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ABSTRACT

We show that the availability of longitudinally polarized electron beams at a 500 GeV Linear Collider would allow, from an analysis of the reaction $e^+e^- \to W^+W^-$, to set stringent bounds on the couplings of a Z' of the most general type. In addition, to some extent it would be possible to disentangle observable effects of the Z' from analogous ones due to competitor models with anomalous trilinear gauge couplings.

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1 Introduction

It has been recently suggested [1] that theoretical models with one extra $Z \equiv Z'$ whose couplings to quarks and leptons are not of the 'conventional' type would be perfectly consistent with all the available experimental information from either LEP1 [2] and SLD [3] or CDF [4] data. Starting from this observation, a detailed analysis has been performed of the detectability in the final two-fermion channels at LEP2 of a Z' whose fermion couplings are arbitrary (but still family independent) [5]. Also, in [5] the problem of distinguishing this model from competitor ones (in particular, from a model with anomalous gauge couplings) has been studied.

The final two-fermion channel is not the only one where virtual effects generated by a Z' can manifest themselves. The usefulness of the final W^+W^- channel in e^+e^- annihilation to obtain improved information on some theoretical properties of such models has been already stressed in previous papers in the specific case of longitudinally polarized beams for models of 'conventional' type (e.g., E_6, LR, *etc.*), showing that the role of polarization in these cases would be essential [6].

The effects of a Z' of 'unconventional' type in the W^+W^- channel have also been considered, and compared with those of models with anomalous gauge couplings [7]. In particular, in [7] it was shown that the benchmark of the model with a Z' would be the existence of a peculiar connection between certain effects observed in the W^+W^- channel and other effects observed in the final lepton- antilepton channel.

The aim of this paper is that of considering whether the search for effects of a 'unconventional' Z' in the W^+W^- channel would benefit from the availability of longitudinal polarization of initial beams, as it is the case for the 'conventional' situation. We shall show in the next Sect. 2 that this is indeed the case, i.e., that in the parameter space the expected experimental sensitivity in the polarized processes is by far better than in the unpolarized case. For what concerns the differentiation from other models, in particular from those with anomalous gauge couplings, we shall also show in Sect. 3 that the characteristic feature of such a Z' would be the existence of certain peculiar properties of different observables, all pertaining to the final W^+W^- channel.

All our discussions assume that longitudinal lepton polarization will be available in the considered examples. In practice, this would be feasible at the future planned $500\,GeV$ linear Collider (LC). Our conclusions for the specific theoretical model that we consider, as summarized in Sect. 4, will therefore be strongly in favour of polarization. An Appendix will be devoted to the derivation of several expressions for the relevant experimental observables.

2 Derivation of the constraints for general Z' parameters

The starting point of our analysis will be the expression of the invariant amplitude for the process

$$e^+ + e^- \to W^+ + W^-. \tag{1}$$

In Born approximation, this can be written as a sum of a t-channel and of an s-channel component. In the Standard Model (SM) case, the latter will be written as follows:

$$\mathcal{M}_s^{(\lambda)} = \left(-\frac{g_{WW\gamma}}{s} + \frac{g_{WWZ}(v - 2\lambda a)}{s - M_Z^2}\right) \times \mathcal{G}^{(\lambda)}(s, \theta), \tag{2}$$

where s and θ are the total c.m. squared energy and W^- production angle; $v = (T_{3,e} - 2Q_e s_W^2)/2s_W c_W$ and $a = T_{3,e}/2s_W c_W$ with $T_{3,e} = -1/2$ and $s_W = \sin\theta_W$, $c_W = \cos\theta_W$ (θ_W is the conventional electroweak mixing angle); $g_{WW\gamma} = 1$ and $g_{WWZ} = \cot\theta_W$; λ denotes the electron helicity ($\lambda = \pm 1/2$ for right/left-handed electrons); finally, $\mathcal{G}^{(\lambda)}(s, \theta)$ is a kinematical coefficient whose explicit form is not essential for our discussion and is omitted for simplicity, as it can be easily found in the literature [8]. Note that, at this stage, we are writing an 'effective' Born approximation that contains both the physical Z couplings and the physical Z mass. We shall systematically ignore extra contributions at one loop. In fact, our purpose is that of evaluating *deviations* from the SM expressions due to one extra Z. In this spirit, we shall also consider the Z' contribution using an 'effective' Born approximation with physical Z' couplings and mass. The more rigorous one-loop treatment would require also the calculation of the, potentially dangerous, QED radiation effects whose study has not yet been performed, to our knowledge, for polarized beams at the LC. We shall assume in the sequel that the results of a rigorous treatment reproduce those of an effective approximation without QED after a suitable, apparatus dependent, calculation as it is the case for the unpolarized case. Then, for the evaluation of the *deviations* due to the Z', the residual purely electroweak one-loop contributions will be safely neglected.

Working in this framework, the effective expression of the invariant amplitude after addition of one extra Z will be written as:

$$\mathcal{M}_s^{(\lambda)} = \left(-\frac{g_{WW\gamma}}{s} + \frac{g_{WWZ_1}(v_1 - 2\lambda a_1)}{s - M_{Z_1}^2} + \frac{g_{WWZ_2}(v_2 - 2\lambda a_2)}{s - M_{Z_2}^2}\right) \times \mathcal{G}^{(\lambda)}(s, \theta). \tag{3}$$

In Eq. (3), we have retained two possible sources of effects. The first one accounts, in full generality, for potential modifications of the SM Z couplings and mass induced by the presence of the additional 'heavy' Z. As a consequence of this fact, the 'light' Z is now denoted as Z_1, and the same convention applies to its couplings v_1, a_1 and g_{WWZ_1}. The second effect is due to the actual extra heavy Z exchange diagram. This will be treated by denoting the physical heavy Z as Z_2 and its physical couplings by analogous notations. With this convention, Z and Z' do not appear in Eq. (3), as they would be 'mathematical' objects that would coincide, in the 'conventional' case, with weak gauge eigenstates whose mixing generates the modifications of the light Z mass and couplings.

It turns out that it is convenient to rewrite Eq. (3) in the following form:

$$\mathcal{M}_s^{(\lambda)} = \left(-\frac{g^*_{WW\gamma}}{s} + \frac{g^*_{WWZ}(v-2\lambda a)}{s-M_Z^2}\right) \times \mathcal{G}^{(\lambda)}(s,\theta), \tag{4}$$

where the 'effective' gauge boson couplings $g^*_{WW\gamma}$ and g^*_{WWZ} are defined as:

$$g^*_{WW\gamma} \equiv 1 + \delta_\gamma \equiv 1 + \delta_\gamma(Z_1) + \delta_\gamma(Z_2), \tag{5}$$

$$g^*_{WWZ} \equiv \cot\theta_W + \delta_Z \equiv 1 + \delta_Z(Z_1) + \delta_Z(Z_2), \tag{6}$$

with

$$\delta_\gamma(Z_1) = v\cot\theta_W\left(\frac{\Delta a}{a} - \frac{\Delta v}{v}\right)(1+\Delta\chi)\,\chi; \quad \delta_\gamma(Z_2) = v\, g_{WWZ_2}\left(\frac{a_2}{a} - \frac{v_2}{v}\right)\chi_2, \tag{7}$$

$$\delta_Z(Z_1) = -\cot\theta_W + g_{WWZ_1}\left(1 + \frac{\Delta a}{a}\right)(1+\Delta\chi)\,; \qquad \delta_Z(Z_2) = g_{WWZ_2}\frac{a_2}{a}\frac{\chi_2}{\chi}. \tag{8}$$

In Eqs. (7) and (8) we have introduced the deviations of the fermionic couplings $\Delta v = v_1 - v$ and $\Delta a = a_1 - a$, and the neutral vector boson propagators (neglecting their widths):

$$\chi(s) = \frac{s}{s-M_Z^2}; \qquad \chi_2(s) = \frac{s}{s-M_{Z_2}^2}; \qquad \Delta\chi(s) = -\frac{2M_Z\Delta M}{s-M_Z^2}, \tag{9}$$

where $\Delta M = M_Z - M_{Z_1}$ is the Z-Z_1 mass-shift, with $\Delta M > 0$ if this effect is due to Z-Z' mixing. From the available experimental results, Δv, Δa and $\Delta M/M_Z$ turn out to be small numbers, which can be treated as a perturbation to the SM. As it will emphasized in the sequel, the general parametrization (4)-(6) is rather useful for phenomenological purposes, in particular to discuss the deviations from the SM in the context of different classes of models contributing with finite deviations (7) and (8).

