



**IMPLEMENTATION OF CONSTITUTIVE  
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INTO THE ABAQUS FINITE ELEMENT CODE -  
SOME PRACTICAL CASES IN HIGH  
TEMPERATURE COMPONENT DESIGN AND  
LIFE ASSESSMENT**

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# **Implementation of constitutive equations for creep damage mechanics into the ABAQUS finite element code - Some practical cases in high temperature component design and life assessment**

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**Abstract.** Constitutive equations for creep damage mechanics are implemented into the finite element program ABAQUS using a user supplied subroutine, UMAT. A modified Kachanov-Rabotnov constitutive equation which accounts for inhomogeneity in creep damage is used.

With a user defined material a number of bench mark tests are analysed for verification. In the cases where analytical solutions exist, the numerical results agree very well. In other cases, the creep damage evolution response appears to be realistic in comparison with laboratory creep tests.

The appropriateness of using the creep damage mechanics concept in design and life assessment of high temperature components is demonstrated.

## **1. Introduction**

In the late fifties, Kachanov[1] introduced the continuum damage mechanics concept. Since then the area has been further developed by researchers such as Rabotnov[2], Lemaitre and Chaboche[3] and Murakami[4] and today it is a well known discipline in the field of mechanics of solid materials. Continuum damage mechanics is a phenomenological way of describing the damage evolution between the virgin state and macroscopic crack initiation. The mechanisms creep, fatigue and ductile fracture of homogeneous materials as well as damage of concrete and damage effects in composite materials can all be described by use of this concept.

In the present paper, the appropriateness of using continuum damage mechanics in high temperature design and life assessment of a circumferential weldment is demonstrated. Constitutive equations, describing the deterioration mechanism of creep, are implemented into the ABAQUS finite element code and by studying three typical weldments, an improved understanding of how a weldment behaves when subjected to creep is achieved. With this knowledge, a more accurate prediction of creep damage evolution, advisable positions for non-destructive examination, time to rupture for the weldment, etc., is given.

## 2. Constitutive equations

A modified Kachanov-Rabotnov constitutive equation which accounts for inhomogeneity in creep damage is used[5]. Neglecting plasticity and primary creep the total strain rate is

$$\frac{d\varepsilon_{ij}^{tot}}{dt} = \frac{d\varepsilon_{ij}^{el}}{dt} + \frac{d\varepsilon_{ij}^{cr}}{dt}, \quad (1)$$

where

$$\frac{d\varepsilon_{ij}^{el}}{dt} = \frac{1+\nu}{E} \left[ \left( \frac{d\sigma_{ij}}{dt} \right) - \frac{\nu}{1+\nu} \left( \frac{d\sigma_{kk}}{dt} \right) \delta_{ij} \right], \quad (2)$$

$$\frac{d\varepsilon_{ij}^{cr}}{dt} = \frac{3}{2} B \sigma_e^{n-1} s_{ij} [(1-\rho) + \rho(1-D)^{-n}], \quad (3)$$

and

$$\frac{dD}{dt} = g \frac{A}{\phi+1} \frac{[\alpha\sigma_l + (1-\alpha)\sigma_e]^\nu}{(1-D)^\phi}, \quad (4)$$

$$D_{crit} = 1 - (1-g)^{1/(\phi+1)}. \quad (5)$$

In the equations given above  $\varepsilon_{ij}^{tot}$ ,  $\varepsilon_{ij}^{el}$ ,  $\varepsilon_{ij}^{cr}$ ,  $\sigma_{ij}$  and  $s_{ij}$  are the total strain, elastic strain, creep strain, stress and stress deviator tensor, respectively.  $\sigma_l$  and  $\sigma_e$  are the maximum principal stress and von Mises stress,  $E$  and  $\nu$  the modulus of elasticity and Poisson's ratio,  $D$  and  $D_{crit}$  the damage variable and critical damage where the material creep life is assumed to be fully utilised when  $D/D_{crit}$  reaches the value one.  $\alpha$  is the material constant relating to the multiaxial rupture criterion which ranges from zero to unity,  $B$ ,  $n$ ,  $A$  and  $\nu$  are the material constants relating to the minimum creep strain rate and rupture behaviour,  $g$ ,  $\phi$  and  $\rho$  the constants accounting for the inhomogeneity of the damage where  $\rho$  represents the volumetric ratio of the damaged phase.

### 3. Implementation into ABAQUS

In the present work the finite element program ABAQUS/Standard[6] is used to perform the creep damage analyses. A modified Kachanov-Rabotnov equation is added to the program library by use of the user subroutine UMAT[7]. The user subroutine is programmed in FORTRAN 77.

