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UN MODELE PDF POUR L'APPROCHE LAGRANGIENNE DES ECOULEMENTS DIPHASIQUES

THE PDF METHOD FOR LAGRANGIAN TWO-PHASE FLOW SIMULATIONS

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SYNTHÈSE :

L'objet de ce rapport est de présenter un modèle PDF (Probability Density Function) et d'illustrer son intérêt pour la modélisation diphasique des écoulements turbulents.

La première partie rappelle l'équivalence entre le point de vue Lagrangien (modélisation des trajectoires) et raisonner sur équation satisfaite par la pdf. On présente ensuite les principes de modélisation. Le modèle retenu pour les particules fluides est celui de Pope (1991) et comprend des équations d'évolution sur les positions, vitesses et dissipations d'un grand nombre de particules.

Le modèle et l'implantation numérique sont validés sur le cas d'une couche de mélange. Un point important est la bonne prise en compte du gradient de pression moyenne qui évite les dérives artificielles. Les résultats montrent la capacité du modèle à simuler l'intermittence externe et aussi à reproduire la déviation des pdfs des vitesses par rapport à la distribution gaussienne.

EXECUTIVE SUMMARY :

A recent turbulence model put forward by Pope (1991) in the context of PDF modelling has been used.

In this approach, the one-point joint velocity-dissipation pdf equation is solved by simulating the instantaneous behaviour of a large number of Lagrangian fluid particles. Closure of the evolution equations of these Lagrangian particles is based on stochastic models and more specifically on diffusion processes. Such models are of direct use for two-phase flow modelling where the so-called fluid seen by discrete inclusions has to be modelled. Full Lagrangian simulations have been performed for shear flows.

It is emphasized that this approach gives far more information than traditional turbulence closures (such as the k-ε model) and therefore can be very useful for situations involving complex physics. It is also believed that the present model represents the first step towards a complete Lagrangian-Lagrangian model for dispersed two-phase flow problems.

THE PDF METHOD FOR LAGRANGIAN TWO-PHASE FLOW SIMULATIONS

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INTRODUCTION

The common approaches to two-phase flow modelling are either via the two-fluid model or via the particle tracking. In the former, both phases are treated as interpenetrating continuous fluids whereas in the latter the discrete phase is simulated by computing a large number of particle trajectories. A third and more theoretical approach is the particle pdf equation (the kinetic equation) as derived by Reeks (1992). From this equation, one can extract the various equations for the different moments and obtain an Eulerian model while a direct Monte-Carlo simulation of the kinetic equation should be equivalent to Lagrangian models. These approaches differ only in the treatment of the particulate phase and still rely upon Eulerian models for the continuous phase. Thus, the central issue (for dilute flows) remains the simulation of the fluid seen by particles along their trajectories. The construction of such a dispersion model generally starts by considering first fluid particles, which corresponds to the case of fluid diffusion, and then by trying to generalize it to the case of discrete particles where inertia and crossing-trajectory effect come into play.

In spite of the wide use of current models, recent theoretical and numerical analyses have revealed serious flaws concerning the cases of diffusion (Mac Innes & Bracco, 1992) and dispersion (Minier & Pozorski, 1992; Pozorski *et al.*, 1993) clearly showing that improvement and clarification should be brought. In the present article, we concentrate on the diffusion case, that is on the modelling of fluid particle behaviour. A number of remarks can be made. When using an Eulerian approach for the prediction of the continuous phase, one is then faced with the task of reconstructing what has been filtered away by the Reynolds averaging process with limited available information. Indeed, classical turbulence models (even the R_{ij}-ϵ) provide only the first two moments of the velocity field and it is difficult not to resort to the Gaussian assumption even though it is known to be at variance with experimental data for nonhomogeneous situations (Townsend, 1976). Furthermore, simulating the trajectories of tagged fluid particles while still computing mean variables with an Eulerian model is not a stringent test. The 'background' mean field may mask or prevent limitations from appearing. On the contrary, a complete Lagrangian simulation where mean quantities are derived from the instantaneous behaviour of Lagrangian particles is obviously a more severe test in order to ascertain the model consistency.