We now focus on the effects of a heavy Z of the previous general kind on polarized observables. Although this is not necessarily a unique choice, we only consider for a first investigation the case of polarized electron beams.

The general expression for the cross section of process (1) with longitudinally polarized electron and positron beams can be expressed as

$$\frac{d\sigma}{d\cos\theta} = \frac{1}{4}\left[(1+P_L)\left(1-\bar{P}_L\right)\frac{d\sigma^+}{d\cos\theta} + (1-P_L)\left(1+\bar{P}_L\right)\frac{d\sigma^-}{d\cos\theta}\right], \tag{10}$$

where P_L and $\bar{P}_L$ are the actual degrees of electron and positron longitudinal polarization respectively, and $\sigma^{\pm}$ are the cross sections for purely right-handed and left-handed electrons. From Eq. (10), the cross section for polarized electrons and unpolarized positrons corresponds to $\bar{P}_L = 0$. The polarized cross sections can be generally decomposed as follows:

$$\frac{d\sigma^{\pm}}{d\cos\theta} = \frac{\pi\alpha_{em}^2\beta_W}{2s}\sum_i F_i^{\pm}\mathcal{O}_i(s,\cos\theta), \tag{11}$$

where: $\beta_W = \sqrt{1-4M_W^2/s} = 2p/\sqrt{s}$, with $p = |\vec{p}|$ the CM momentum of the W; $F_i^{\pm}$ are combinations of couplings involving in particular the deviations from the SM couplings, e.g., of the kind previously introduced; $\mathcal{O}_i$ are functions of the kinematical variables. To make the paper self-contained, we list the explicit expressions of the relevant $F_i^{\pm}$ and $\mathcal{O}_i$ in the Appendix.

In practice, we shall denote by σ^L and σ^R the cross sections corresponding, in Eq. (10), to the values $P_L = -0.9$ and $P_L = 0.9$ respectively. Such degrees of longitudinal polarization should be realistically obtainable at the LC [9].

Our analysis proceeds in the this way. Following the suggestions of previous dedicated searches [10], the sensitivity of σ^L and σ^R to δ_γ and δ_Z is assessed numerically by dividing the angular range $|\cos\theta| \leq 0.98$ into 10 equal 'bins', and defining a χ^2 function in terms of the expected number of events $N(i)$ in each bin:

$$\chi^2 = \sum_i^{bins}\left[\frac{N_{SM}(i) - N(i)}{\delta N_{SM}(i)}\right]^2, \tag{12}$$

where the uncertainty on the number of events $\delta N_{SM}(i)$ combines both statistical and systematic errors as

$$\delta N_{SM}(i) = \sqrt{N_{SM}(i) + (\delta_{syst}N_{SM}(i))^2}, \tag{13}$$

(we assume $\delta_{syst} = 2\%$). In Eq. (12), $N(i) = L_{int}\sigma_i\varepsilon_W$ with L_{int} the time-integrated luminosity and ($z = \cos\theta$):

$$\sigma_i \equiv \sigma(z_i, z_{i+1}) = \int_{z_i}^{z_{i+1}}\left(\frac{d\sigma}{dz}\right)dz, \tag{14}$$

Also, ε_W is the efficiency for W^+W^- reconstruction, for which we take the channel of lepton pairs ($e\nu + \mu\nu$) plus two hadronic jets, giving $\varepsilon_W \simeq 0.3$ from the relevant branching ratios. An analogous procedure is followed to evaluate $N_{SM}(i)$.

As a criterion to derive the constraints on the coupling constants in the case where no deviations from the SM were observed, we impose that $\chi^2 \leq \chi^2_{crit}$, where χ^2_{crit} is a number that specifies the chosen confidence level. With two independent parameters in Eqs. (5) and (6), the 95% CL is obtained by choosing $\chi^2_{crit} = 6$ [11].

From the numerical procedure outlined above, we obtain the allowed bands for δ_γ and δ_Z determined by the polarized cross sections σ^R and σ^L (as well as σ^{unpol}) depicted in Fig. 1, where $L_{int} = 50\, fb^{-1}$ has been assumed.

One can see from inspection of Fig. 1 that the role of polarization is essential in order to set meaningful finite bounds. Indeed, contrary to the unpolarized case, which evidently by itself does not provide any finite region for δ_γ and δ_Z (unless one of the two parameters is fixed by some further assumption), from the combined and *intersecting* bands relative to σ^L and σ^R one can derive the following 95% CL allowed ranges

$$\begin{aligned} -0.002 < \delta_\gamma < 0.002 \\ -0.004 < \delta_Z < 0.004. \end{aligned} \tag{15}$$

Eq. (15) gives the most general constraint that would be derivable at the LC for a 'unconventional' Z' with polarized electron beams. It should be stressed that the constraint is completely model independent. To have a feeling of how polarization works in more specific cases, we have also considered the familiar situation of an extra Z of extended gauge origin, in particular generated by a previous E_6 symmetry [12]. Denoting by ϕ the Z-Z' mixing angle defined by following the conventional prescriptions, Eqs. (7) and (8) would read now:

$$\delta_\gamma = v g_{WWZ} \chi \phi \left(\frac{a'}{a} - \frac{v'}{v}\right)\left(1 - \frac{\chi_2}{\chi} + \Delta\chi\right)\chi, \tag{16}$$

$$\delta_Z = \cot\theta_W \left[\phi \frac{a'}{a}\left(1 - \frac{\chi_2}{\chi}\right) + \Delta\chi\right], \tag{17}$$

with v' and a' fixed by the specific model, and in general

$$\tan^2\phi = \frac{M_Z^2 - M_{Z_1}^2}{M_{Z_2}^2 - M_Z^2} \simeq \frac{2M_Z \Delta M}{M_{Z_2}^2}. \tag{18}$$

Consequently, in the $(\delta_\gamma, \delta_Z)$ plane of Fig. 1 each model is now represented, in linear approximation in ϕ, by a line of equation:

$$\delta_Z = \delta_\gamma \frac{1}{v\chi} \frac{(a'/a)}{(a'/a) - (v'/v)}. \tag{19}$$

Such relation does not depend on either ϕ or M_{Z_2}, but only on the ratios of fermionic couplings.

In Fig. 2 we depict, as an illustration, the cases corresponding to the models currently called η, χ and ψ. From this figure, two main conclusions can be drawn: *i*) polarization systematically reduces the allowed range for the model parameters, in some cases (ψ, η models) more spectacularly than in other ones (χ model), and *ii*) depending on the considered model, different polarization values are relevant, i.e., for the η and ψ cases σ^R is essential while for the χ model σ^L provides the main constraint.