The user subroutine is called for at each material integration point at every iteration of each increment. When it is called, it is provided with the material state, i.e. stress, solution dependent state variables, temperature etc., at the start of the increment and with the strain increment and the time increment. The subroutine updates the stresses to their values at the end of the increment and calculates the Jacobian matrix, i.e.  $\partial\Delta\sigma_{ij}/\partial\Delta\varepsilon_{ij}$ . Since most constitutive models require the storage of solution dependent state variables, ABAQUS provides possibilities to allocate storage for any number of such variables for each integration point. The damage parameter,  $D$ , is treated in this manner and is updated in the user subroutine during each increment.

Since the damage and stress increments cannot be expressed in closed form, the non-linear equations are solved numerically, using additional routines. Fig. 1 shows the communication between ABAQUS and the separate files.

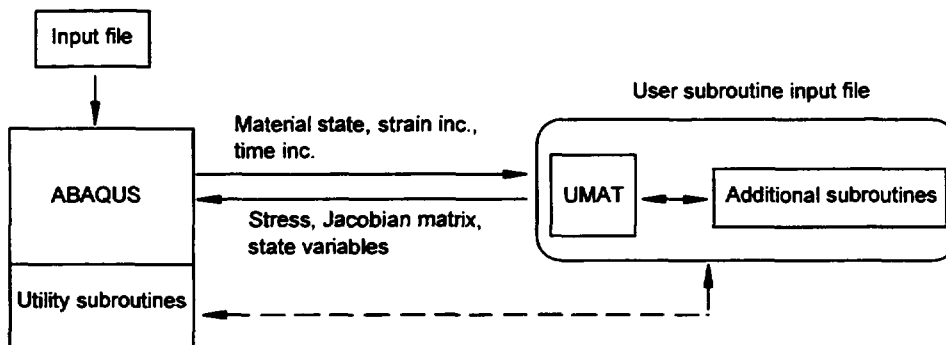


Fig. 1. Communication between ABAQUS and the separate files.

In the present work UMAT is formulated strictly for three dimensional continuum elements. The first step is a purely elastic step and the second step is the creep damage response step. An explicit time

integration scheme is used with a central difference operator[8] according to

$$\frac{d}{dt}(f_{t+\frac{1}{2}\Delta t}) = \frac{\Delta f}{\Delta t}, \quad (6)$$

$$f_{t+\frac{1}{2}\Delta t} = f_t + \frac{\Delta t}{2}, \quad (7)$$

where  $f$  is an arbitrary function,  $f_t$  its value at the beginning of the increment,  $\Delta f$  the change of the function over the increment and  $\Delta t$  the time increment. Discretisation of the constitutive equations (1) and (4), results in a set of six coupled non-linear equations which are numerically solved by using a globally convergent Newton method[9].

With a user defined material a number of bench mark tests were analysed for verification. In the cases where analytical solutions exist, the numerical results agreed very well. In other cases, the creep damage evolution response seemed to be realistic in comparison with laboratory creep tests.

#### **4. High temperature design and life assessment by use of the continuum damage mechanics concept**

In components where geometrical and/or material discontinuities are present, continuum damage mechanics simulations are particularly useful in order to understand the creep behaviour of the component. Using this concept, stress redistribution due to the damage evolution can be taken into account and a more profound understanding of how the component behaves when subjected to creep is achieved[10-14].

##### ***4.1 Finite element modelling and simulation***

In the present investigation, the creep damage evolution in a circumferential V-shaped weldment in a piping system is investigated, see Fig. 2. The outer diameter and the wall thickness of the pipe are 500 and 40 mm, respectively. The welded pipe is subjected to an internal pressure resulting in a nominal hoop stress of 110 MPa and an axial stress of 50.2 MPa.

Three different combinations of creep properties of the weldment constituents, i.e. parent metal (PM), weld metal (WM) and heat

affected zone (HAZ), are studied by altering the characteristics of the weld metal. These three combinations represent three typical weld systems that can be found in power plants of today.

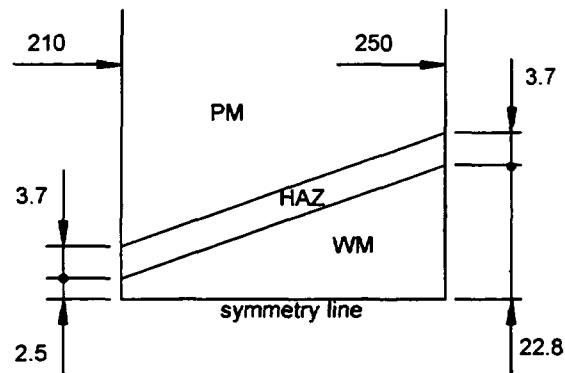


Fig. 2. Geometry of pipe with weldment.