Actually, complete Lagrangian models have been proposed in the context of pdf modelling. Although they do not seem to be well known for two-phase flow problems, they have reached a rather high degree of sophistication. A detailed description of pdf models is outside the scope of the present paper since comprehensive presentations have already been published (Pope 1985, 1994b; Dopazo 1994). Pdf methods have become popular mainly for problems involving chemical reactions since they treat convection and source terms (however non-linear and complicated) exactly. A first aspect of the present article is to underline that they are applicable to Lagrangian two-phase flow modeling. As an example, the issue of spurious drifts which has recently attracted attention (Mac Innes & Bracco, 1992) was settled some years ago (Pope, 1987). It was shown that the existence of spurious drifts is simply related to an inadequate (or non-existent) taking into account of the mean pressure gradient in the particle equation of motion. In other words, spurious drifts signify that the mean Navier-Stokes equation is not satisfied.

The Lagrangian model retained here is the one developed by Pope and coworkers (Haworth & Pope, 1986, 1987; Pope & Chen, 1990 and Pope, 1991). In the following, the relation between pdf equations and Lagrangian models is outlined. The stochastic models for particle velocity and dissipation are briefly described. Application of the model to the case of free shear flows is presented in order to show its ability to reproduce important characteristic turbulence features such as deviations of velocity pdf from Gaussianity as well as internal and external intermittency.

STOCHASTIC MODELLING

In this section, we present only the main lines of reasoning and limit ourselves to what is believed to be the key points. There are different ways to derive the pdf equation. In the following, we have used a short-cut method which starts directly from the Lagrangian point of view since it is in line with the present context.

Pdf equation and Lagrangian model

If we consider a random process $Y(t)$ whose rate of change is $A(t)$, the time evolution equation of the pdf $f(y,t)$, equivalent to the Liouville equation in classical statistical mechanics, is :

$$\frac{\partial f(y,t)}{\partial t} = -\frac{\partial}{\partial y}[\langle A(t)\,|\,Y(t) = y\rangle f(y,t)]. \tag{1}$$

The above equation is readily extended to the case of vector state $\mathbf{Y}$. For particles for which state variables are the location $\mathbf{x}(t)$ and the velocity $\mathbf{U}(t)$, we have :

$$\begin{aligned}\frac{\partial f(\mathbf{x},\mathbf{V},t)}{\partial t} &= -V_i\frac{\partial f(\mathbf{x},\mathbf{V},t)}{\partial x_i} \\ &\quad -\frac{\partial}{\partial V_i}[\langle F\,|\,\mathbf{x},\mathbf{V}\rangle f(\mathbf{x},\mathbf{V},t)]. \end{aligned} \tag{2}$$

In the above equation appears the expected value of the force acting on the particle conditioned on the present state $\mathbf{x}(t)=\mathbf{x}$, $\mathbf{U}(t)=\mathbf{V}$. It is thus seen that modelling (when the conditional rate of change is not closed) and solving a pdf equation is closely related to the modelling of particle evolution equations. This can be applied to the Navier-Stokes equations which can be written in a mixed Lagrangian-Eulerian way. Following a fluid particle whose location and velocity are denoted by a "+" superscript, we have :

$$\begin{aligned} d\mathbf{x}^+ &= \mathbf{U}^+(t)\,dt, \\ dU_i^+ &= \left(-\frac{1}{\rho}\frac{\partial\langle p\rangle}{\partial x_i}+\nu\nabla^2\langle U_i\rangle\right)^+ dt + \\ &\quad \left(-\frac{1}{\rho}\frac{\partial p'}{\partial x_i}+\nu\nabla^2 u_i\right)^+ dt + g_i\,dt \end{aligned} \tag{3}$$

where $\mathbf{g}$ stands for an external force field. Reynolds decomposition into the mean and the fluctuating part has been applied to the Eulerian instantaneous velocity $\mathbf{U}(\mathbf{x},t)$ and the pressure field $p(\mathbf{x},t)$: $\mathbf{U}=\langle\mathbf{U}\rangle+\mathbf{u}$ and $p=\langle p\rangle+p'$. The corresponding pdf satisfies the following equation :

$$\begin{aligned} \frac{\partial f}{\partial t}+V_i\frac{\partial f}{\partial x_i}+\left(-\frac{1}{\rho}\frac{\partial\langle p\rangle}{\partial x_i}+\nu\nabla^2\langle U_i\rangle+g_i\right)\frac{\partial f}{\partial V_i} &= \\ \frac{\partial}{\partial V_i}[\langle -\frac{1}{\rho}\frac{\partial p'}{\partial x_i}+\nu\nabla^2 u_i\,|\,\mathbf{x},\mathbf{U}=\mathbf{V}\rangle f] & \end{aligned} \tag{4}$$

