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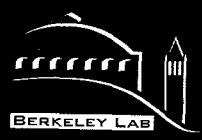

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Crystalline Structure in the Confined-Deconfined Mixed Phase: Neutron Stars as an Example

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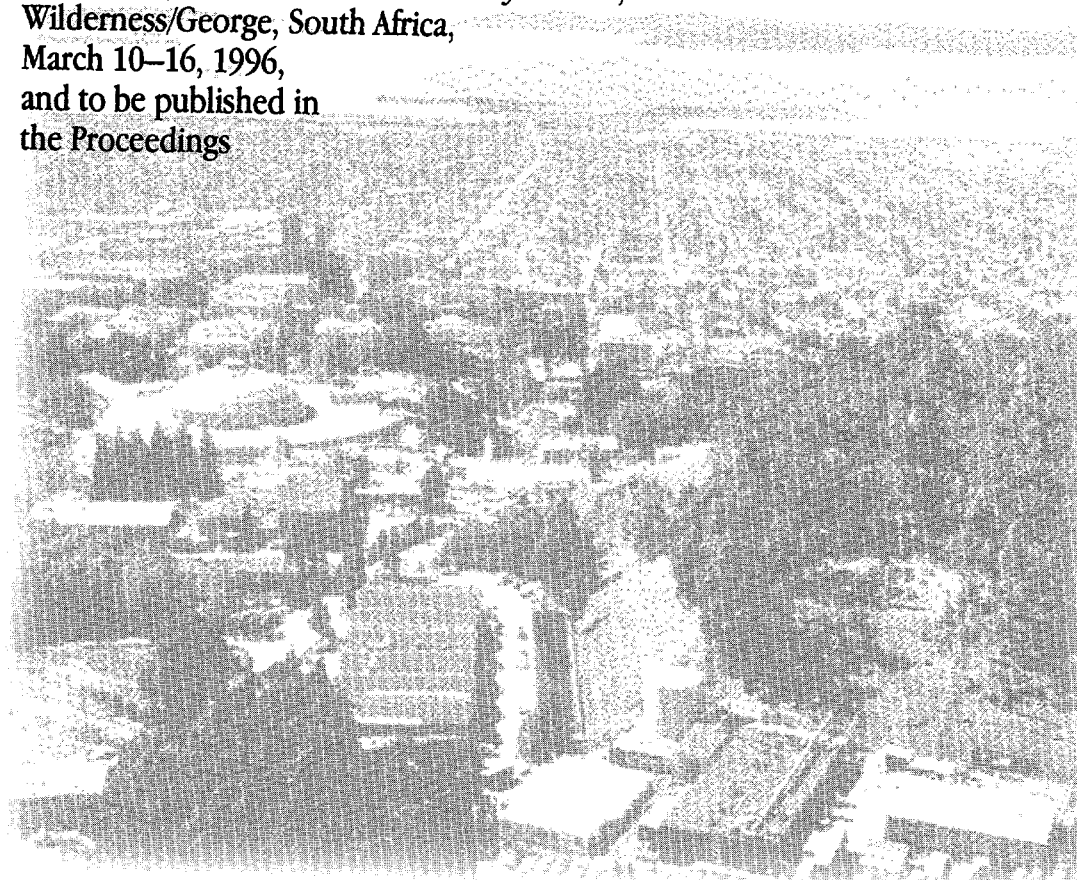

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CRYSTALLINE STRUCTURE IN THE CONFINED-DECONFINED MIXED PHASE: NEUTRON STARS AS AN EXAMPLE †

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ABSTRACT

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We review the differences in first order phase transition of single and multi-component systems, and then discuss the crystalline structure expected to exist in the mixed confined deconfined phase of hadronic matter. The particular context of neutron stars is chosen for illustration. The qualitative results are general and apply for example to the vapor-liquid transition in subsaturated asymmetric nuclear matter.