As a simple quantitative illustration of these features, we can consider in more detail the case of the η model. As it can be read from Fig. 2, for this case the bounds obtainable from σ^{unpol} only are the following:

$$\begin{array}{c} -0.005 < \delta_\gamma < 0.005 \\ -0.003 < \delta_Z < 0.003. \end{array} \tag{20}$$

The use of σ^R allows the improvement of Eq. (20) to the more stringent bounds:

$$\begin{array}{c} -0.002 < \delta_\gamma < 0.002 \\ -0.001 < \delta_Z < 0.001. \end{array} \tag{21}$$

The ranges of δ_γ and δ_Z allowed to the specific models in Fig. 2 can be translated into limits on the mixing angle ϕ and the heavier gauge boson mass M_{Z_2}, using Eqs. (16)-(19). Continuing our illustrative example of the η model, in this case the resulting allowed region (at the 95% CL) in the (ϕ, M_{Z_2}) plane is limited by the thick solid line in Fig. 3. We have orientatively chosen for ΔM in Eq. (9) an upper limit of about $150\,MeV$ from LEP data [13, 14], although the limiting curves do not appreciably depend on the specific value of this quantity. Also, the indicative current lower bound on M_{Z_2} from direct searches, as well as the bound obtainable from full exploitation of the e^+e^- annihilation into lepton-antilepton [15, 16], are reported in Fig. 3. For a comparison, the dashed line in Fig. 3 represents the maximal region allowed to ϕ by the model-and process-independent relation (18), with the same upper bound on ΔM. As one can conclude from Fig. 3, the W^+W^- channel with polarized electron beams represents a quite sensitive, and independent, source of information on deviations from the SM model due to the extra Z, which can be combined with that provided by the final leptonic channel and nicely complements it.

The previous discussion should have shown, hopefully in a clear and simple way, the advantages of longitudinal electron polarization at the LC in order to study the effects of a model with an extra Z of the most general kind in the reaction (1) at a linear electron-positron collider. Actually, our analysis has focused on the derivation of bounds, starting from the (negative) assumption that no effects, i.e., no deviations from the SM predictions are observed within the expected accuracy on the cross section. In the next section we shall take, instead, the (positive) attitude of assuming that certain deviations from the SM are observed in σ^L and/or σ^R. In such a case, it might be possible to identify, to some extent, the relevant model originating the observed deviation.

3 Comparison with a model with anomalous gauge couplings

It has been already pointed out [7, 10] that a model with one extra Z would produce virtual manifestations in the final W^+W^- channel at the LC that in principle could mimic those of a model (of completely different origin) with anomalous trilinear gauge couplings. As shown by Eqs. (4)-(8), this is due to the fact that the effects of the extra Z can be reabsorbed into a redefinition of the VWW couplings ($V=\gamma,\ Z$). Therefore, the identification of such an effect, if observed at the LC, becomes a relevant problem.

We discuss this question for the specific case of a model where anomalous trilinear gauge boson couplings are present. Using the notations of [8], the relevant trilinear VWW component which conserves $U(1)_{e.m.}$, C and CP can be written as ($e=\sqrt{4\pi\alpha_{em}}$):

$$\begin{aligned}\mathcal{L}_{eff} &= -ie\left[A_\mu\left(W^{-\mu\nu}W^+_\nu - W^{+\mu\nu}W^-_\nu\right) + F_{\mu\nu}W^{+\mu}W^{-\nu}\right] - ie\,x_\gamma\,F_{\mu\nu}W^{+\mu}W^{-\nu}\\ &- ie\left(\cot\theta_W+\delta_Z\right)\left[Z_\mu\left(W^{-\mu\nu}W^+_\nu - W^{+\mu\nu}W^-_\nu\right) + Z_{\mu\nu}W^{+\mu}W^{-\nu}\right]\\ &- ie\,x_Z\,Z_{\mu\nu}W^{+\mu}W^{-\nu} + ie\,\frac{y_\gamma}{M_W^2}\,F^{\nu\lambda}W^-_{\lambda\mu}W^{+\mu}_{\ \ \nu} + ie\,\frac{y_Z}{M_W^2}\,Z^{\nu\lambda}W^-_{\lambda\mu}W^{+\mu}_{\ \ \nu},\end{aligned} \tag{22}$$

where $W^\pm_{\mu\nu}=\partial_\mu W^\pm_\nu-\partial_\nu W^\pm_\mu$ and $Z_{\mu\nu}=\partial_\mu Z_\nu-\partial_\nu Z_\mu$. The relation of the couplings in Eq. (22) to those more directly connected with W static properties is

$$x_\gamma\equiv\Delta k_\gamma=k_\gamma-1;\quad y_\gamma\equiv\lambda_\gamma, \tag{23}$$

$$\delta_Z\equiv g_{WWZ}-\cot\theta_W;\ x_Z\equiv\Delta k_Z\left(\cot\theta_W+\delta_Z\right)=\left(k_Z-1\right)g_{WWZ};\ y_Z\equiv\lambda_Z\cot\theta_W, \tag{24}$$

where μ_W and are Q_W the W magnetic and quadrupole electric moments, respectively:

$$\mu_W=\frac{e}{2M_W}(1+k_\gamma+\lambda_\gamma);\qquad Q_W=-\frac{e}{M_W^2}(k_\gamma-\lambda_\gamma), \tag{25}$$

and a similar interpretation holds for the WWZ couplings.

At the tree-level, the SM values of these couplings are

$$\delta_Z=x_\gamma=x_Z=y_\gamma=y_Z=0. \tag{26}$$

One can notice that, owing to the assumed $U(1)_{e.m.}$ invariance, no effects of the δ_γ kind are present in Eq. (22). This property, which will be essential for our identification purposes, also characterizes models derived from an effective $SU(2)\times U(1)$ invariant Lagrangian, if only dimension six operators are retained [17]. Conversely, $\delta_\gamma\neq 0$ (at $s\neq 0$) would be allowed by a Z'.

For our analysis, we would have now to account for the deviations from the SM induced by the various anomalous couplings on polarized observables, considering that the general model in Eq. (22) introduces five independent parameters. Thus, regardless of the attempt to distinguish this case from the simpler one of an extra Z (where only two parameters are involved), the determination of suitable experimental observables, depending on reduced subsets of anomalous gauge boson couplings, would represent an important issue by itself which deserves a separate treatment.

To illustrate a 'minimal' identification program, as anticipated in the previous section we assume that a virtual signal has been detected to a given, conventionally fixed confidence level, in either σ^L or σ^R, or both. In our notations, that would be expressed as:

$$\frac{\Delta\sigma^{L,R}}{\delta\sigma^{L,R}} \equiv \frac{\sigma^{L,R}_{exp} - \sigma^{L,R}_{SM}}{\delta\sigma^{L,R}_{SM}} \geq \kappa, \tag{27}$$

where $\delta\sigma$ is the expected statistical uncertainty on the cross section an the value of κ corresponds to an assigned number of standard deviations.

As the next step, we try to define an observable which is 'orthogonal' to the Z' model, in the sense that such variable should depend only on those four couplings (x_V, y_V) that are specific of the Lagrangian (22), but not on δ_Z which would induce an effect in common with the Z' model of Sect. 2.