Actually, since we have followed a Lagrangian point of view, the pdf f represents the transitional pdf $f_L(\mathbf{x},\mathbf{V},t\,|\,\mathbf{x}_0,\mathbf{V}_0,t_0)$ of particles whose initial conditions are $\mathbf{x}_0,\mathbf{V}_0$ at time t_0. However, the Eulerian pdf is easily retrieved through the relation (Pope, 1985) :

$$f_E(\mathbf{x},\mathbf{V},t)=\int f_L(\mathbf{x},\mathbf{V},t|\mathbf{x}_0,\mathbf{V}_0,t_0)f_E(\mathbf{x}_0,\mathbf{V}_0,t_0)d\mathbf{x}_0d\mathbf{V}_0 \tag{5}$$

and thus follows the same evolution equation than f_L.

Modelling principles

From the previous paragraph, it is clear that modelling and solving the pdf equation can be addressed from the Lagrangian point of view by modelling the *instantaneous* behaviour of fluid particles. The variables attached to any particle are the location, velocity and dissipation rate, $\mathbf{x}$, $\mathbf{U}$, and ϵ, respectively. The introduction of the dissipation rate along particle trajectory (Pope & Chen, 1990) allows the turbulent timescale to be calculated from the model. The evolution equations of the modelled particles, disregarding the mean viscous and the gravity terms, can be written as :

$$d\mathbf{x}^* = \mathbf{U}^*(t)\,dt \tag{6}$$

$$dU_i^* = -\frac{1}{\rho}\frac{\partial\langle p\rangle}{\partial x_i}dt + dA_i^*(t) \quad (7)$$
$$d\epsilon^* = dA_\epsilon^*(t)\,. \quad (8)$$

The first equation shows that convection is treated exactly. Consequently, there is no need for a gradient diffusion hypothesis. In the particle equation of motion, the fluctuating part of the force has to be modelled whereas the evolution equation of the dissipation has to be modelled altogether since it only involves small-scale quantities. The name "stochastic particles" is used since the unknown terms $A^*(t)$ are based on stochastic processes, more precisely on diffusion processes. Application of these processes is suggested by Kolmogorov hypotheses and physical reasoning. Indeed, the sample functions of a diffusion process A_t are continuous (contrary to so-called jump processes) and can be expressed as solutions of stochastic differential equations (Arnold 1974):

$$dA_t = D(t, A_t)dt + \sqrt{B(t, A_t)}dW \quad (9)$$

where $D(t, A_t)$ and $B(t, A_t)$ are respectively the so-called drift and diffusion functions. They represent the conditional mean and variance of the increment of A_t, $dA_t = A_{t+dt} - A_t$, conditioned on the present state $A_t = A_0$:

$$\begin{aligned} \langle dA_t \mid A_t = A_0\rangle &= D(t, A_0)\,dt \\ \langle dA_t^2 \mid A_t = A_0\rangle &= B(t, A_0)\,dt, \end{aligned} \quad (10)$$

while higher-order moments are $o(dt)$; dW stands for the Wiener process (Gaussian white noise). Once again, modelling the evolution of the process samples (Ito's point of view) is strictly equivalent to modelling the pdf equation (Kolmogorov's point of view) which is then a Fokker-Planck equation :

$$\frac{\partial f}{\partial t} = -\frac{\partial}{\partial A_i}[\,D_i(t, A_t)f\,] + \frac{1}{2}\frac{\partial^2}{\partial A_i \partial A_j}[\,B_{ij}(t, A_t)f\,]. \quad (11)$$

Stochastic model for velocity

As just explained, fluid particle velocities are modelled by a diffusion process which writes:

$$dU_i^* = -\frac{1}{\rho}\frac{\partial\langle p\rangle}{\partial x_i}dt + D_i dt + \sqrt{C_0\epsilon^*}\,dW_i. \quad (12)$$