1 Introduction

First order phase transitions are very familiar only in one-component substances such as water. The transition in multi-component substances is very different [1,2]. In such systems the common pressure and all other properties of each phase varies with the proportion of the phases in equilibrium. This has unique consequences in certain situations, such as in the presence of a gravitational field. More than that, when one of the independent components is electrically charged, the two phases in equilibrium may form a Coulomb lattice of the rare phase immersed in the dominant one [2]. These are quite general theorems and do not depend on details of models. They will hold for diverse systems such as in the gas-liquid transition in nuclear physics, in the confined-deconfined transition in nuclear collisions and the same transition in neutron stars as well as in chemical systems. However in the first two cases mentioned, the ideal equilibrium situation is probably never achieved, and the observed results will depend on such things as how central or peripheral the collision was, how much of the colliding nuclei thermalized and had time to reach equilibrium in every sense, and so on. Nevertheless, it is important to keep in mind, even for such systems as the gas-liquid nuclear transition, what the ideal transition would look like. *There is no plateau* as is often referred to in the ideal case *except* for isospin symmetric systems.

Our aim here is to briefly recapitulate the physical reason for the different behavior of a first order phase transition in single- and multi-component substances, and then to compute the varying geometry of the crystalline structure as a function of proportion of the phases in equilibrium. We shall do this in the

context of the confined-deconfined phase transition in neutron star matter— matter that is charge neutral and in equilibrium with respect to all baryon and quark species. The results would be qualitatively similar for the liquid-vapor transition in sub-saturated nuclear matter if equilibrium were achieved.

2 Degrees of Freedom in Multi-component System

Consider a substance composed of two conserved 'charges' or independent components— Q of one kind, B of the other. In the case of a neutron star, these could denote the net electric charge number (in units of e) and baryon charge number. Let the substance be closed and in a heat bath. Define the concentration by $c = Q/B$. The ratio is fixed *only* as long as the system remains in one pure phase or the other! When in the mixed phase the concentration in each of the regions of one phase or the other may be different and they are restricted *only* by the conservation on the total numbers,

$$c_1 = Q_1/B_1, \quad c_2 = Q_2/B_2, \qquad (Q_1 + Q_2 = Q, \quad B_1 + B_2 = B)\,. \tag{1}$$

If the internal forces can lower the energy of the system by rearranging the concentration, they will do so. The essential point is that conservation laws in chemical thermodynamics are global, not local. The concentrations, c_1, c_2 will be optimized by the internal forces of the system at each proportion of the phases, so that the properties of the phases are *not* constant in the mixed phase. This contrasts with the liquid-vapor transition in water.

The mathematical proof of the above properties is not nearly so illuminating as the physical verbal proof above, but we give it for completeness. The Gibbs condition for phase equilibrium is that the chemical potentials μ_b, μ_q corresponding to B and Q, temperature T and the pressures in the two phases be equal,

$$p_1(\mu_b, \mu_q, T) = p_2(\mu_b, \mu_q, T) \tag{2}$$

As discussed, the condition of *local* conservation is stronger than required. We apply the weaker condition of *global* conservation,

$$< \rho > \equiv (1-\chi)\rho_1(\mu_b, \mu_q, T) + \chi\rho_2(\mu_b, \mu_q, T) = B/V\,, \tag{3}$$

$$(1-\chi)q_1(\mu_b, \mu_q, T) + \chi q_2(\mu_b, \mu_q, T) = Q/V\,, \quad \chi = V_2/V\,. \tag{4}$$

Given a temperature, the above three equations serve to determine the two independent chemical potentials and V for a *specified* volume fraction χ of phase '2' in equilibrium with phase '1'. We note that the condition of global

conservation expressed by (3) and (4) is compatible, together with (2), with the number of unknowns to be determined. It would *not* be possible to satisfy Gibbs conditions if *local* conservation were demanded, for that would replace (4) by *two* equations, such as $q_1(\mu_b, \mu_q, T) = Q_1/V_1$, $q_2(\mu_b, \mu_q, T) = Q_2/V_2$, and the problem would be over determined. Note also that the solution of the equations is different for each proportion of the phases χ so that all properties, including the common pressure, vary through the mixed phase.