Table 1 shows the material parameters used for the three cases. The constants are based on creep tests of weldments carried out at the Swedish Institute for Metals Research[15]. In [16] it was suggested that  $\alpha$  equals 0.43 for the ferritic steels 0.5Cr0.5Mo0.25V and 2.25Cr1Mo why the same value is used for the parent material in the present paper. For the weld and HAZ metals, no data for  $\alpha$  were available and hence the same value as that of the parent material is used.

Table 1. Constants in constitutive equations for PM, HAZ, matched WM, creep-soft WM and creep-hard WM, respectively.

Constant	PM	HAZ	WM Matched	WM Creep-soft	WM Creep-hard
$B$	1.940e-15	1.540e-13	1.772e-14	8.860e-14	5.907e-15
$n$	4.354	3.925	3.870	3.870	3.870
$A$	8.325e-13	4.365e-12	3.800e-13	4.749e-13	3.304e-13
$\nu$	3.955	3.750	4.110	4.110	4.110
$g$	0.961	0.955	0.965	0.965	0.965
$\phi$	1.423	2.017	0.6517	0.6517	0.6517
$\rho$	0.393	0.280	0.0985	0.0985	0.0985
$\alpha$	0.43	0.43	0.43	0.43	0.43
$E$	160000	160000	160000	160000	160000
$\nu$	0.3	0.3	0.3	0.3	0.3

The structure is modelled by use of the 20-node solid element C3D20R and axisymmetrical boundary conditions are utilised.

#### *4.2 Results from damage simulation of a matched weldment*

Fig. 3 shows the hoop stress in the matched weldment at different times characterising the stress redistribution that takes place during the life of the weldment. Due to a higher minimum creep strain rate of the HAZ than that of the parent and weld metal, the HAZ region is off-loaded. This off-loading begins as soon as the creep process starts. In case the tertiary creep would be neglected, a steady state stress field would eventually be established.

The first plot in Fig. 3 at 4920 hours, shows the hoop stress just before the tertiary effects start to influence the stresses in the weldment region, i.e. the stress field that comes closest to a steady state. The following stress plots show the hoop stress at times when the tertiary effects have a substantial influence. As seen at the end of the creep life of the weldment, the hoop stress in the HAZ starts to increase again. This is explained by the fact that the creep strain rate of the parent and weld metal has become larger than that of the HAZ due to higher creep damage in the PM and WM.

In Fig. 4, the creep damage evolution is shown. As seen from the damage plots, the creep damage is least developed in the HAZ region even though the creep rupture strength of the HAZ is lower than that of the other two weldment constituents. The explanation to this is the off-load of the HAZ as mentioned earlier. The damage evolution in the parent material is somewhat similar to that of the weld metal as expected. Fully damaged material is first found in the weld metal close to the HAZ and final rupture occurs at 11020 hours.

#### *4.3 Results from damage simulation of a creep-soft weldment*

As for the matched weldment, redistribution of stresses takes place due to differences in creep strain rates between the weldment constituents. In this case, the material discontinuity between the HAZ and the weld metal is reduced compared to the former case. The first stress plot in Fig. 5 shows the 'steady state' stress field where the HAZ and weld metal are off-loaded due to their higher minimum creep strain rate. As