Closure of the diffusion term is consistent with the Kolmogorov theory (1941) which yields that the second order Lagrangian structure function

$$D_{L(ij)}(\Delta t) = \langle\, [u_i(t+\Delta t) - u_i(t)]\,[u_j(t+\Delta t) - u_j(t)]\,\rangle$$

in the inertial range is given by $\delta_{ij}C_0\epsilon^*\Delta t$ (Monin & Yaglom, 1971) where the instantaneous dissipation rate ϵ^* is used according to the refined Kolmogorov hypothesis (1962). Because of the assumption of local isotropy of turbulent flows at Re sufficiently high, the diffusion term retains the isotropic form even for inhomogeneous cases, due to the use of isotropic Wiener process. The total drift term has been separated into the mean pressure gradient and a secondary term D_i which must meet the zero-mean condition, $\langle D_i\rangle = 0$. This ensures that the mean Navier-Stokes equation is respected and, in turn, that no spurious drifts exist. This term is closed by reference to second-order modelling taking into account internal intermittency and writes (Pope, 1991) :

$$D_i = -(\frac{1}{2} + \frac{3}{4}C_0)\langle\omega\rangle\frac{\tilde{k}}{k}u_i^* + G_{ij}^a u_j^*$$
$$-\frac{3}{4}C_0[\tilde{A}_{ij}^{-1}(\omega^* u_j^* - \langle\omega u_j\rangle) - (\frac{\tilde{k}}{k})A_{ij}^{-1}\langle\omega\rangle u_j^*]. \qquad (13)$$

In the latter relation, G_{ij}^a (where the subscript ^{a} stands for anisotropic) represents a second-order tensor which is function of the mean velocities and of the Reynolds stresses. The normalized Reynolds stress tensor A and the turbulent kinetic energy k are given by :

$$k = \frac{1}{2}\langle u_i u_i\rangle \quad \text{and} \quad A_{ij} = \langle u_i u_j\rangle/(\frac{2}{3}\langle k\rangle), \qquad (14)$$

while $\tilde{A}$ and $\tilde{k}$ are dissipation-weighted means :

$$\tilde{k} = \frac{1}{2}\langle u_i u_i \epsilon\rangle/\langle\epsilon\rangle \quad \text{and} \quad \tilde{A_{ij}} = \langle u_i u_j \epsilon\rangle/(\frac{2}{3}\langle\epsilon\rangle\tilde{k}). \qquad (15)$$

Stochastic model for dissipation

The evolution equation for turbulent energy dissipation rate attached to a fluid particle is a generalization of the one proposed by Pope & Chen (1990). It makes use of the refined Kolmogorov hypothesis which says that $\epsilon^*(t)$ is a lognormally-distributed random variable. The variable $\chi^*(t) = \ln(\epsilon^*(t)/\langle\epsilon(t)\rangle)$ is then Gaussian and can be modelled by a diffusion process. The dissipation equation is developed not in terms of the dissipation ϵ itself but rather in terms of the turbulence frequency (relaxation rate) ω which is defined by :

$$\omega(\mathbf{x}, t) = \epsilon(\mathbf{x}, t)/\tilde{k}(\mathbf{x}, t)\,.$$

The complete version for the general case of inhomogeneous turbulence writes (Pope 1991):

$$d\omega^* = -\omega^*\langle\omega\rangle dt\left\{S_\omega + C_\chi\left[\ln\left(\frac{\omega^*}{\langle\omega\rangle}\right) - \langle\frac{\omega}{\langle\omega\rangle}\ln(\frac{\omega}{\langle\omega\rangle})\rangle\right]\right\}$$
$$+ \langle\omega\rangle^2 h\,dt + \omega^*\sqrt{2C_\chi\langle\omega\rangle\sigma^2}\,dW\,. \qquad (16)$$

In this equation, σ^2 is the variance of χ, C_χ is a constant. Next, h represents a drift term and models the mechanism through which entrained laminar particles ($\omega^* = 0$) become turbulent. The source term h is larger near the edges of a turbulent stream where the flow is highly intermittent and decreases near the centre of such flows where a fully turbulent state is approached. It is related to the intermittency factor γ which can be defined as the local proportion of turbulent particles by $h = C_{\omega 3}(1-\gamma^{1/2})^2$. S_ω stands for the normalized decay rate of $\langle\omega\rangle$ and is modelled by reference to the standard equation of $\langle\epsilon\rangle$:

$$S_\omega = (C_{\epsilon 2} - 1) - (C_{\epsilon 1} - 1)\frac{P}{\langle\epsilon\rangle}, \qquad (17)$$

where P is the turbulent kinetic energy production term.