An illustrative, simple, example of such quantity can be worked out by starting from the introduction of the polarized observables, along the lines proposed in [18]:

$$\sigma A^-_{FB} = \int_0^1 \frac{d\sigma^-}{d\cos\theta} d\cos\theta - \int_{-1}^0 \frac{d\sigma^-}{d\cos\theta} d\cos\theta, \tag{28}$$

and

$$\sigma A^\pm_{CE}(z^*) = \int_{-z^*}^{z^*} \frac{d\sigma^\pm}{d\cos\theta} d\cos\theta - \left[\int_{z^*}^{1} \frac{d\sigma^\pm}{d\cos\theta} d\cos\theta + \int_{-1}^{-z^*} \frac{d\sigma^\pm}{d\cos\theta} d\cos\theta\right]. \tag{29}$$

Similar to (11), one can expand (28) and (29) as

$$\sigma A^-_{FB} = \frac{\pi^2\alpha^2_{em}\beta_W}{2s}\left[F^-_0 \mathcal{O}_{0,FB} + F^-_2 \mathcal{O}_{2,FB} + \left(F^-_6 + F^-_7\right)\mathcal{O}_{6,FB}\right], \tag{30}$$

$$\sigma A^\pm_{CE}(z^*) = \frac{\pi\alpha_{em}\beta_W}{2s}\sum_i F^\pm_i \mathcal{O}_{i,CE}(z^*), \tag{31}$$

where the explicit expressions of couplings and corresponding kinematical coefficients can be obtained from the Appendix. One can notice, also, that $\sigma A^+_{FB} = 0$. The δ_Z contribution in Eqs. (30) and (31) is contained in $F^\pm_1$ and F^-_2 (recall that $F^+_2 = 0$). Therefore, the simplest observable 'orthogonal' to δ_Z is represented, at $\sqrt{s} = 500\,GeV$, by the quantity

$$\sigma A^+_{CE}(z^* \simeq 0.4). \tag{32}$$

Indeed, numerical inspection of the formulae in the Appendix shows that, at $z^* \simeq 0.4$ (and $\sqrt{s} = 500\,GeV$) the coefficient $\mathcal{O}_{1,CE}$ vanishes, leaving a pure dependence of (32) from (x^+_V, y^+_V) but not from δ_Z. Clearly the position of this zero is entirely determined by M_W and the CM energy $\sqrt{s}$. The possibility to measure (32) strongly depends on the angular resolution for the W and its decay products.

Another example, which would eliminate both F_1^- and F_2^- in Eqs. (30) and (31), should be the specific combination:

$$Q^- = \sigma\mathcal{A}_{FB}^- - \frac{\mathcal{O}_{2,FB}}{\mathcal{O}_{2,CE}(0.4)}\left(\sigma\mathcal{A}_{CE}^-(0.4)\right) \simeq \sigma\mathcal{A}_{FB}^- + 0.029\cdot\sigma\mathcal{A}_{CE}^-(0.4). \qquad (33)$$

Still another possibility could be represented, in principle, by the combination

$$P^+(z^*) = \sigma^+ - \frac{\mathcal{O}_1}{\mathcal{O}_{1,CE}(z^*)}\left(\sigma\mathcal{A}_{CE}^+(z^*)\right), \qquad (34)$$

with σ^+ the total cross section for right-handed electrons and arbitrary $z^* \not\simeq 0.4$. The coefficients $\mathcal{O}_1$, $\mathcal{O}_{1,CE}$ and $\mathcal{O}_{2,CE}$ can be easily calculated from the formulae given in the Appendix. Concerning the dependence on the remaining anomalous couplings, in the linear approximation to the $F_i^\pm$ (see Eq. (A4)) the combination (33) depends on (x_V^-, y_V^-), while (34) is determined by (x_V^+, y_V^+) similar to (32), but with different parametrical dependence hence with different sensitivities on the anomalous couplings.

In Figs. 4-6 we depict the statistical significance of the variables (32)-(34) as a function of the relevant anomalous couplings. For any observable O, such significance is defined as the ratio $\mathcal{S} = \Delta O/(\delta O)_{stat}$ where $\Delta O = O(x_V, y_V) - O^{SM}$ and $(\delta O)_{stat}$ is the statistical uncertainty attainable on O. The different sensitivities to the various couplings x_V and y_V can be directly read from these figures and, as one can see, in certain cases they can be substantial. Figs. 4-6 are obtained by varying one of the parameters at a time and setting all the other ones at the SM values.

By definition, the observables (32)-(34) require 100% longitudinal electron polarization, a situation that will not be fully obtained in practice. However, the presently planned degree of electron polarization at the NLC, $|P_L| \simeq 0.90 - 0.95$ [9, 19] is high enough that Eq. (10) can represent a satisfactory approximation to evaluate such observables. Clearly, such approximation would be substantially improved if also positron polarization were available, e.g., $|\bar{P}_L| \simeq 0.6$ [19], because in that case in Eq. (10) the coefficient of σ^+ could be emphasized over that of σ^- by a large factor, or viceversa.

Moreover, we can remark that the independence of Q in Eq. (33) from δ_Z holds for both $\pm$ cases, therefore ultimately also for the case of unpolarized beams.

Concerning a possible discrimination between the Z' model of Sect. 2 and the model considered in this section, a strategy could be the following. If a signal is observed in either σ^L and/or σ^R and also in at least one of the 'orthogonal' observables defined above, we can conclude that it is due to the model with anomalous gauge couplings, and we can try to derive the values of some of them by properly analyzing the observed effects [20]. If, conversely, only σ^L and/or σ^R show an effect, we are left with the possibility that both models are responsible for such deviations. In this situation, we still have a simple tool to try to distinguish among the two models, which uses the observation that, under the assumption that only δ_γ and δ_Z are effective, the expressions of the consequent deviations of the integrated cross sections σ^L and σ^R are the following:

$$\Delta\sigma^L \simeq \Delta\sigma^- \propto \delta_\gamma - \delta_Z g_e^L \chi, \tag{35}$$

$$\Delta\sigma^R \simeq \Delta\sigma^+ \propto \delta_\gamma - \delta_Z g_e^R \chi, \tag{36}$$

where both δ_γ and δ_Z have been taken nonvanishing, and $g_e^{L,R} = v \pm a$ are the left- and right-handed electron couplings, respectively. Recalling that $\delta_\gamma = 0$ in the case of anomalous trilinear gauge boson couplings, using the experimental value of $s_W^2 \simeq 0.23$, one has for such a model the very characteristic feature

$$\Delta\sigma^L \simeq \left(1 - \frac{1}{2s_W^2}\right)\Delta\sigma^R = -1.17\Delta\sigma^R, \tag{37}$$

where the explicit expressions of g_e^L and g_e^R have been used. If, on the contrary, the effect is due to a model with a Z', no *a priori* relationship exists between $\Delta\sigma^L$ and $\Delta\sigma^R$. Accordingly, from inspection of these two quantities, if they are found not to be related by Eq. (37) to a given confidence level one would conclude that the observed effect should be due to the general extra Z discussed in Sect. 2. Then, depending on the actual values of the experimental deviations, a determination of the two parameters δ_γ and δ_Z might be carried on.

Actually, if the deviations of $\sigma_{L,R}$ satisfy the correlation Eq. (37), a small residual ambiguity would remain. Although the possibility that in a model with both δ_γ and δ_Z nonvanishing the correlation Eq. (37) is satisfied just by chance seems rather unlikely, one cannot exclude it *a priori*. Should this be the real situation, further analysis, e.g., in the different final fermion-antifermion channel would be required. The discussion of this essentially unlikely case can be performed, but is beyond the purpose of this paper.

4 Concluding remarks

We have shown in this paper that the availability of longitudinal electron beam polarization at the LC would be very useful for the study of the most general model with one extra Z from an analysis of the final W^+W^- channel. In principle, it would also be possible to discriminate this model from a rather 'natural' competitor one where anomalous gauge boson couplings are present. This could be done by analyzing suitable experimental variables, all defined in the same W^+W^- final channel.