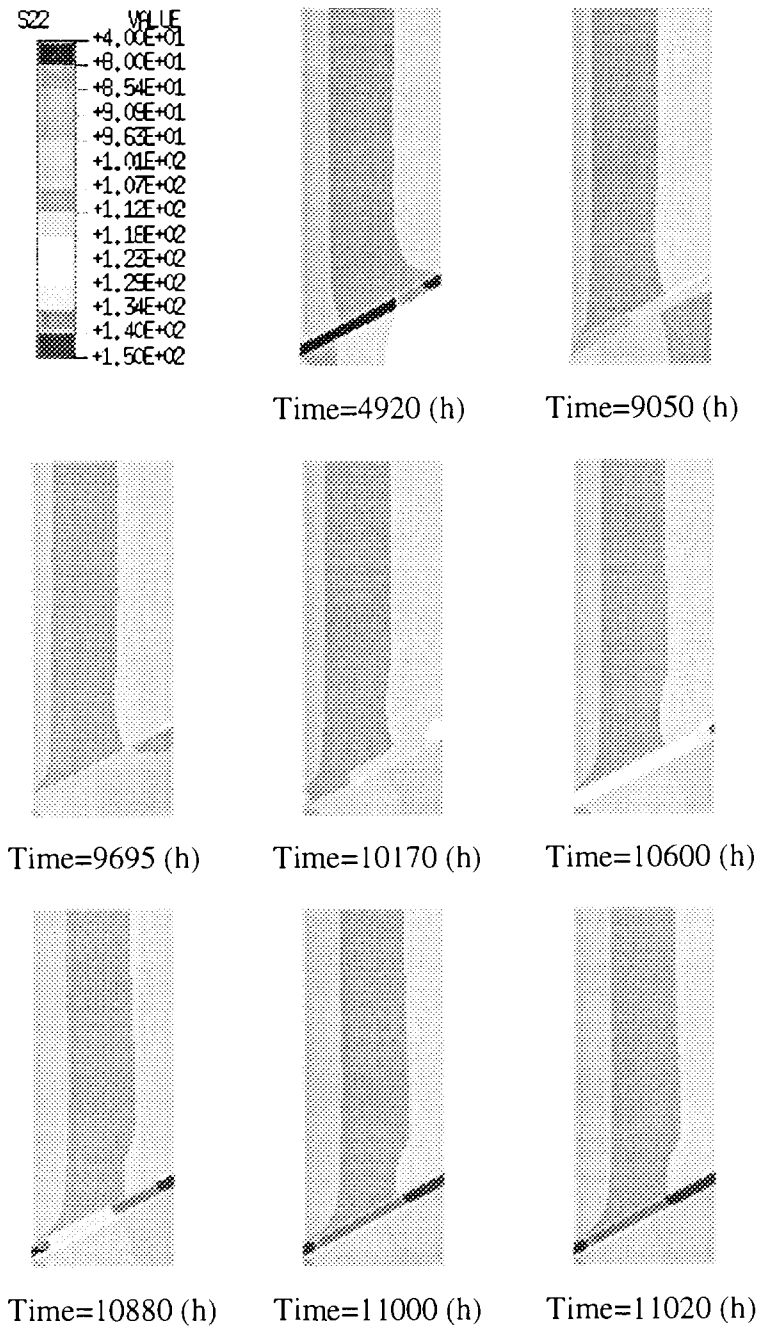


Fig. 3. Hoop stress in the matched weldment (MPa).

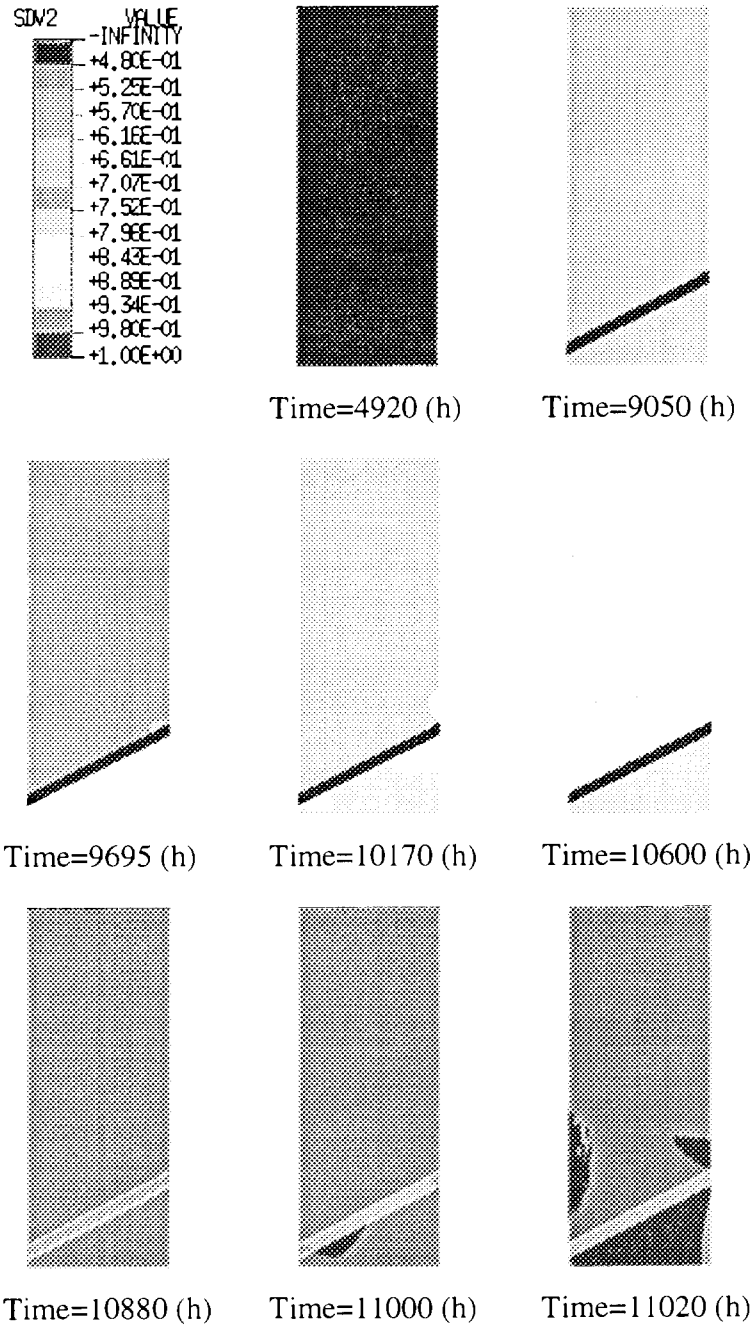


Fig. 4. Creep damage ratio,  $D/D_{crit}$ , in the matched weldment.