The constants of the model are $C_{\epsilon 1} = 1.37$, $C_{\epsilon 2} = 1.9$, $C_{\omega 3} = 1$, $C_\chi = 1.6$, $\sigma^2 = 1$, $C_0 = 3.5$.

Resulting mean equations

As indicated above, modeling the governing equations for the stochastic particle dynamics amounts to closing the joint velocity-dissipation pdf equation. From this equation or directly from the instantaneous evolution equations, one can derive the transport equations satisfied by the statistical moments. The first two write:

$$\frac{\partial \langle U_i \rangle}{\partial x_i} = 0 \,, \tag{18}$$

$$\frac{\partial \langle U_i \rangle}{\partial t} + \langle U_j \rangle \frac{\partial \langle U_i \rangle}{\partial x_j} + \frac{\partial}{\partial x_j} \langle u_i u_j \rangle = -\frac{1}{\rho} \frac{\partial \langle p \rangle}{\partial x_i} + \nu \nabla^2 \langle U_i \rangle \tag{19}$$

and are thus exact, while the second-order (Reynolds stress) equation is :

$$\begin{aligned}
\frac{\partial}{\partial t}\langle u_i u_j \rangle \quad &+ \quad \langle U_k \rangle \frac{\partial}{\partial x_k} \langle u_i u_j \rangle + \frac{\partial}{\partial x_k} \langle u_i u_j u_k \rangle \\
&+ \quad \langle u_i u_k \rangle \frac{\partial \langle U_j \rangle}{\partial x_k} + \langle u_j u_k \rangle \frac{\partial \langle U_i \rangle}{\partial x_k} \\
&= \quad \langle u_i D_j \rangle + \langle u_j D_i \rangle + C_0 \tilde{k} \langle \omega \rangle \delta_{ij} \\
&= \quad -(1 + \frac{3}{2} C_0) \frac{\langle \epsilon \rangle}{k} (\langle u_i u_j \rangle - \frac{2}{3} k \delta_{ij}) \\
&\qquad + G^a_{il} \langle u_j u_l \rangle + G^a_{jl} \langle u_i u_l \rangle - \frac{2}{3} \langle \epsilon \rangle \delta_{ij}
\end{aligned} \tag{20}$$

The equation for the mean $\langle \omega \rangle$ is easily derived from the instantaneous one:

$$\langle \frac{d\omega}{dt} \rangle = \frac{\partial \langle \omega \rangle}{\partial t} + \langle U_i \rangle \frac{\partial \langle \omega \rangle}{\partial x_i} + \frac{\partial}{\partial x_i} \langle u_i \omega \rangle = -\langle \omega \rangle^2 (S_\omega - h) \,. \tag{21}$$

It is thus seen that the present model is directly equivalent to second-order modelling (Pope, 1994a) with the further advantage that triple moments are in a closed form and do not have to be modelled. When the anisotropic matrix G^a is put to zero, the model gives the Rotta return to isotropy term. Actually, the relation to Reynolds stress equation is normal. In the momentum equation, we have to model the fluctuating force acting on particles and, in particular, the fluctuating pressure gradient. The work performed by this force appears only at the second level in the hierarchy of moment equations and it is thus not surprising that identification is made at this stage. We have limited ourselves to the equations for low-order moments of $\mathbf{U}$ and ω. However, it should be emphasized that the pdf method provides a full description of one-point turbulence statistics. All Lagrangian and Eulerian one-point statistical quantities can be obtained but no two-point information can be extracted. In particular, the Eulerian length scales cannot be derived from the present model. There is, however, some information on small-scale velocity gradients through the computation of instantaneous dissipation along particle trajectory. The stochastic model for $\omega(t)$ takes into account internal intermittency and, from that point of view, contains the statistical signature of small-scale coherent structures.