The interesting property of polarized observables in the W^+W^- channel should be joined to analogous interesting features that are characteristic of polarization asymmetries in the final two-fermion channel, whose general discussion has been presented recently [21].

All these facts allow us to conclude that polarization at the LC would be, least to say, a highly desirable opportunity.

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Appendix

Limiting to CP conserving couplings, to generally describe the deviations from the SM of the cross section for process (1) of interest here, we have to account for the effect of six parameters, i.e., the five anomalous gauge couplings (δ_Z, x_V and y_V) of Eq. (22) plus the deviation δ_γ possible in the Z' model. Moreover, in this case the index i in Eq. (11) runs from 1 to 11.[3]

Referring to the expression of polarized differential cross sections $d\sigma^\pm$ in Eq. (11), one can easily realize that the F_i there are coveniently expressed in terms of the combinations of anomalous coupling constants defined as follows:

$$\delta_V^\pm = \delta_\gamma - \delta_Z \, g_e^\pm \, \chi; \quad x_V^\pm = x_\gamma - x_Z \, g_e^\pm \, \chi; \quad y_V^\pm = y_\gamma - y_Z \, g_e^\pm \, \chi, \tag{A1}$$

where

$$g_e^+ = v - a = \tan\theta_W; \qquad g_e^- = v + a = g_e^+ \left(1 - \frac{1}{2s_W^2}\right), \tag{A2}$$

and v, a and the Z propagator χ have been previously defined in Sect. 2 with regard to Eqs. (2) and (3). Introducing the combination

$$g_s^\pm = 1 - \cot\theta_W \, g_e^\pm \, \chi, \tag{A3}$$

we have:

$$\begin{gathered}
F_0^- = \frac{1}{16 s_W^4}; \quad F_1^\pm = \left(g_s^\pm + \delta_V^\pm\right)^2 \quad F_2^- = -\frac{1}{2s_W^2}\left(g_s^- + \delta_V^-\right), \\
F_3^\pm = \left(g_s^\pm + \delta_V^\pm\right) x_V^\pm \simeq g_s^\pm x_V^\pm; \quad F_4^\pm = \left(g_s^\pm + \delta_V^\pm\right) y_V^\pm \simeq g_s^\pm y_V^\pm, \\
F_6^- = -\frac{1}{4s_W^2} x_V^-; \qquad F_7^- = -\frac{1}{4s_W^2} y_V^-, \\
F_9^\pm = \frac{1}{2}\left(x_V^\pm\right)^2; \quad F_{10}^\pm = \frac{1}{2}\left(y_V^\pm\right)^2; \quad F_{11}^\pm = \frac{1}{2}\left(x_V^\pm y_V^\pm\right).
\end{gathered} \tag{A4}$$

All other F's vanish. Clearly, the polarized cross sections will depend on on the anomalous parameters as $\sigma^+ \equiv \sigma^+(\delta_V^+, x_V^+, y_V^+)$ and $\sigma^- \equiv \sigma^-(\delta_V^-, x_V^-, y_V^-)$.

With $t = M_W^2 - \frac{s}{2}(1 - \beta_W \cos\theta)$, and β_W defined previously, the corresponding kinematical coefficients $\mathcal{O}_i(s, \cos\theta)$ that appear in Eq. (11) are [18]:

$$\mathcal{O}_0 = 8\left[\frac{2s}{M_W^2} + \frac{\beta_W^2}{2}\left(\frac{s^2}{t^2} + \frac{s^2}{4M_W^4}\right)\sin^2\theta\right],$$

$$\mathcal{O}_1 = \frac{\beta_W^2}{8}\left[\frac{16s}{M_W^2} + \left(\frac{s^2}{M_W^4} - \frac{4s}{M_W^2} + 12\right)\sin^2\theta\right],$$

[3]The general expansion including also CP violating couplings can be found, e.g., in [18].

$$\mathcal{O}_2 = 16\left(1+\frac{M_W^2}{t}\right)+8\beta_W^2\left[\frac{s}{M_W^2}+\frac{1}{16}\left(\frac{s^2}{M_W^4}-\frac{2s}{M_W^2}-\frac{4s}{t}\right)\sin^2\theta\right],$$

$$\mathcal{O}_3 = \frac{\beta_W^2 s^2}{2M_W^4}\left[1+\frac{6M_W^2}{s}-\left(1-\frac{2M_W^2}{s}\right)\cos^2\theta\right]; \qquad \mathcal{O}_4=\frac{4\beta_W^2 s}{M_W^2},$$

$$\mathcal{O}_6 = \frac{\beta_W s^3}{2tM_W^4}\left[-\beta_W\left(1+\frac{6M_W^2}{s}\right)+\left(1+\frac{4M_W^2}{s}-\frac{16M_W^4}{s^2}\right)\cos\theta\right.$$
$$\left.+\beta_W\left(1-\frac{2M_W^2}{s}\right)\cos^2\theta-\beta_W^2\cos^3\theta\right],$$

$$\mathcal{O}_7 = \frac{4\beta_W s^2}{tM_W^2}\left[-\beta_W+\left(1-\frac{2M_W^2}{s}\right)\cos\theta\right],$$

$$\mathcal{O}_9 = \frac{\beta_W^2 s^2}{2M_W^4}\left[1+\frac{2M_W^2}{s}-\left(1-\frac{2M_W^2}{s}\right)\cos^2\theta\right],$$

$$\mathcal{O}_{10} = \frac{\beta_W^2 s^2}{M_W^4}\left[1+\frac{M_W^2}{s}-\left(1-\frac{M_W^2}{s}\right)\cos^2\theta\right]; \quad \mathcal{O}_{11}=\frac{2\beta_W^2 s}{M_W^2}\left(1+\cos^2\theta\right). \qquad (A5)$$

Then, defining

$$C=\frac{2M_W^2-s}{2p\sqrt{s}}=-\frac{1+\beta_W^2}{2\beta_W}; \quad L_{CE}(z^*)=\ln\frac{C+1}{C-1}-2\ln\frac{C+z^*}{C-z^*},$$

the coefficients $\mathcal{O}_{i,CE}(z^*)$ in Eq. (31) have the following expressions:

$$\mathcal{O}_{0,CE} = 32\left[(2z^*-1)\left(\frac{s}{M_W^2}-1\right)+\left(z^*-\frac{z^{*3}}{3}-\frac{1}{3}\right)\frac{\beta_W^2 s^2}{8M_W^4}\right.$$
$$\left.-2z^*\frac{C^2-1}{C^2-z^{*2}}+1-CL_{CE}(z^*)\right],$$

$$\mathcal{O}_{1,CE} = \beta_W^2\left[4(2z^*-1)\frac{s}{M_W^2}+\frac{1}{2}\left(z^*-\frac{z^{*3}}{3}-\frac{1}{3}\right)\left(\frac{s^2}{M_W^4}-\frac{4s}{M_W^2}+12\right)\right],$$

$$\mathcal{O}_{2,CE} = 2(2z^*-1)\left(20+\frac{8\beta_W^2 s}{M_W^2}-\frac{8M_W^2}{s}\right)+\frac{2\beta_W^2 s^2}{M_W^4}\left(1-\frac{2M_W^2}{s}\right)\left(z^*-\frac{z*^3}{3}-\frac{1}{3}\right)$$
$$-16\frac{M_W^2}{s}\left(2+\frac{M_W^2}{s}\right)\frac{1}{\beta_W}L_{CE}(z^*),$$