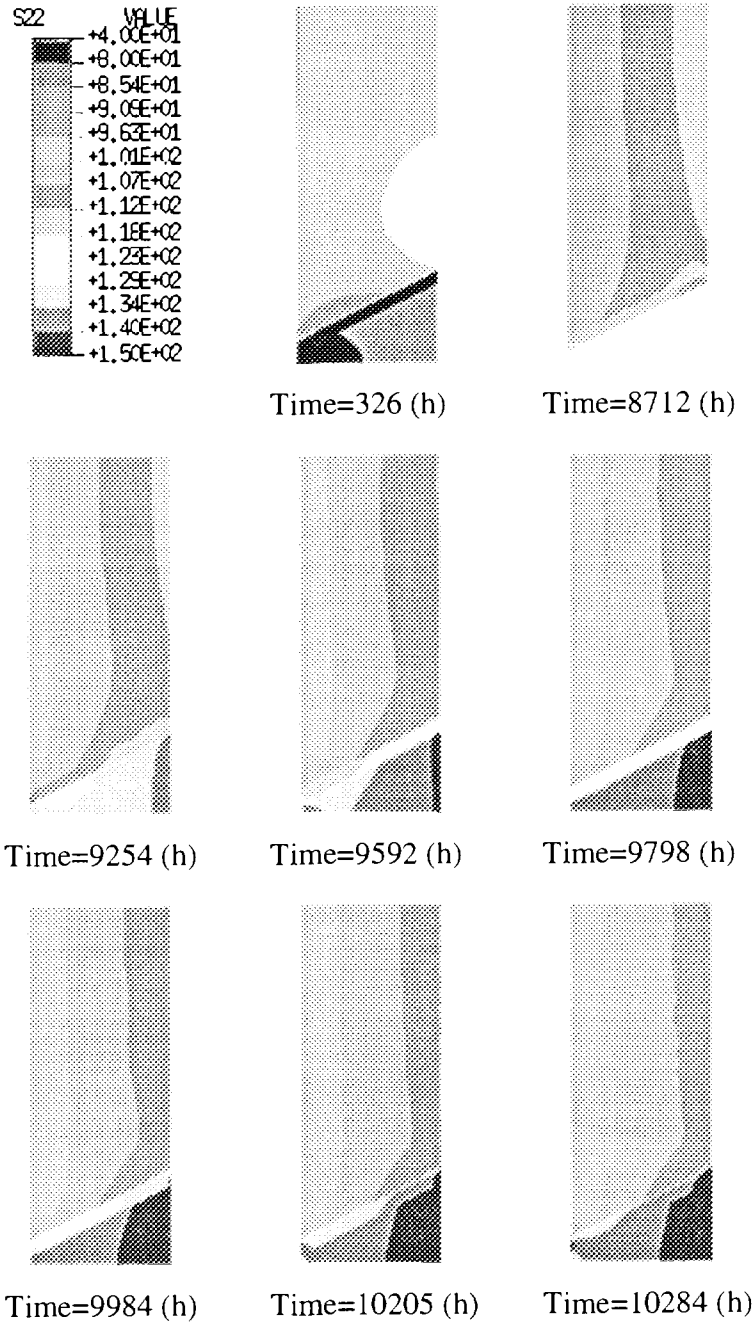


Fig. 5. Hoop stress in the creep-soft weldment (MPa).