Numerical solution We give only the salient points since details of the numerical method as well as its implementation can be found elsewhere (Pope, 1985; Pozorski & Minier, 1994). A fractional time step method is used. The first step corresponds to the application of the stochastic models without moving the particles in physical space. The second one corresponds to convection and mean pressure gradient effects. A correct calculation of the mean pressure gradient is essential in order to avoid spurious drifts. The pressure gradient is not calculated as the solution of a Poisson equation but in order to ensure the consistency condition. If each particle represents a fixed amount of mass Δm, this condition writes:

$$\int \langle \mathcal{F}_N \rangle \, dV d\theta = M \langle \delta(x - x_n) \rangle = \rho \tag{22}$$

where $M = \sum_{n=1}^{N} \Delta m$ is the total mass in the domain and $\mathcal{F}_N = \rho f_N$ is the discrete mass density function of an ensemble of N particles

$$\mathcal{F}_N(V, \theta, x, t) = \Delta m \sum_{n=1}^{N} \delta(x - x_n)\delta(V - U_n)\delta(\theta - \epsilon_n). \tag{23}$$

Its expected value must equal the true one $\mathcal{F} = \rho f$:

$$< \mathcal{F}_N(V, \theta, x, t) >= \mathcal{F}(V, \theta, x, t). \tag{24}$$

In other words, for incompressible flows, the consistency condition states that the particle number density must be constant. This is ensured by a correct calculation of the mean pressure gradient.

EXAMPLES OF RESULTS FOR FREE SHEAR FLOWS

Numerical simulations have been performed for free shear flows: the mixing layer and the plane wake (see Pozorski & Minier 1994). The case of the mixing layer has been studied more in detail; various turbulence statistics ranging from the mean velocity components up to the fourth-order moments of fluctuating velocity have been calculated. Validity of the model (and of the numerical scheme) is assessed by checking that a self-similar regime has been reached and by comparison with experimental data. The typical number of stochastic particles used was about 100 000 and the computation time was about 45 min. on the 40 MFLOPS workstation. The mixing layer was simulated with the two free stream velocities being equal to 1m/s and 1.5m/s respectively. The computed spreading rate is 0.036 which compares well with experimental values which can be expressed by (Townsend, 1976) :

$$S = \simeq 0.185 \frac{U_{up} - U_{low}}{U_{up} + U_{low}} \sim 0.037. \tag{25}$$

As is usually the case, low-order moments are the easiest to be reproduced faithfully. Figure 1 presents the four non-zero Reynolds stress components; agreement with experimental results is satisfactory, especially for the variables judged particularly important: the shear stress $\langle uv \rangle$ (directly related to the spreading of the layer) and the streamwise component $\langle u^2 \rangle$ (which takes part in the turbulent energy production from the mean velocity gradients). Higher-order moments have been computed such as triple correlations and skewness and flatness factors of the velocity components (Figure 2). In the analysis of the numerical outcomes, emphasis is put on the possibility of the model to simulate detailed turbulence quantities. Indeed, at a given location across the layer, various one-point and Lagrangian turbulence statistics can be extracted and analysed. For example, the pdf of the turbulence frequency ω has been computed in two locations across the mixing layer (Figure 3). Two important features of the model are readily recognized. First, in the region near the center of the layer, the pdf is log-normal. This simply reproduces the assumption that has been made when constructing the stochastic equation for the turbulence frequency. Then, near the edge of the layer, two classes of particles can be distinguished: a group of turbulent ones (as in the previous case) and another group of laminar particles, represented on the plot by the Dirac delta at $\omega = 0$. This illustrates the capability of the model to simulate the external intermittency. Various ways of calculating external intermittency are presented in Figure 4 and compared with the experimental results of Wygnanski & Fiedler (1970). One-point velocity pdfs have also been considered and are plotted in Figure 5. Results are in qualitative agreement with the experimental analysis of Champagne *et al.* (1976) and show that the model is able to reproduce deviations from the normal distribution which is an important factor in inhomogeneous situations.

