# PRODUCTION FUNCTION APPLICATION ATTEMPT IN ELECTRICITY GENERATION FORECASTING

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Summary

The paper presents the attempt of the production function application for forecasting the electricity generation in a few years term. The modified Cobb-Douglas' function was used to prepare the prognosis of the electrical energy generation in Poland up to 2010. The test calculation results based on the procedure of electricity generation model parameters estimations are presented.

Key word index: Production function, Forecasting the Electricity Generation

## 1. Introduction

The fast development of forecasting as a branch of science is observed due to significant importance of a prognosis for planning the modern economy progress. Statistical and econometrical methods are distinguished in forecasting that describe process of economic development based on data in the shape of statistical time string. In the frame of forecasting, the method and model form is chosen considering up-to-date process course. On the next stage, by selected method, the conclusion of the presumable course is drawn and prognosis error is evaluated. The selection of trend function form ,which decides upon prognosis accuracy, develops significant problems, especially in medium and long term prognosis. The paper presents the attempt of solving this difficulty building electricity generation model by application of production function.

## 2. Modification of the production function

The classical form of the Cobb-Douglas production function neglecting the chance factor is given by:

$$P=\alpha_0 X_1^{\alpha_1} X_2^{1-\alpha_1} \qquad (1)$$

where:

| | | |
|---|---|---|
| P | - | production, |
| $X_1$ | - | capital, |
| $X_2$ | - | labor, |
| $\alpha_1$ | - | capital production elasticity, |
| $\alpha_1=1-\alpha_1$ | | labor production elasticity |

In such an embrace, it is a static function where elasticity of substitution factor is equal one. The production changes may result only by increase of $X_1$, $X_2$ factors, regarding invariable generating method. During long term generating process various technological and management modifications may occur influencing production level. Therefore, the necessity of considering the time factor (t) exists, defining the influence of introducing technical and management progress into generating process. The dynamic production model (modified classical Cobb-Douglas' function) takes the form:

$$P=f_0(t)\, X_1^{f_1(t)} X_2^{f_2(t)} \qquad (2)$$

where:

| | | |
|---|---|---|
| $f_0(t)=\alpha_0 e^{rt}$ | - | the result of introducing technical and management progress, |
| $f_1(t)$ | - | $\alpha_1$ parameter trend function, |
| $f_2(t)$ | - | $\alpha_2$ parameter trend function. |

Trend functions of $\alpha_1$ and $\alpha_2$ may be presented in the linear mathematical form. The modification of Cobb-Douglas' function consisting on including to the variables group additional variables detail describing capital $X_1$ and labor $X_2$ seems to be especially interesting. Such model takes the form:

$$P=\alpha_0 \left(\prod_i X_{1i}^{\alpha_{1i}}\right) \left(\prod_i X_{2i}^{\alpha_{2i}}\right) e^{\gamma t} \qquad (3)$$

where:

$\prod X_{1i}^{\alpha 1i} = X_{11}^{\alpha 11} X_{12}^{\alpha 12} \ldots$ variables describing capital of production, capital structure, etc.

$\prod X_{2i}^{\alpha 2i} = X_{21}^{\alpha 21} X_{22}^{\alpha 22} \ldots$ variables describing labor structure, qualification of staff, etc.

The interesting modification of the production function ( called CES production function) has been done by Arrow, Chenery, Minhas and Solow. The new CES function assumes constant elasticy of substitution. It is vast described in available literature.

### 3. The application of production function for forecasting the electricity generation

Considering statistically-econometrical point of view, the production function is not a function sensu stricto but the typical regression model in multiplicative form including the chance factor in addition to variables. Applying general simplified form of Cobb-Douglas function without assumption of constant production scale, we obtained:

$$P=\alpha_0 X_1^{\alpha_1} X_2^{\alpha_2} e^{\xi} \qquad (4)$$

where:

$\alpha_1 > 0$, $\alpha_2 > 0$, $\xi$ - the chance factor.

The test calculations were done for hard coal fired Power Plants, based on generation data supplied in Main Statistical Office of Poland (GUS) publications.

The model of electricity generation was defined from electricity generation time series in 1980-90 regarding capital of productivity power plant (expressed as capacity installed - $X_1$ in [MW]) and employment ( expressed as time of using capacity installed - $X_2$ in [h/a]) :

$$A_r = 11.218\ X_1^{0.28} X_2^{0.72} \qquad (5)$$

where:

$A_r$ - annual electricity generation , GW∘h

To estimate $\alpha_1$ , $\alpha_2$ parameters level the Rosenbroock's optimalization procedure was used. The estimation error was less than 2 %.

The formula as below is obtained developing the prognosis of the increase the electricity generation in relation to the base year ($A_{r0}$) by $\lambda$ [%] :

$$A_{ri} = \lambda^{\alpha_1 + \alpha_2} A_{r0} \qquad (6)$$

where:

$A_{ri}$ - electricity generation in i-th year,

The test calculation results are presented in the table below:

| develop ment variant | 1990 /base year/ | 1995 | 2000 | 2005 | 2010 |
|---|---|---|---|---|---|
| low | $\lambda$ [%] | 1.4 | 15.0 | 30.0 | 49.0 |
| | $A_{r0}$ [GWh] | 99682 | 113168 | 128059 | 146941 |
| high | $\lambda$ [%] | 7.1 | 27.6 | 50.2 | 70.8 |
| | $A_{r0}$ [GWh] | 113397 | 135298 | 159477 | 181541 |

## 4. Conclusions

The modified Cobb-Douglas' production function enables to evaluate level of electricity generation for medium and long term prognosis in an easy and simple way.

The presented method is distinguished by high accuracy relating to classical methods despite application relative short time series.

Several advantages of such a method enable its application in studies considering development policy of electricity generations.

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