\begin{aligned} \frac{dN_c}{dt} &= \frac{\rho - \beta - a_{cr} a_{rc}}{\Lambda_c} N_c + \frac{a_{rc}}{\ell_r} N_r + \lambda C \\ \frac{dC}{dt} &= \frac{\beta}{\Lambda_c} N_c - \lambda C \\ \frac{dN_r}{dt} &= \frac{a_{cr}}{\Lambda_c} N_c - \frac{N_r}{\ell_r} \end{aligned} \qquad [9]$$

In the concept of a transfer function, reactivity is considered as input and neutron population in core or reflector as output variables. Furthermore, the equations should be linearized in order to make them Laplace transformable (ref. 4). The only non-linearity is present in the product of reactivity and core neutron population. This term is linearized in the usual way by considering small deviations from the stationary value N_{co} and neglecting second-order terms. The linear equations are then Laplace-transformed, the delayed-neutron precursor term is eliminated, and transfer functions are obtained by taking quotients of output and input Laplace transforms:

$$C(s) = \frac{1}{N_{co}} \frac{\delta N_c(s)}{\delta\rho(s)} = \frac{1}{\Lambda_c s + \dfrac{a_{cr} a_{rc} \ell_r s}{1+\ell_r s} + \dfrac{\beta s}{s+\lambda}} \qquad [10a]$$

$$R(s) = \frac{1}{N_{ro}} \frac{\delta N_r(s)}{\delta\rho(s)} = \frac{C(s)}{(1 + \ell_r s)} \qquad [10b]$$

C(s) and *R(s)* denote the reactivity transfer functions (RTFs) of core and reflector, respectively; *s* is the Laplace transform variable. In the terms of systems theory, the core RTF has three poles and two zeros, whereas the reflector RTF has three poles and one zero.

As expected, the conventional point-kinetic RTF can be obtained as a special case of [10a] by taking $\ell_r = 0$. Physically a neutron lifetime in the reflector being zero can be interpreted as that neutrons leaking from the core are either instantaneously reflected to the core or instantaneously absorbed in the reflector, so that the neutron kinetics of the core is not influenced by the reflector.

It is interesting to note that the physical effect of the reflector on the core RTF is equivalent to adding an extra delayed-neutron group with average lifetime ℓ_r and effective fraction $a_{cr}a_{rc}$; this fraction can be rather high as will be shown in the next paragraph. The lifetime of these pseudo delayed neutron precursors (on the order of 10 milliseconds) is short compared to the fastest decaying real precursors (lifetime about 0.26 s) and also short compared to the reduced generation time of the system; this implies that the reflector effect will manifest itself at frequencies higher than the break frequency of the one-point RTF.

4. Model parameters

Accurate values for the model parameters should be obtained from detailed transport calculations on the system. Realistic guesses to study the implications of the two-point model can, however, be obtained rather easily. The key parameter is the infinite system reactivity, which for PROTEUS cores is about 0.40 (k-infinity ranges from 1.65 to 1.75). The criticality condition [4] relates reflector albedo to core leakage fraction. This relation is shown in figure 1. In this figure we see that the reflector albedo is surely below 0.7; at the latter value the reactor is even critical if all the core neutrons leak out before partly (70 %) returning. As a representative value for the reflector albedo 0.5 can be taken in view of the reflective properties of graphite and the geometry of PROTEUS. The concurrent core leakage fraction of 0.8 is in accordance with the small core dimensions and the large neutron migration area. Consequently, in this case the product of reflector albedo and core leakage fraction is 0.4 (figure 1), which provides us through equation [7] with a relation between infinite core generation time, reflector lifetime and system generation time.