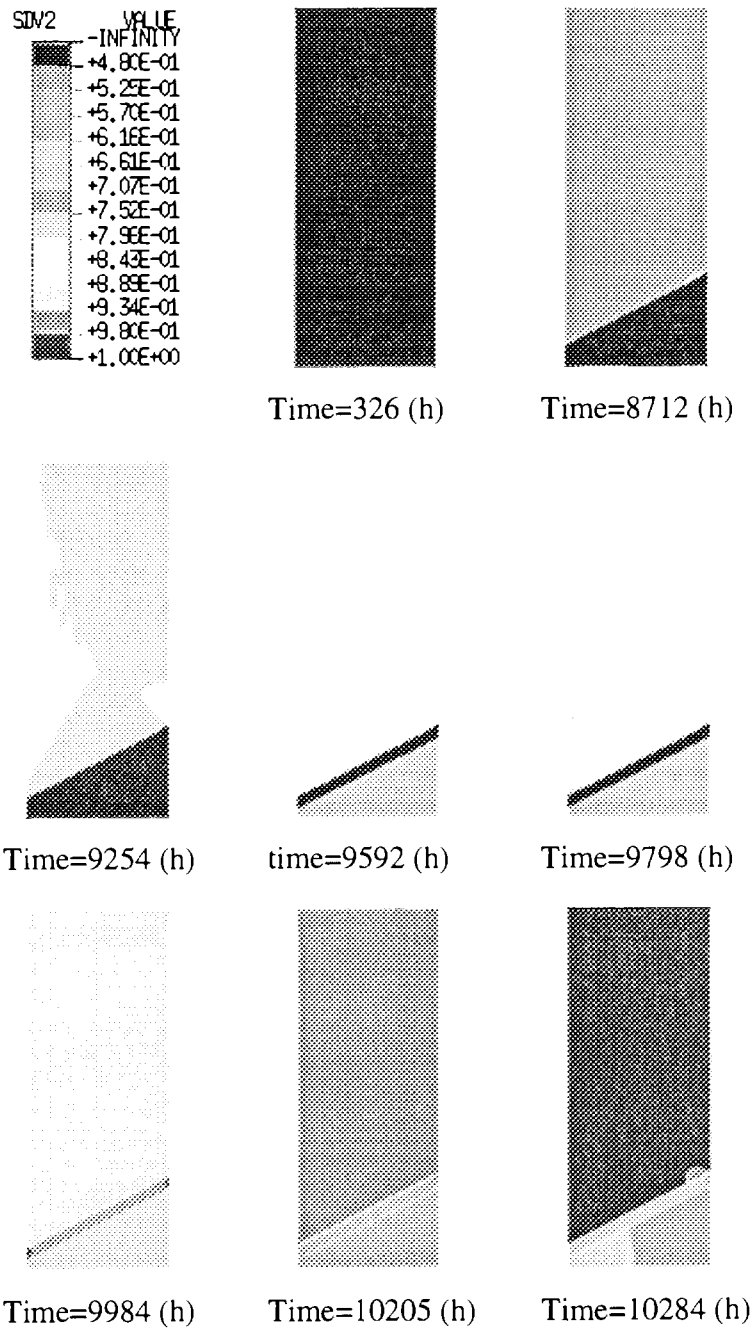


Fig. 6. Creep damage ratio,  $D/D_{crit}$ , in the creep-soft weldment.