$$\mathcal{O}_{3,CE} = \frac{\beta_W^2 s^2}{M_W^4}\left[(2z^*-1)\left(1+\frac{6M_W^2}{s}\right)+\frac{1}{3}\left(1-2z^{*3}\right)\left(1-\frac{2M_W^2}{s}\right)\right],$$

$$\mathcal{O}_{4,CE} = \frac{8\beta_W^2 s}{M_W^2}(2z^*-1),$$

$$\mathcal{O}_{6,CE} = \frac{2}{3}\left(1-2z^{*3}\right)\frac{\beta_W s^2}{M_W^4}+2(2z^*-1)\frac{s^2}{M_W^4}\left(1+\frac{4M_W^2}{s}-\frac{16M_W^4}{s^2}\right)$$
$$-\frac{32M_W^2}{s}\frac{1}{\beta_W}L_{CE}(z^*),$$

$$\mathcal{O}_{7,CE} = \frac{16s}{M_W^2}\left[(2z^* - 1)\left(1 - \frac{2M_W^2}{s}\right) - \frac{2M_W^4}{s^2}\frac{1}{\beta_W}L_{CE}(z^*)\right],$$

$$\mathcal{O}_{9,CE} = \frac{\beta_W^2 s^2}{M_W^4}\left[(2z^* - 1)\left(1 + \frac{2M_W^2}{s}\right) + \frac{1}{3}\left(1 - 2z^{*3}\right)\left(1 - \frac{2M_W^2}{s}\right)\right],$$

$$\mathcal{O}_{10,CE} = \frac{2\beta_W^2 s^2}{M_W^4}\left[(2z^* - 1)\left(1 + \frac{M_W^2}{s}\right) + \frac{1}{3}\left(1 - 2z^{*3}\right)\left(1 - \frac{M_W^2}{s}\right)\right],$$

$$\mathcal{O}_{11,CE} = \frac{8\beta_W^2 s}{M_W^2}\left(z^* + \frac{z^{*3}}{3} - \frac{2}{3}\right). \qquad (A6)$$

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Figure captions

Fig. 1 Allowed bands for δ_γ and δ_Z (95% CL) from σ^L and σ^R at $\sqrt{s} = 500\, GeV$ and $L_{int} = 50\, fb^{-1}$, and combined allowed domain. Also the domain determined by σ^{unpol} alone is reported.

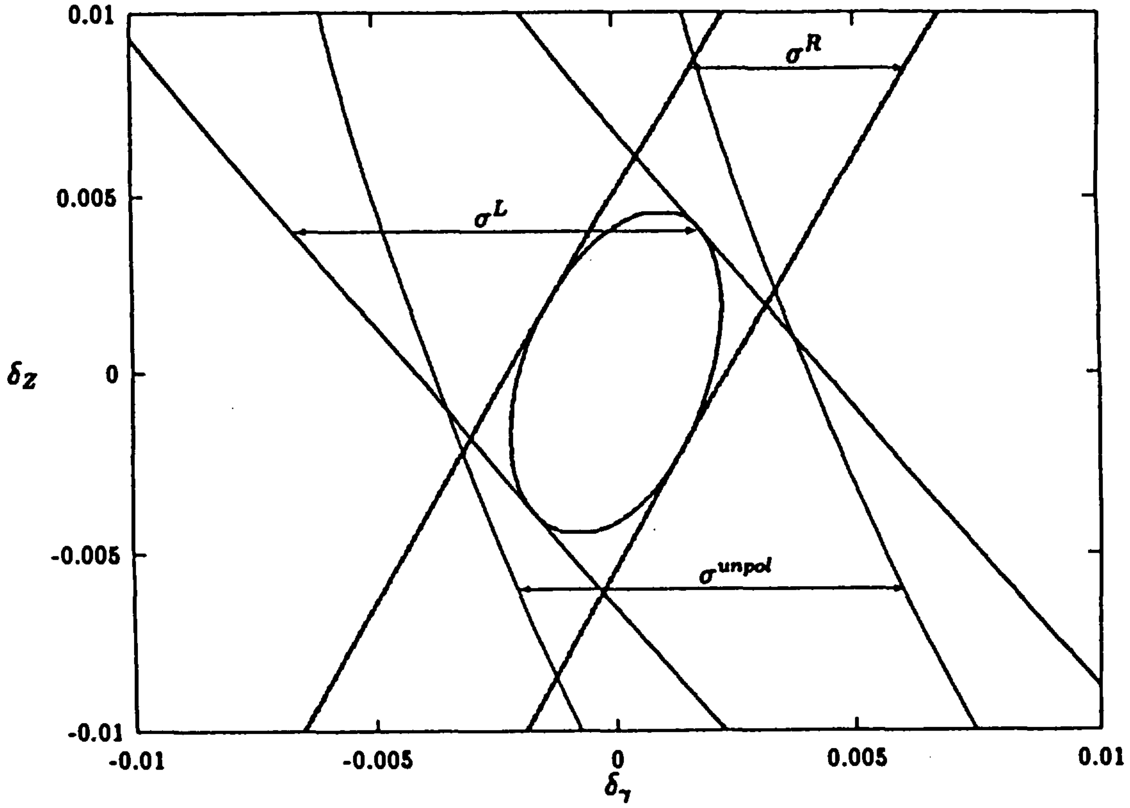
Fig. 2 Same as Fig. 1, with the straight lines (19) for the η, χ and ψ models superimposed.

Fig. 3 Allowed domains (95% CL) on (ϕ, M_{Z_2}) for the η model.

Fig. 4 Statistical significance in x_V^+ of the observables $\sigma \mathcal{A}_{CE}^+$ of Eq. (32) and P^+ of Eq. (34) with $z^* = 0.5$.

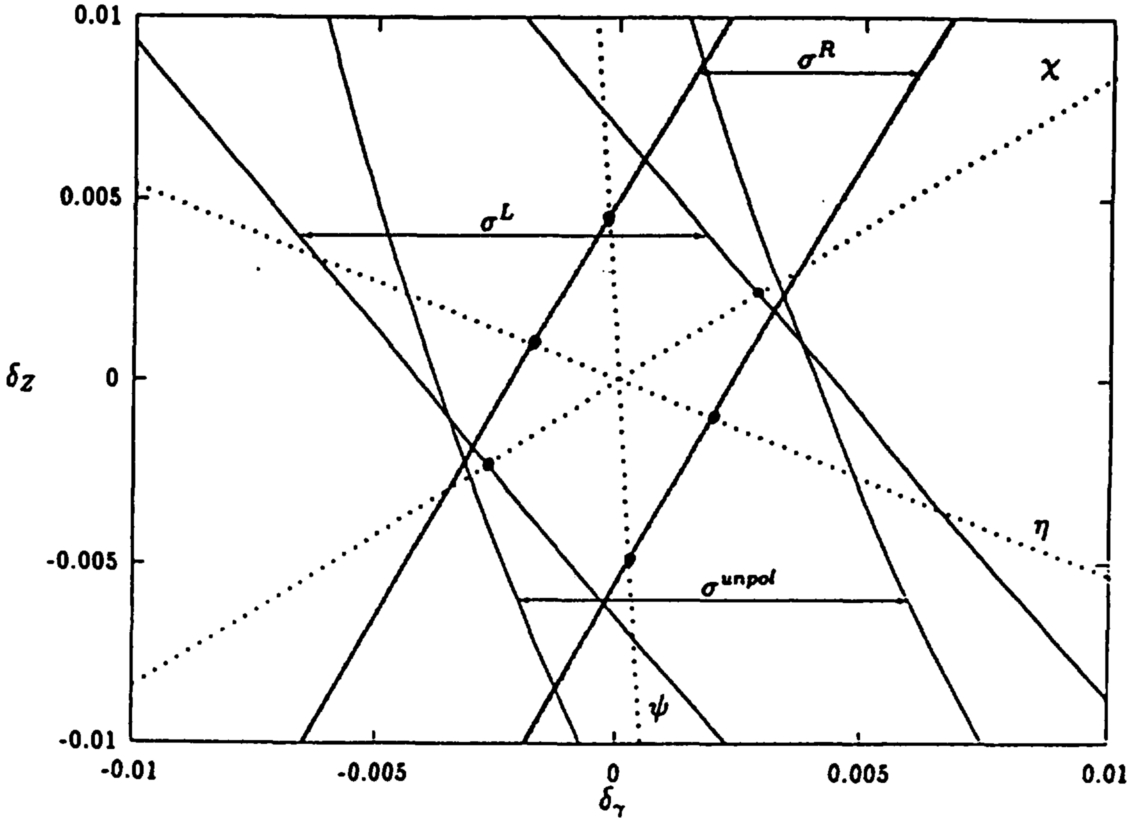
Fig. 5 Statistical significance in y_V^+ of the observables $\sigma \mathcal{A}_{CE}^+$ of Eq. (32) and P^+ of Eq. (34) with $z^* = 0.5$.