The internal force that can exploit the degree of freedom opened by *global* charge neutrality and which is closed to one in which *local* neutrality is artificially enforced, is the isospin restoring force experienced by the confined phase of hadronic matter. The hadronic regions of the mixed phase can arrange to be more isospin symmetric than in the pure phase by transferring charge to the quark phase in equilibrium with it. Symmetry energy will be lowered thereby at only a small cost in rearranging the quark Fermi surfaces. Thus the mixed phase region of the star will have *positively* charged regions of nuclear matter and *negatively* charged regions of quark matter.

3 Structure in the Mixed Phase

The Coulomb interaction will tend to break the regions of like charge into smaller ones, while this is opposed by the surface interface energy. Their competition will be resolved by *forming a lattice of the rare phase immersed in the dominant one whose form, size and spacing will minimize the sum of surface and Coulomb energies.*. In other words, a crystalline lattice will be formed.

Consider a Wigner-Seitz cell of radius R containing the rare phase object of radius r and an amount of the dominant phase that makes the cell charge neutral. The whole medium can be considered as made of such non-interacting cells. The solution to problems involving a competition between Coulomb and surface interface energies is universal. The radius of the rare phase immersed in the other and the minimum of the sum of Coulomb and surface energies, in the case of three geometries, slabs, rods and drops are

$$\frac{1}{r^3} = \frac{4\pi[q_H(\chi) - q_Q(\chi)]^2 e^2 f_d(x)}{\sigma d}, \quad d = 1, 2, 3, \tag{5}$$

$$\frac{E_C + E_S}{V} = 6\pi x \Big(\frac{[\sigma d (q_H(\chi) - q_Q(\chi))e]^2 f_d(x)}{16\pi^2} \Big)^{1/3}, \tag{6}$$

where, q_H, q_Q are the charge densities of hadronic and quark matter (in units of e) at whatever proportion χ being considered. We have denoted the volume fraction of quark matter V_Q/V by χ. The ratio of droplet (rod, slab) to cell

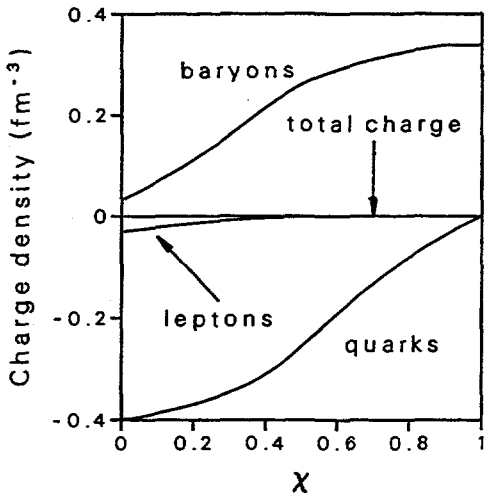


Figure 1: The charge densities in the mixed phase carried by regions of quark and hadronic matter, as well as leptons which permeate all regions in our approximation. Multiplied by the respective volumes occupied, the total charge adds to zero.

volume is called $x = (r/R)^d$. It is related to χ by $\chi = x$ when hadronic matter is the background (ie. dominant) phase. The quark droplets (rods, slabs) have radius r and the spacing between centers is R. In the case of drops or rods, r is their radius and R the half distance between centers while for slabs, r is the half thickness. In the opposite situation where quark matter is the background, $1 - \chi = x$ is the fraction of hadronic matter which assumes the above geometric forms.

The function $f_d(x)$ is given in all three cases by,

$$f_d(x) = \frac{1}{d+2}\left[\frac{1}{(d-2)}(2 - dx^{1-2/d}) + x\right], \tag{7}$$

where the apparent singularity for $d = 2$ is well behaved in the limit.