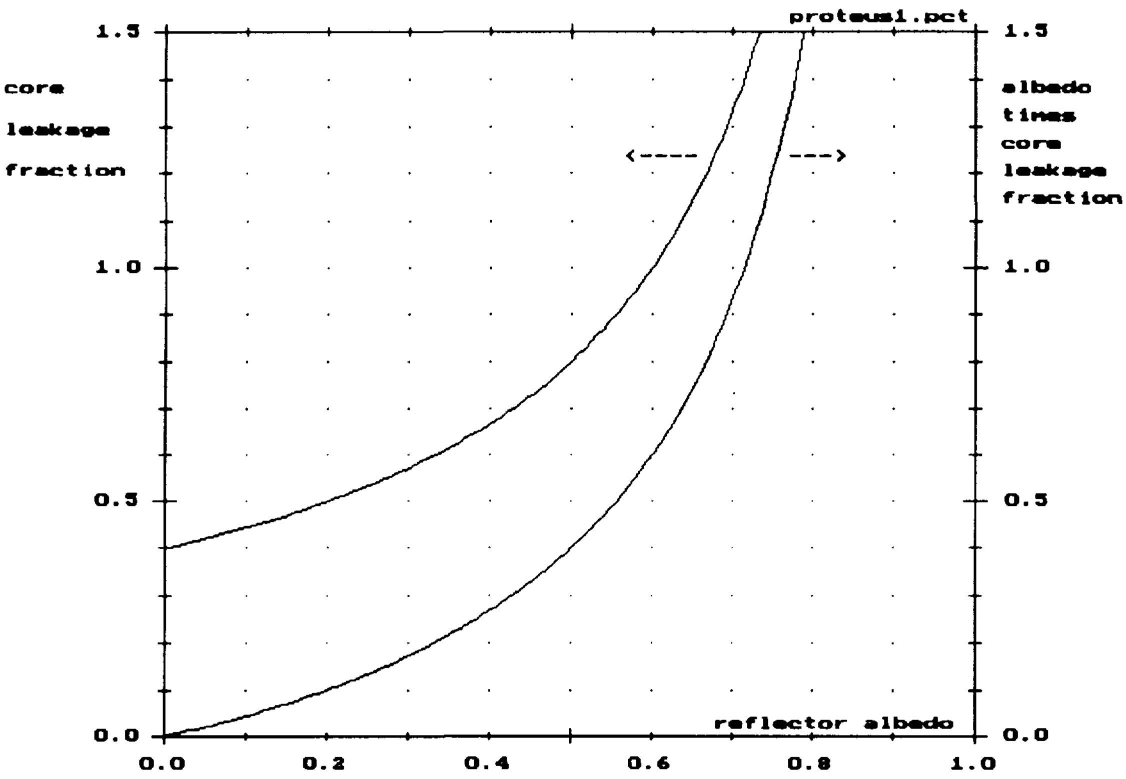

Figure 1. Core leakage fraction and product of core leakage fraction and reflector albedo as a function of the reflector albedo for a critical PROTEUS-reactor.

As a next step we take a fixed system neutron generation time of 1.95 ms; together with a beta-effective of 0.00723 this gives a reduced generation time of 0.27 seconds which is representative for PROTEUS (ref. 5). The table shows three combinations of parameters that were used for further analysis. The combination with zero neutron lifetime in the reflector is equivalent to a one-point reactor model.

Table. Parameters for model analysis

Case	Λ_c (ms)	ℓ_r (ms)	N_r/N_c	ψ_r/ψ_c
1	2	0	0	0
2	0.4	4	8	0.5
3	0.8	3	3	0.5

The ratio of neutron importances is constant and equal to 0.5 being identical to the reflector albedo (eq. [6]). The neutron population in the reflector is 8 and 3 times the core population, respectively. These ratios seem not to be unrealistic, because the average neutron density in the reflector is not far below the density in the core whereas the reflector volume is about 25 times the core volume. For the purpose of further assessment of the model these parameters seem adequate.

5. Results and discussion

The reactivity transfer functions for the core and the reflector are shown in figures 2 to 5 for the cases presented in the previous paragraph. One delayed neutron group was assumed (with a decay constant of 0.08 s^{-1}) because delayed neutron effects can be neglected in the frequency range where the reflector effect is important. However, the use of one condensed delayed neutron group has the consequence that in our results the transfer functions have a clear break point associated with the neutron generation time, whereas in PROTEUS this break point is less clear due to interference with the fastest decaying delayed neutron precursor group. So, at lower frequencies our RTFs have a flattering form.

In figures 2 and 3 the moduli of the RTFs are shown. The core RTF has an upward deviation compared to the one-point model; this is due to the additional zero in the transfer function associated with the neutron lifetime in the reflector. The break point for this effect is the inverse of this lifetime, so the effect is only apparent at higher frequencies (radian frequency > 200 s^{-1}). The reflector RTF has a downward deviation, which is less strong than for the core. For the frequency range of experimental interest (< 100 s^{-1}) the one-point model seems to be adequate to describe the modulus behaviour of the RTF. From the equations [10a] and [10b] it can be inferred that one pole of the RTF is still positioned at the inverse of the reduced generation time, with the generation time given by eq. [7] (so the *system generation time* and not the *core generation time*).