Fig. 6 Statistical significance in x_V^-, y_V^- of the variable Q^- of Eq. (33).



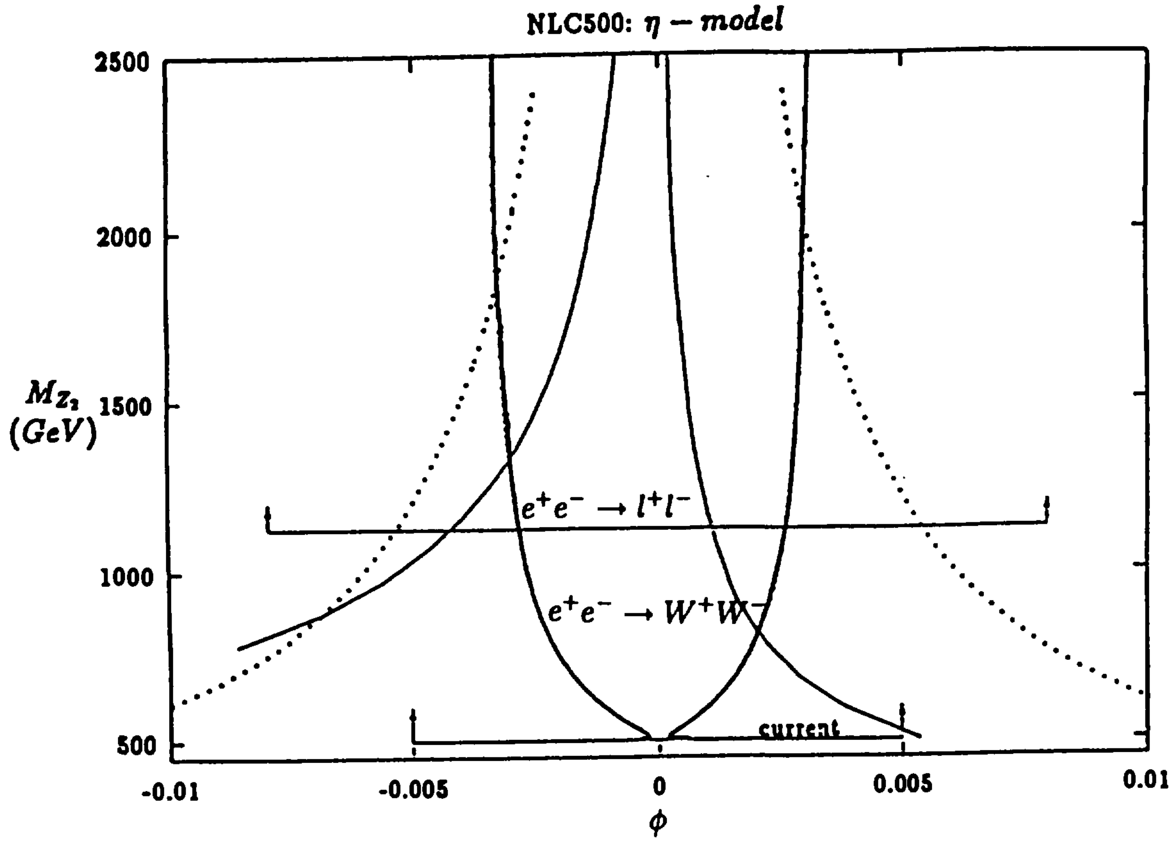
0.01
0.005
0
-0.005
-0.01
δ_Z
-0.01
-0.005
0
0.005
0.01
δ_γ
σ^R
σ^L
σ^{unpol}

Fig.1



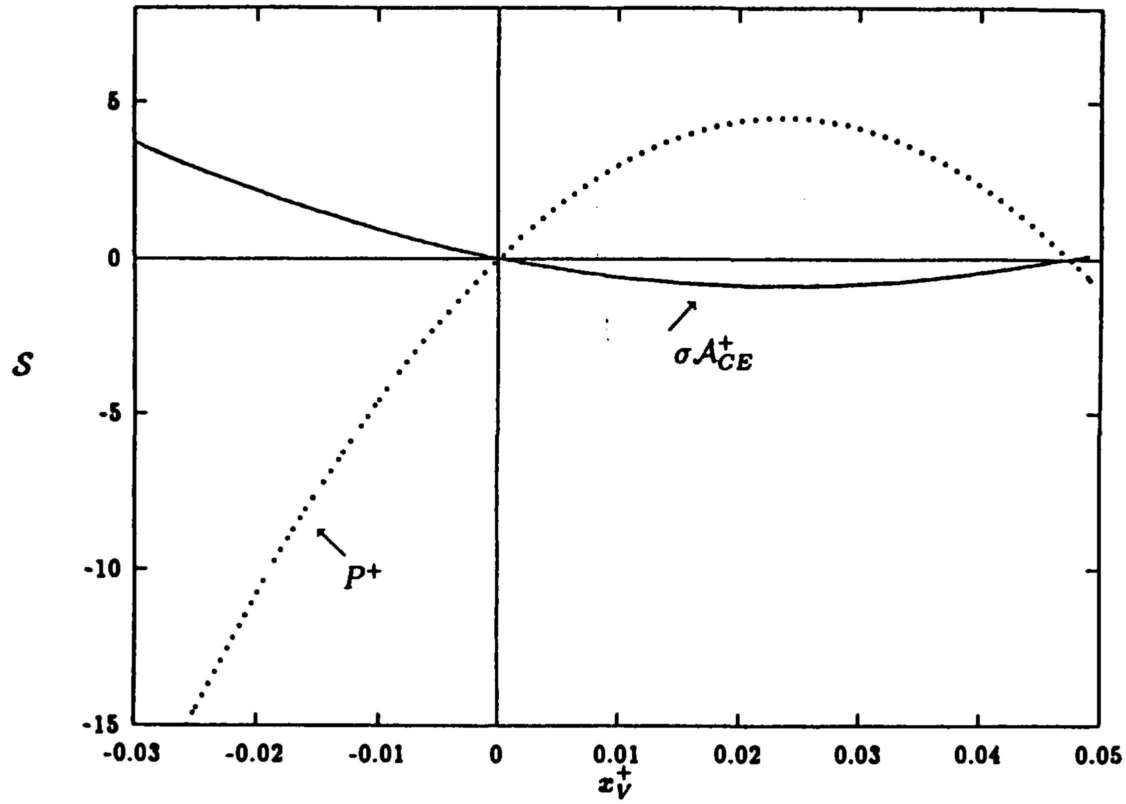
0.01
0.005
0
-0.005
-0.01
δ_Z
-0.01
-0.005
0
0.005
0.01
δ_γ
σ^R
χ
σ^L
η
σ^{unpol}
ψ