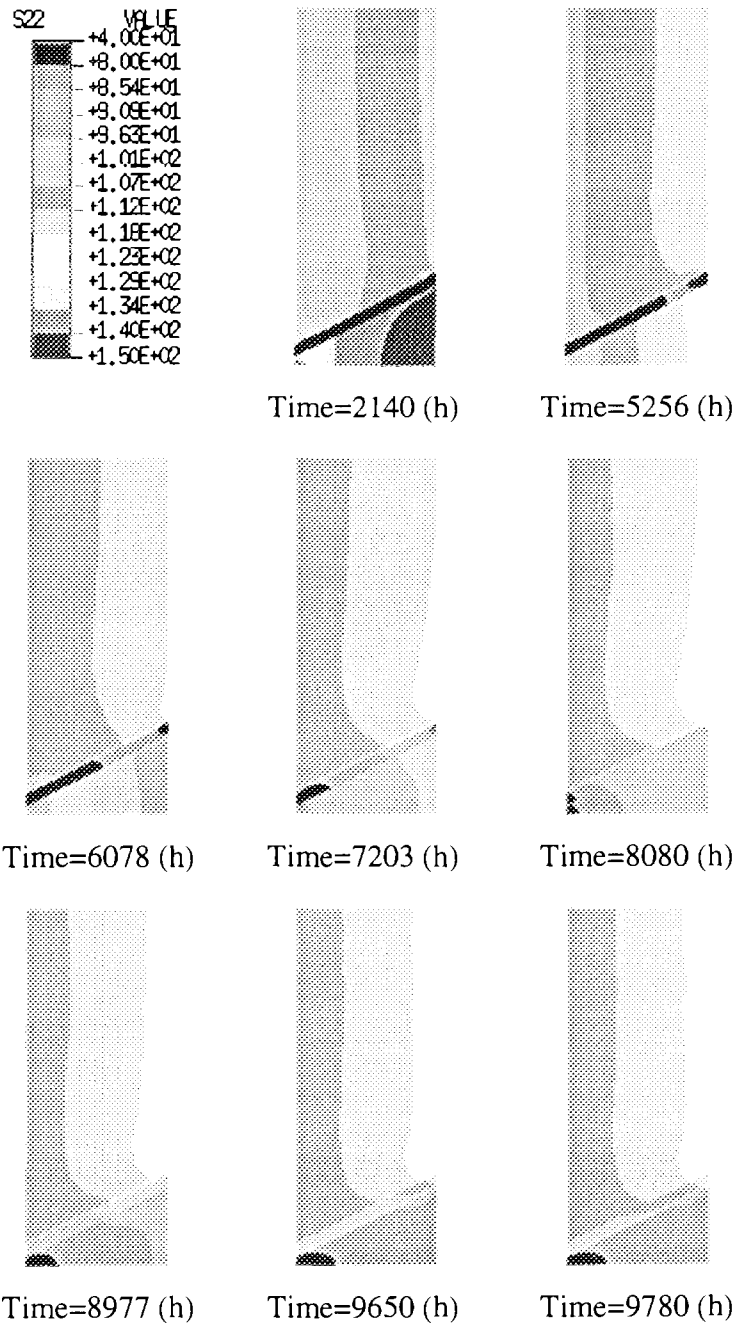


Fig. 7. Hoop stress in the creep-hard weldment (MPa).

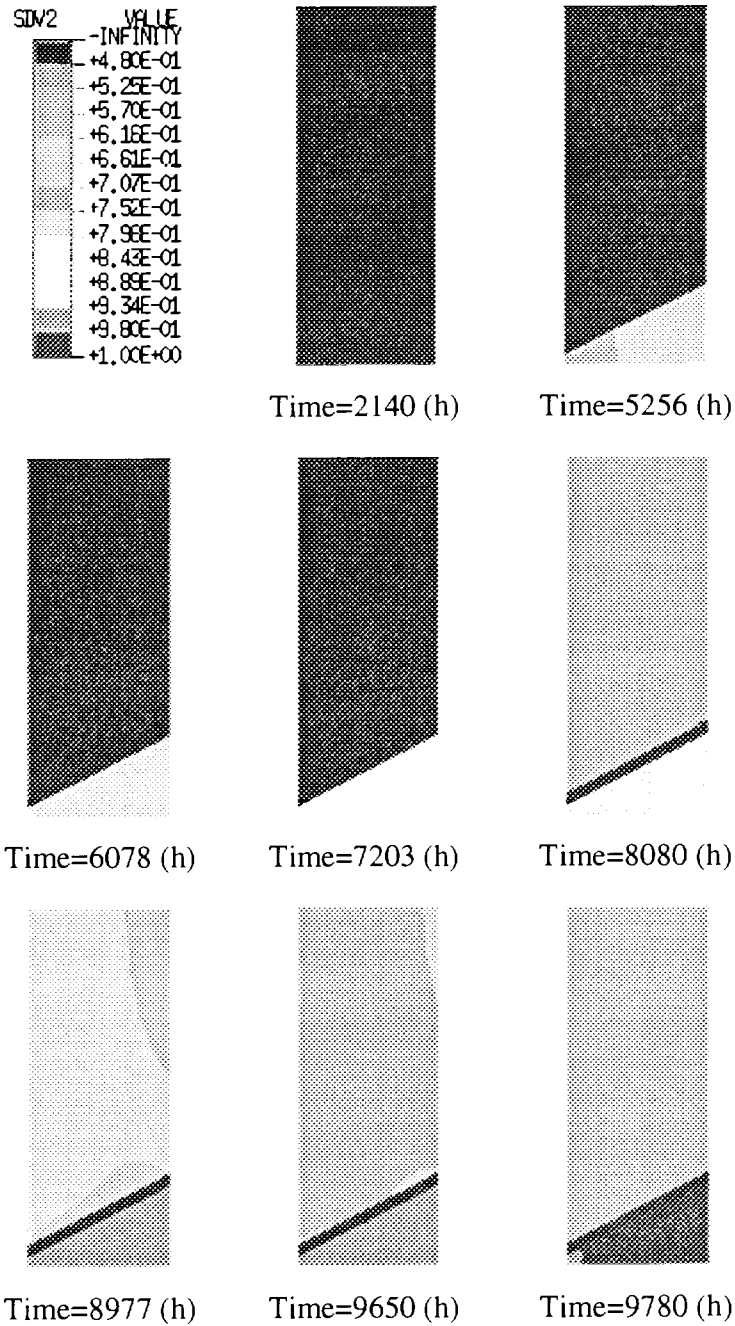


Fig. 8. Creep damage ratio,  $D/D_{crit}$ , in the creep-hard weldment.

a result of the redistribution of stresses, the parent material in the vicinity of the HAZ is on-loaded.

In the following stress plots in Fig. 5, the influence of creep damage on the hoop stress is shown. At the end of the life of the weldment, the parent material is heavily damaged as seen in Fig. 6, resulting in a substantial increase in creep strain rate. This change in strain rate results in the fact that the weld metal is on-loaded, i.e. at the end of the life of the creep-soft weldment it becomes creep-hard.