The effects are more clearly seen in the phase behaviour, shown in figures 4 and 5. From 10 rad/s upward there is an increasing phase difference between core and reflector. The reflector shows an increasing phase difference up to 180 degrees with break points arising from the effective neutron generation time and the neutron lifetime in the reflector. In the phase of the core RTF the reflector effect manifests itself in a phase 'hump' due to the fact that the returning neutrons from the reflector are 'faster' than the fission chains in the core. From these results it is expected that experimentally the influence of the reflector can be studied best by measuring phase differences between core and reflector response to reactivity perturbations. Such phase differences are of course the most clear indication for failure of the one-point model. One should be aware that the previous results refer to a *critical system*. In a *subcritical* system the break point associated with the pole is

positioned at higher frequency and might interfere with the zero associated with the reflector effect. Such interference may lead to overestimation of subcriticality for in-core measurements and underestimation in case of in-reflector measurements.
It should be kept in mind that all the results presented here are based on estimated values for system parameters. Before considering to make a more thorough analysis it is advisable to check experimentally if phase differences exist between the core and the reflector RTF. Because deterministic perturbations with sufficiently high frequencies might be difficult to achieve, noise studies should be applied, possibly with an artificial reactivity noise source.

References

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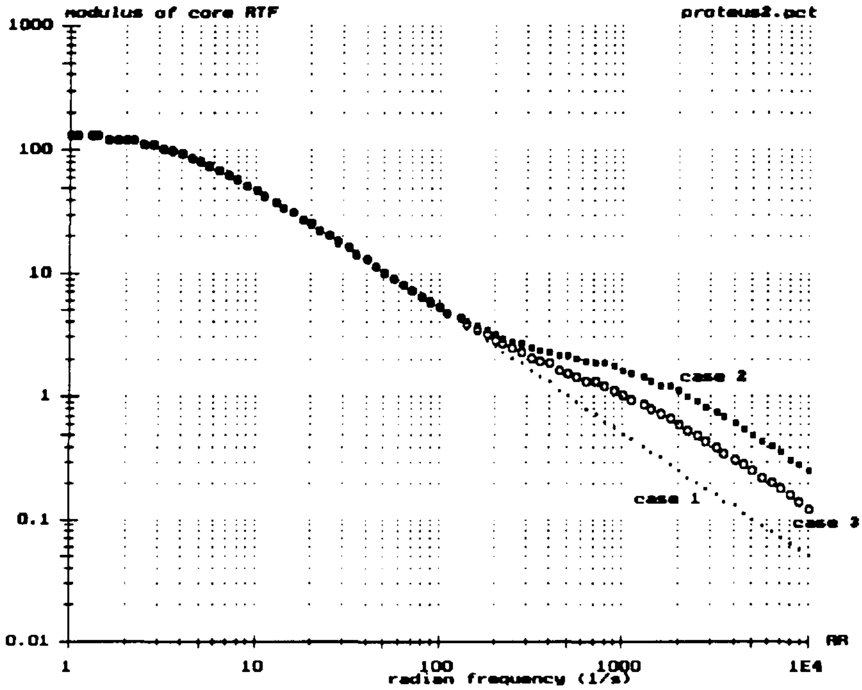

Figuur 2. Moduli of reactivity transfer functions of the core.

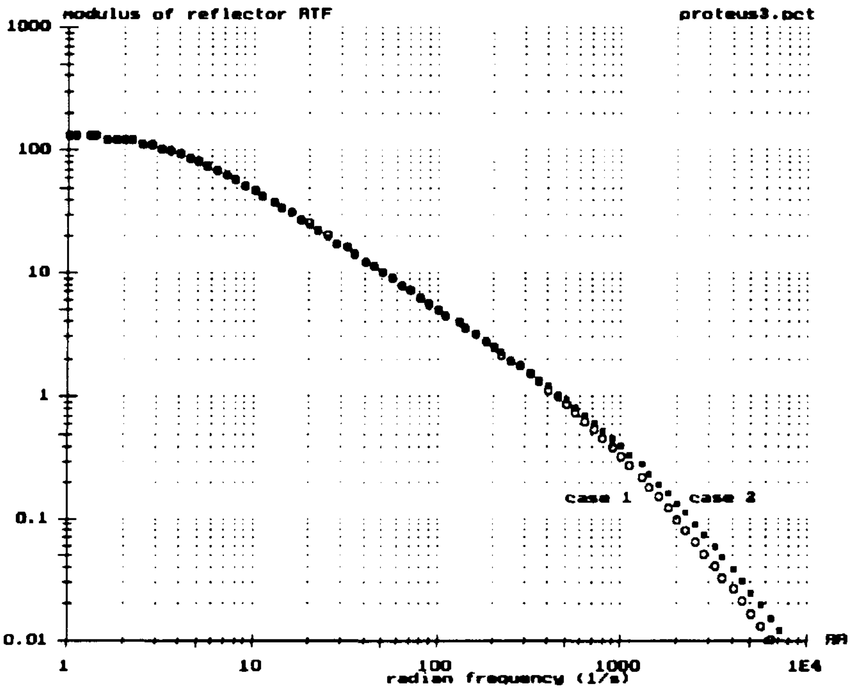

Figure 3. Moduli of reactivity transfer functions of the reflector.