Fig.2



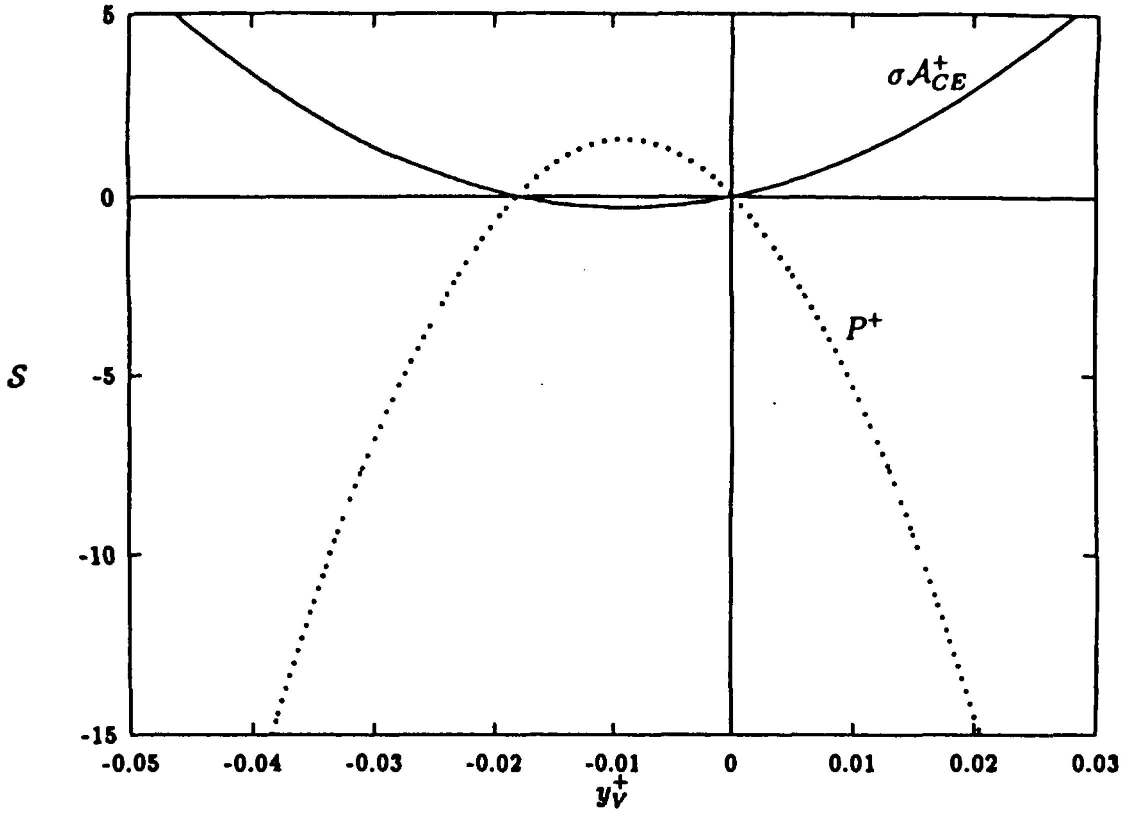
NLC500: $\eta - model$
2500
2000
1500
1000
500
M_{Z_2} (GeV)
$e^+e^- \to l^+l^-$
$e^+e^- \to W^+W^-$
current
-0.01
-0.005
0
0.005
0.01
ϕ

Fig.3



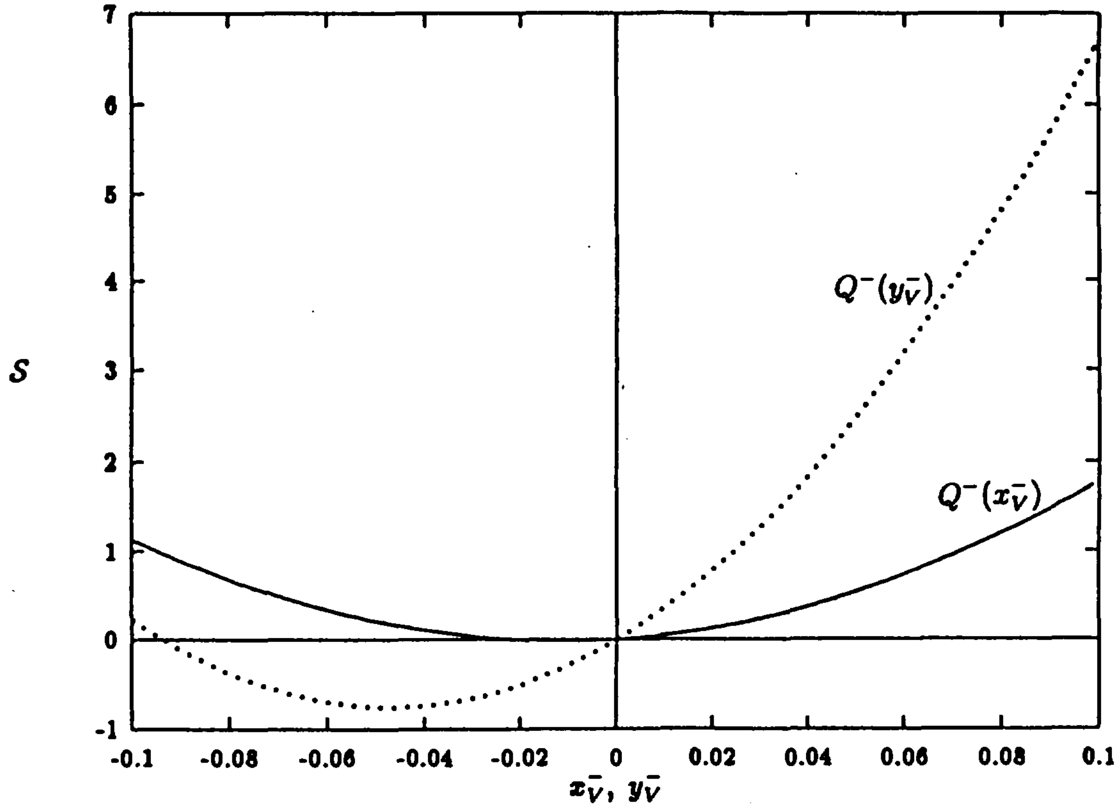
5
0
-5
-10
-15
S
σA^+_{CE}
P^+
-0.03
-0.02
-0.01
0
0.01
0.02
0.03
0.04
0.05
x^+_V

Fig.4



σA^{+}_{CE}
P^{+}
S
5
0
-5
-10
-15
-0.05
-0.04
-0.03
-0.02
-0.01
0
0.01
0.02
0.03
y^{+}_{V}

Fig.5



$Q^{-}(y^{-}_{V})$
$Q^{-}(x^{-}_{V})$
S
7
6
5
4
3
2
1
0
-1
-0.1
-0.08
-0.06
-0.04
-0.02
0
0.02
0.04
0.06
0.08
0.1
x^{-}_{V}, y^{-}_{V}

Fig.6