The failure of the creep-soft weldment occurs in the parent material after 10284 hours, see Fig. 6, despite the fact that the creep rupture strength of the parent material is higher than that of the WM and HAZ. As for the matched weldment, the explanation for this is the resulting stress redistribution.

#### *4.4 Results from damage simulation of a creep-hard weldment*

For the creep-hard weldment, the minimum creep strain rate of the weld metal is lower than that of the parent material. An effect of this is that on-loading of the weld metal and off-loading of the HAZ and the parent material in the vicinity of the weld take place when the weldment is subjected to creep. The first plot in Fig. 7 shows the redistributed hoop stresses at 2140 hours which is just before the tertiary phase is entered in the weld metal. The following plots show how the tertiary effects influence the hoop stress field as a function of time. At the end of the life of the weldment, the weld metal is heavily damaged as seen in Fig. 8, resulting in a substantial increase in creep strain rate. This change in strain rate results in the fact that the weld metal is off-loaded, i.e. at the end of the life of the creep-hard weldment it becomes creep-soft.

The failure of the creep-hard weldment occurs in the weld metal after 9780 hours, see Fig. 8, despite the fact that the creep rupture strength of the weld metal is higher than that of the PM and HAZ. As for the matched and the creep-soft weldment, the explanation for this is the resulting stress redistribution.

#### *4.5 Summary of damage simulations*

The results of the simulations show that the creep damage evolution and time to rupture differ between the three cases. For example, time to rupture for the creep-hard weldment is more than 10 % lower than that

of the matched one. Changing the material creep properties of the weldment constituents or the over all loading condition, the differences may even be further enhanced.

In design of pressure vessel and piping systems, a factor of safety of 1.5 is applied on primary stresses[17]. For many low alloy steels, this margin is equivalent to a factor of safety of 5 on rupture time. Applying this margin on the present cases, the design life of the matched, creep-soft and creep-hard weldment would be 2204, 2057 and 1956 hours, respectively.

Uniaxial creep testing of the five different weldment constituents at a stress of 110 MPa, i.e. the nominal hoop stress in the present analyses, would give a rupture time of 10137, 5067, 10717, 8576 and 12326 hours for the PM, HAZ, matched WM, creep-soft WM and the creep-hard WM, respectively. In most high temperature design codes of today, the weldment creep properties are not considered. Practically, this means that at positions where weldments are located, the parent material properties are used which here results in a design life of 2027 hours. Basing the design life of the weldments on the weakest zone, i.e. the HAZ, would give 1013 hours which is far too conservative. In the ASME Code Case N-47[18], the ratio of the creep rupture strength of the weld metal and the parent material is considered, assuming that the design life of a creep-soft weldment is shorter than that of a creep-hard one. With the material characteristics of the weld systems used in the present study, the opposite prevails.

Only by studying the whole weld system with correct material properties for the weldment constituents, rational design and life assessment procedures can be achieved.

## **5. Discussion**

The stress and damage plots show that continuum damage mechanics simulations form a very useful basis for the interpretation of how components, with geometrical and/or material discontinuities, behave in the creep regime. This knowledge can be very valuable when choosing positions for non-destructive examination and when improving existing design codes and life assessment procedures for high temperature components. It is understood that correct material data for different parts of a structure are necessary for the understanding of its creep behaviour. Concerning weldments, the lack of creep property data, particularly for the HAZ, is still an obstacle. Collection of data



together with further testing, including multi-axial and long term creep testing, is an important task. Experimental verification is also an essential task in the evaluation of the possibility to use the continuum damage mechanics concept in predicting the creep behaviour of high temperature components.

To analyse every component in a high temperature plant by use of the continuum damage mechanics technique will probably not be the most convenient way of designing a complete system. Instead, design diagrams can be established by performing parametric analyses of known systems, e.g. different weld systems with known creep properties. These design diagrams are then easily used by the designer.

As a result of welding, residual stresses are introduced. Without post weld heat treatment a tensile membrane stress of yield point magnitude can be present. When the welded component is subjected to creep, the residual stresses are reduced through relaxation. The time it takes for the stresses to relax to a negligible level is essentially a function of material and temperature. In the present study, the residual stresses are not considered. It is assumed that the stress redistribution, due to the weldment mis-match, dominates over the residual stresses. This is the fact as long as the temperature is sufficiently high above the creep limit.

In design and particularly life assessment of high temperature components, the understanding of crack initiation and crack propagation is essential. Where macro cracks emanate from bulk creep cavitation, i.e. macro cracks formed by coalescence of micro cracks, the concept presented in this paper is applicable. However, for cracks that start from a single defect, for example a fusion line defect in a weldment, the fracture mechanics concept or the continuum damage mechanics concept, using a local approach, must be used.

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