The surface tension should be self-consistent with the two models of matter, quark and hadronic, in equilibrium with each other. The densities of each phase change as their proportion does [2,1]. So the surface energy is not a constant. Gibbs studied the problem of surface energies, and as a gross approximation, one can deduce that it is given by the difference in energy densities of the substances in contact times a length scale typical of the surface thickness [3], in this case of the order of the strong interaction range, $L = 1$ fm. In other words, the surface interface energy should depend on the proportion of phases in phase equilibrium, just as everything else does.

$$\sigma = \text{const} \times [\epsilon_Q(\chi) - \epsilon_H(\chi)] \times L\,. \tag{8}$$

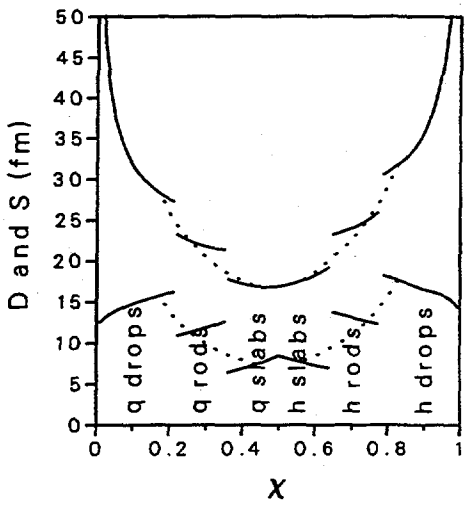


Figure 2: Diameter (lower curves) and Spacing (upper curves) of rare phase immersed in the dominant as a function of the proportion of quark phase. Geometries are identified as drops, rods, slabs, and composition as q (quark) or h (hadronic). Dots are a continuous dimensionality interpolation of the discrete shapes.

Following our deduction that a Coulomb lattice should exist in the mixed hadron-quark phase[2,1], Heiselberg, Pethick and Staubo investigated the dependence of the geometrical structure on the surface tension[4]. They adopted a selection of values from various sources, none of them computed self-consistently. The constant in (8) should be chosen so that the structured phase lies below the unstructured one. Heiselberg et al found their energy difference to be about 10 MeV. We choose the constant accordingly.

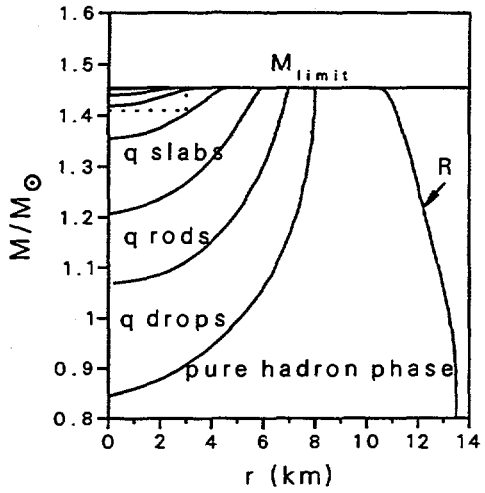
The sum of surface and Coulomb energies scale with the surface energy coefficient as $\sigma^{1/3}$ independent of geometry. Therefore the location in the star where the geometry changes from one form to another is independent of σ.

4 Bulk Description of the Phases

The geometrical structure of the mixed phase occurs against the background of the bulk structure. The energy and pressure are dominated by the bulk properties of matter. The description of the hadronic phase of neutron star matter has been discussed in detail elsewhere [5]. We include all the charge states of the lowest baryon octet, $(p, n, \Lambda, \Sigma^+, \Sigma^-, \Sigma^0, \Xi^-, \Xi^0)$.

The nuclear properties that define the values of the couplings are the binding, $B/A = -16.3$ MeV, saturation density, $\rho_0 = 0.153$ fm^{-3}, and symmetry energy coefficient, $a_{\rm sym} = 32.5$ MeV, compression modulus $K = 240$ MeV and nucleon effective mass at saturation $m^*_{\rm sat}/m = 0.78$. The hyperon couplings can be constrained by levels in hypernuclei, the binding of the Λ in nuclear

Figure 3: Boundaries of the various crystalline structures and radius of the star are shown for stars of mass shown on the y-axis. Detail of the dotted boxed area is shown in Fig. 4.



matter, and from neutron star masses[6]. We shall assume that all hyperons in the octet have the same coupling as the Λ. They are expressed as a ratio to the above mentioned nucleon couplings, $x_\sigma = g_{H\sigma}/g_\sigma$, $x_\omega = g_{H\omega}/g_\omega$, $x_\rho = g_{H\rho}/g_\rho$. The first two are related to the Λ binding by a relation derived in[6]; the third can be taken equal to the second by invoking vector dominance. We adopt the value of $x_\sigma = 0.6$ and corresponding x_ω taken from[6].