CONCLUSION

Pope's model for the velocity-dissipation pdf has been applied and results similar to those reported (Pope 1991) have been obtained. Since the numerical algorithm has been developed independently, these results can been regarded as a confirmation of the validity of the model. This model contains a high level of physical information on turbulence through the introduction of a model for the dissipation rate. It takes into account internal intermittency and is able to simulate external intermittency therefore allowing the laminar/turbulent regions of free shear flows to be calculated. A number of results derived from the model have been presented. They include second order moments, the intermittency factor, the pdfs of velocity and dissipation rate.

A first objective was to indicate that methods and models developed in the context of so-called pdf methods (along with the theoretical work performed) are of potential use for two-phase flow modelling. They constitute a sound starting point since they correspond to the diffusion case and have already been shown to perform accurately. An underlying aim of the present paper was to emphasize the strict equivalence between Lagrangian models and pdf equations. Lagrangian models represent Monte-Carlo simulation of the pdf equation. In that regard, the normalising condition that the pdf must respect is equivalent to a proper calculation of the mean pressure gradient and thus to the zero-divergence condition for the velocity field.

It is also believed that the approach used here opens the way to a full Lagrangian-Lagrangian simulation of two-phase flows. When solid particles are present, one has then to model the successive fluid velocities encountered by the particles along their trajectory. The problem is compounded by particle inertia and crossing-trajectory effect. Since no two-point information can be extracted from the present approach, the point is to derive a model for the fluid seen by the particles starting from a sound Lagrangian model for fluid particles. In that regard, the problem is similar to the situation of classical Eulerian-Lagrangian models. Yet, the pdf method provides far more information than simply the first two moments of the velocity. Special care should be taken when generalizing diffusion models to dispersion case: simulations should not be based on one single sample trajectory as demonstrated in a previous paper (Pozorski *et al.*, 1993). It is hoped that the solutions proposed there for homogeneous turbulence can be extended to inhomogeneous cases.

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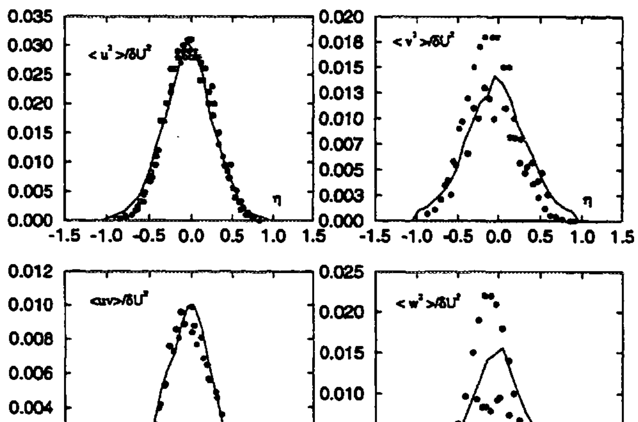

Figure 1: Reynolds stress components plotted as a function of the normalized cross-stream location $\eta = y/x/\delta$ where δ is the spreading rate. Comparison with the experiments of Wygnanski & Fiedler ($\bullet$), Patel ($\star$) and Champagne *et al.* ($\sqcap$).

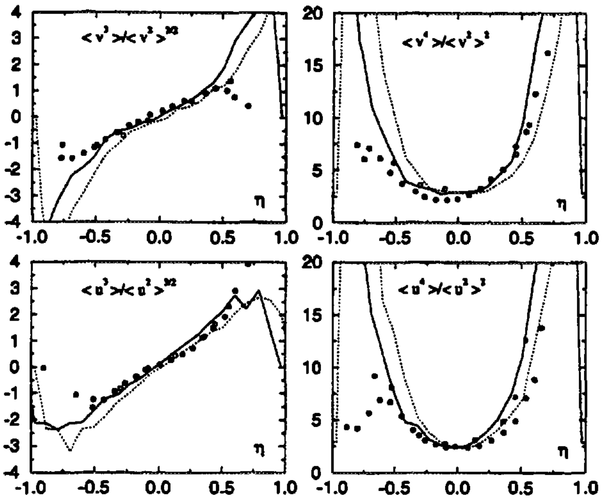

Figure 2: Skewness and flatness factors of the streamwise and the cross-stream velocity components. Comparison with the Wygnanski & Fiedler (•) and the Champagne *et al.* (⊓) experiments.