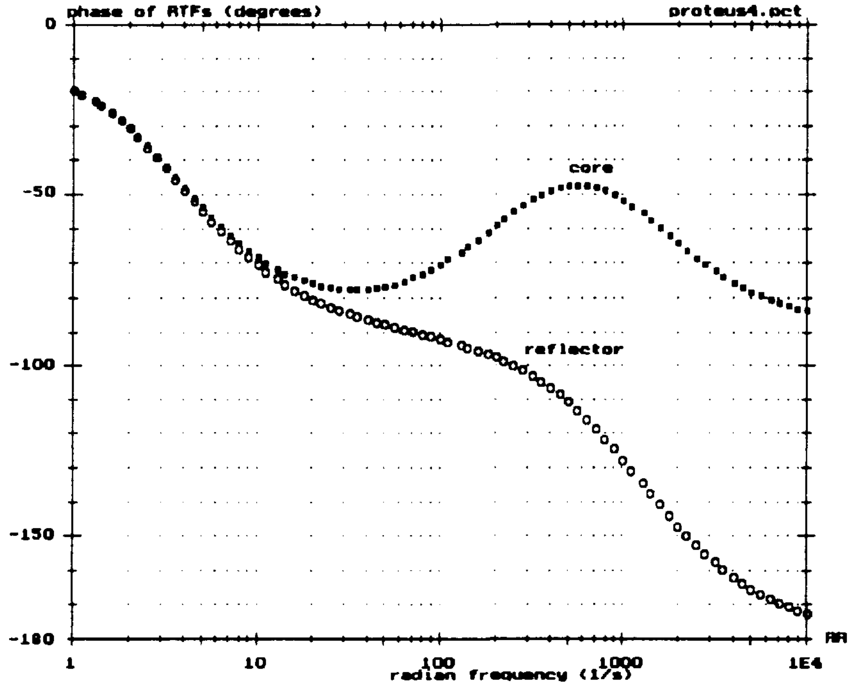

Figure 4. Phases of reactivity transfer functions of core and reflector for case 2.

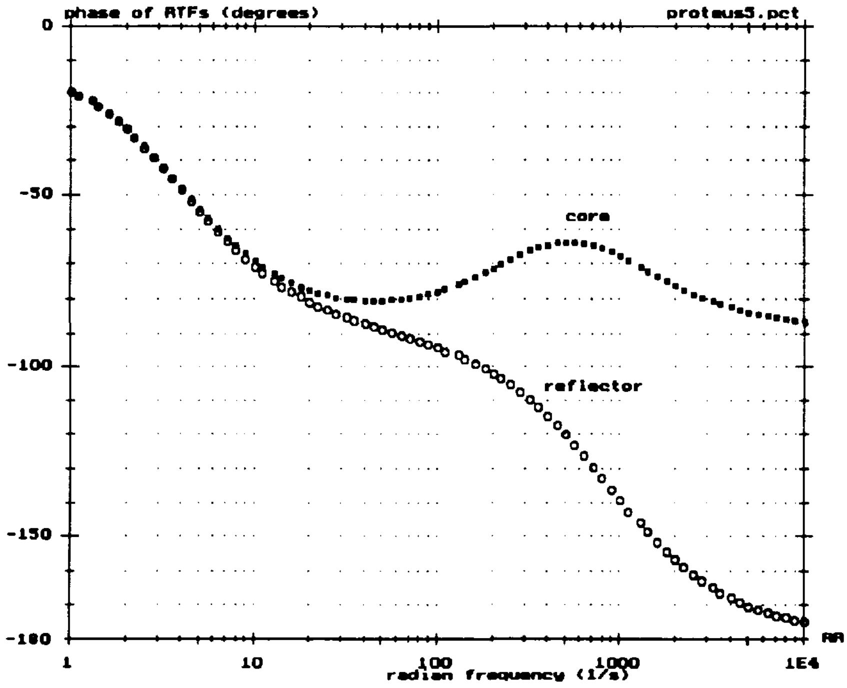

Figure 5. Phases of reactivity transfer functions of core and reflector for case 3.