To describe quark matter we use a simple version of the bag model with $B^{1/4} = 180\,\mathrm{MeV}$ and quark masses $m_u = m_d = 0$, $m_s = 150\,\mathrm{MeV}^2$. Because of the long time-scale, strangeness is not conserved in a star. The quark chemical potentials for a system in chemical equilibrium are therefore related to those for baryon number and electron by $\mu_u = \mu_c = (\mu_n - 2\mu_e)/3$, $\mu_d = \mu_s = (\mu_n + \mu_e)/3$.

5 Varying Crystalline Structure

To illustrate the rearrangement of the electric charge concentration between the quark and baryonic regions of the mixed phase, we show the charge density in each region, and the electron charge density, assumed to be uniform throughout the Wigner-Seitz cell, as functions of the proportion of quark matter in Fig. 1. Quark matter, which in the absence of baryonic matter ($\chi = 1$) is charge neutral in a star, carries a high negative charge density when there is little of it and it is in equilibrium with baryonic matter. The latter acquires an ever increasing density as the quantity of quark matter with which it can

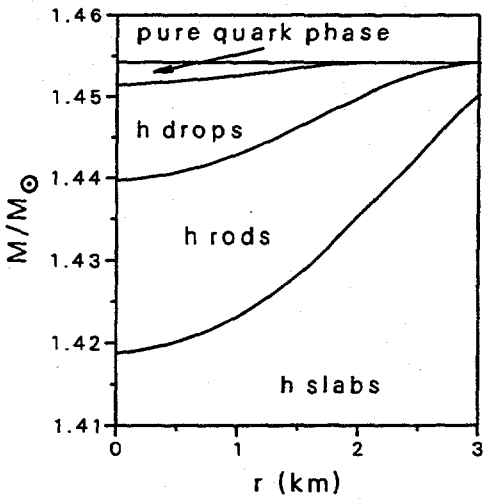


Figure 4: Detail of the dotted boxed area of Fig. 3

balance electric charge grows. This illustrates how effectively the symmetry driving force acts to optimally rearrange charge.

As shown above, because one of the conserved quantities is the electric charge, having long range, an order will be established in the mixed phase, the size of the objects of the rare phase and their spacing in the dominant one, being determined by the condition for a minimum sum of Coulomb and surface energy. In Fig. 2 the diameter D and spacing S is shown by the lower and upper curves as a function of proportion of quark phase. It is noteworthy that at the limit of the pure phases corresponding to $\chi = 0$ or 1, the spheres of rare phase are of finite diameter, but spaced far apart. The size of the objects is between 7 and 15 fm. As noted previously the location in χ of the geometries is independent of σ, but the size and spacings scale as $\sigma^{1/3}$.

6 Crystal Structure in the Cores of Neutron Stars

In the environment of a particular mass star χ is a function of radial coordinate. The various structures occur at various radial locations in stars of different mass. In Fig. 3 the radial location of the boundaries are shown for the stellar masses shown on the y-axis. Stars of low mass have no mixed phase, as expected. With increasing mass, quark drops appear and at still higher mass the full complement of geometries appear in both quark and hadronic form. For stars very close to the limiting mass an inner sphere of pure quark mat-

ter appears having a radius of 2 km at the mass limit. Generally the region spanned by the crystalline structures is of the order of 8 km. We note the transition from pure hadronic to mixed phase occurs at the rather low density of about $2\rho_0$, as was found also by several other authors [4,7].

7 Summary

It is almost certain that a solid region in a pulsar will play a role in the period glitch phenomenon, which is highly individualistic from one pulsar to another. We have suggested that this high degree of individual behavior may be due to the extreme sensitivity on stellar mass of the radial extent of the solid region [8]. The sensitivity arises because of the rather flat radial profile of the pressure and energy density in neutron stars, so that a small change in central density and therefore a small change in stellar mass, moves a transition pressure a considerable distance in the radial direction in the star.

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