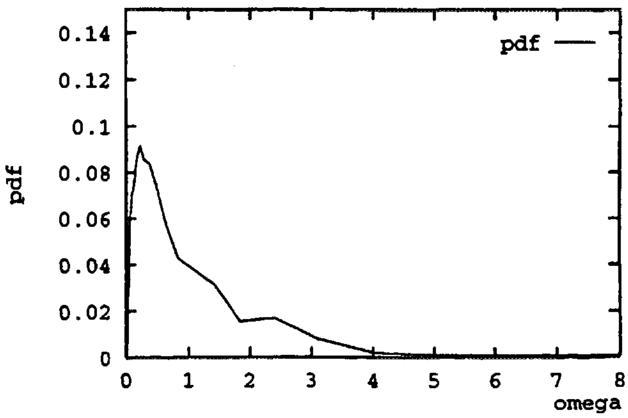

Figure 3: Pdf of the dissipation rate near the center of the mixing layer.

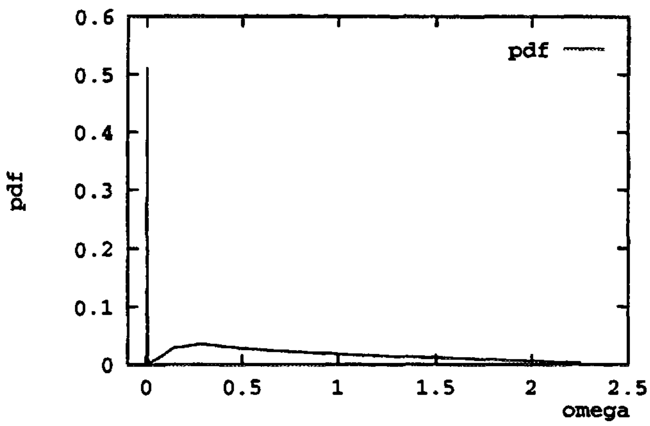

Figure 4: Pdf of the dissipation rate at the edge of the mixing layer.

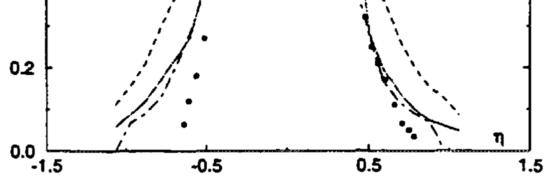

Figure 5: External intermittency coefficient. Comparison with the Wygnanski & Fiedler (•) experiment. Solid line: direct computation based on the number of turbulent particles in each cell. Dotted line: the evaluation of $\langle\omega^{1/2}\rangle/\langle\omega\rangle^{1/2}$. Dot-dashed line: the ratio $k/\tilde{k}$.

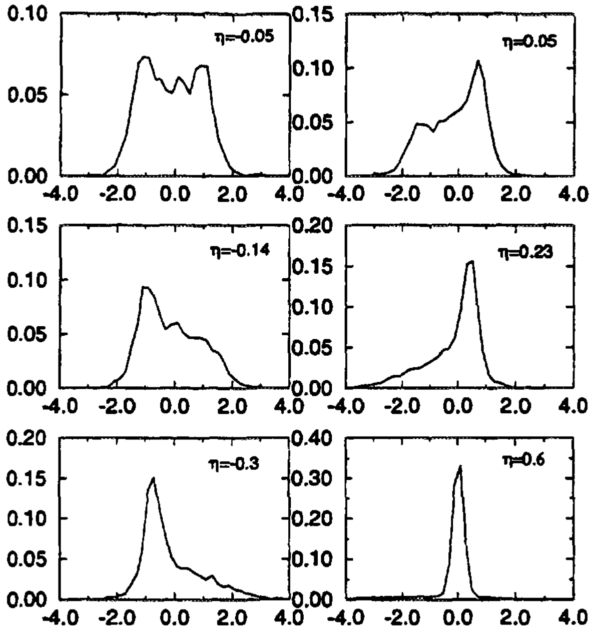

Figure 6: Histograms of the streamwise velocity component at different locations across the mixing layer. The distributions are centred and plotted as a function of the normalized value $(u - \langle u \rangle)/\langle u^2 \rangle^{1/2}$.